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SYMMETRY OF THE MAXWELL AND MINKOWSKI EQUATIONS SYSTEM

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Abstract. We study the symmetry of Maxwell's equations for external moving media together with the additional Minkowski constitutive equations (or Maxwell–Minkowski equations). We established that the system is conformally invariant.

1. Introduction

Symmetry properties of Maxwell equations in vacuum was studied in detail by Lorentz, Poincare, Bateman, Cuningham [1, 2].

Maximal local Lie group of invariance of linear equations for electromagnetic fields in vacuum is 16 parameters group containing 15 parameter conformal group as a subgroup [3]. It was proved in [4] that the Maxwell equations in the medium, which form a system of first order partial differential equations for vectors $\vec{D}$, $\vec{B}$, $\vec{E}$ and $\vec{H}$, admit infinite symmetry. Thus, the system of equations

$$\frac{\partial \vec{D}}{\partial t} = \operatorname{rot} \vec{H} - \vec{j}, \qquad \operatorname{div} \vec{D} = \rho \tag{1}$$

$$\frac{\partial \vec{B}}{\partial t} = -\operatorname{rot} \vec{E}, \qquad \operatorname{div} \vec{B} = 0 \tag{2}$$

if $\vec{j} = 0, \rho = 0$ is invariant under the infinite-dimensional Lie algebra with basis elements

$$X = \xi^{\mu}(k)\frac{\partial}{\partial x_{\mu}} + \eta^{E^a}\frac{\partial}{\partial E^a} + \eta^{B^a}\frac{\partial}{\partial B^a} + \eta^{D^a}\frac{\partial}{\partial D^a} + \eta^{H^a}\frac{\partial}{\partial H^a} \tag{3}$$

where

$$\begin{aligned}
\eta^{E_1} &= \xi_0^3 B_2 - \xi_0^2 B_3 - (\xi_1^1 + \xi_0^0)E_1 - \xi_1^2 E_2 - \xi_1^3 E_3 \\
\eta^{E_2} &= -\xi_0^3 B_1 - \xi_0^1 B_3 - (\xi_2^2 + \xi_0^0)E_2 - \xi_2^1 E_1 - \xi_2^3 E_3 \\
\eta^{E_3} &= \xi_0^2 B_1 - \xi_0^1 B_2 - (\xi_3^3 + \xi_0^0)E_3 - \xi_3^2 E_2 - \xi_3^1 E_1
\end{aligned}$$