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COMPLEX STRUCTURES IN ELECTRODYNAMICS

STOIL DONEV and MARIA TASHKOVA

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Abstract. In this paper we show that the basic external (i.e., not determined by the equations) object in Maxwell vacuum equations is a *complex structure*. In the three-dimensional standard form of Maxwell equations this complex structure $\mathcal{I}$ participates implicitly in the equations and its presence is responsible for the so called *duality invariance*. We give a new form of the equations showing explicitly the participation of $\mathcal{I}$. In the four-dimensional formulation the complex structure is extracted directly from the equations, it appears as a linear map Φ in the space of two-forms on $\mathbb{R}^4$. It is shown also that Φ may appear through the equivariance properties of the new formulation of the theory. Further we show how this complex structure Φ combines with the Poincaré isomorphism $\mathfrak{P}$ between the two-forms and two-tensors to generate all well known and used in the theory (pseudo)metric constructions on $\mathbb{R}^4$, and to define the conformal symmetry properties. The equations of Extended Electrodynamics (EED) do not also need these pseudometrics as beforehand necessary structures. A new formulation of the EED equations in terms of a generalized Lie derivative is given.

1. Introduction

We begin with two examples, showing that meeting with implicitly participating objects in some equations of mathematical physics is not an unknown phenomenon. Recall the wave D'Alembert equation (in standard form)

$$U_{tt} - c^2 \left(U_{xx} + U_{yy} + U_{zz} \right) = 0.$$

Except the constant c, no external objects participate (at first sight, explicitly) in this equation. During the first half of 20th century a new understanding of this equation was created, namely, that a new external object participates implicitly in it and it is the pseudoeuclidean metric tensor $g^{\mu\nu}$, $-g^{11} = -g^{22} = -g^{33} = g^{44} = 1$ on $\mathbb{R}^4$, so that the true form of this equation should read

$$g^{\mu\nu} \frac{\partial^2 U}{\partial x^\mu \partial x^\nu} = 0.$$