JGSP **6** (2006) 55–66

JOURNAL OF
Geometry and Symmetry
in Physics

HARMONIC POLYNOMIALS ON THE POINCARÉ DODECAHEDRAL THREE-MANIFOLD

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Communicated by Gerald A. Goldin

Abstract. With Lie-algebraic methods we obtain the set of orthogonal harmonic polynomials on the Poincaré dodecahedral three-manifold. The expansion in these polynomials of temperature fluctuations of the cosmic microwave background tests this manifold as a candidate for cosmic topology.

1. Introduction

The global topology of the three-space is not fixed by Einstein general relativity, since this is formulated in terms of local differential equations. Einstein's first static cosmological models used for the space-part of the universe a simply connected sphere $\mathbb{S}^3$. With present-day cosmological information it becomes possible to propose and test multiply-connected topologies for the space-part of the universe. J.-P. Luminet *et al.* [7] and J. Weeks [10] propose to explore the topology of three-space from temperature fluctuations of the **cosmic microwave background** (CMB). These fluctuations are measured with very high precision.

As one way to test the topology, one can try to expand the temperature fluctuations of the CMB into harmonic polynomials of a chosen topological three-manifold, for example the Poincaré dodecahedral manifold $\mathcal{P}$. The topology will be verified if the harmonic polynomials of the manifold suffice to expand these fluctuations. We shall see that the restriction of the topology from $\mathbb{S}^3$ to $\mathcal{P}$ results in strong and specific selection rules for the harmonic poplynomials.

To implement such an expansion, a prerequisite is to explicitly characterize the harmonic polynomials on $\mathcal{P}$. The details of such a characterization are given in Kramer [5] and gr-qc/0410094. Here we describe the main steps of the analysis.