

ON A CERTAIN SUBCLASS OF STARLIKE FUNCTIONS WITH NEGATIVE COEFFICIENTS

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Abstract: We introduce the class $\overline{H}(\alpha, \beta)$ of analytic functions with negative coefficients. In this paper we give some properties of functions in the class $\overline{H}(\alpha, \beta)$ and we obtain coefficient estimates, neighborhood and integral means inequalities for the function $f(z)$ belonging to the class $\overline{H}(\alpha, \beta)$. We also establish some results concerning the partial sums for the function $f(z)$ belonging to the class $\overline{H}(\alpha, \beta)$.

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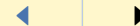
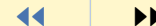
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Functions

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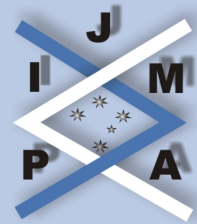
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1. Introduction

Let A denote the class of functions of the form

$$(1.1) \quad f(z) = z + \sum_{k=2}^{\infty} a_k z^k,$$

which are analytic in the unit disc $U = \{z : |z| < 1\}$. And let S denote the subclass of A consisting of univalent functions $f(z)$ in U .

A function $f(z)$ in S is said to be starlike of order α if and only if

$$\operatorname{Re} \left(\frac{z f'(z)}{f(z)} \right) > \alpha \quad (z \in U),$$

for some α ($0 \leq \alpha < 1$). We denote by $S^*(\alpha)$ the class of all functions in S which are starlike of order α . It is well-known that

$$S^*(\alpha) \subseteq S^*(0) \equiv S^*.$$

Further, a function $f(z)$ in S is said to be convex of order α in U if and only if

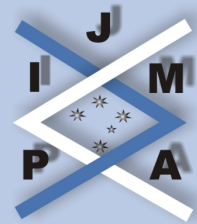
$$\operatorname{Re} \left(1 + \frac{z f''(z)}{f'(z)} \right) > \alpha \quad (z \in U),$$

for some α ($0 \leq \alpha < 1$). We denote by $K(\alpha)$ the class of all functions in S which are convex of order α .

The classes $S^*(\alpha)$, and $K(\alpha)$ were first introduced by Robertson [8], and later were studied by Schild [10], MacGregor [4], and Pinchuk [7].

Let T denote the subclass of S whose elements can be expressed in the form:

$$(1.2) \quad f(z) = z - \sum_{k=2}^{\infty} a_k z^k \quad (a_k \geq 0).$$



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We denote by $T^*(\alpha)$ and $C(\alpha)$, respectively, the classes obtained by taking the intersections of $S^*(\alpha)$ and $K(\alpha)$ with T ,

$$T^*(\alpha) = S^*(\alpha) \cap T \quad \text{and} \quad C(\alpha) = K(\alpha) \cap T.$$

The classes $T^*(\alpha)$ and $C(\alpha)$ were introduced by Silverman [11].

Let $H(\alpha, \beta)$ denote the class of functions $f(z) \in A$ which satisfy the condition

$$\operatorname{Re} \left(\frac{\alpha z^2 f''(z)}{f(z)} + \frac{z f'(z)}{f(z)} \right) > \beta$$

for some $\alpha \geq 0$, $0 \leq \beta < 1$, $\frac{f(z)}{z} \neq 0$ and $z \in U$.

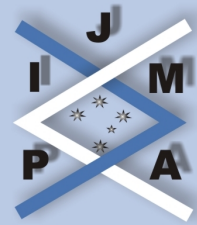
The classes $H(\alpha, \beta)$ and $H(\alpha, 0)$ were introduced and studied by Obradovic and Joshi [5], Padmanabhan [6], Li and Owa [2], Xu and Yang [14], Singh and Gupta [13], and others.

Further, we denote by $\overline{H}(\alpha, \beta)$ the class obtained by taking intersections of the class $H(\alpha, \beta)$ with T , that is

$$\overline{H}(\alpha, \beta) = H(\alpha, \beta) \cap T.$$

We note that

$$\overline{H}(0, \beta) = T^*(\beta) \quad (\text{Silverman [11]}).$$



2. Coefficient Estimates

Theorem 2.1. A function $f(z) \in T$ is in the class $\overline{H}(\alpha, \beta)$ if and only if

$$(2.1) \quad \sum_{k=2}^{\infty} [(k-1)(\alpha k+1) + (1-\beta)] a_k \leq 1 - \beta.$$

The result is sharp.

Proof. Assume that the inequality (2.1) holds and let $|z| < 1$. Then we have

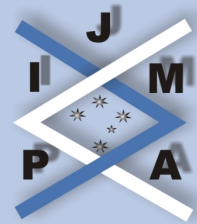
$$\begin{aligned} \left| \frac{\alpha z^2 f''(z)}{f(z)} + \frac{z f'(z)}{f(z)} - 1 \right| &= \left| \frac{-\sum_{k=2}^{\infty} (k-1)(\alpha k+1) a_k z^{k-1}}{1 - \sum_{k=2}^{\infty} a_k z^{k-1}} \right| \\ &\leq \frac{\sum_{k=2}^{\infty} (k-1)(\alpha k+1) a_k}{1 - \sum_{k=2}^{\infty} a_k} \leq 1 - \beta. \end{aligned}$$

This shows that the values of $\frac{\alpha z^2 f''(z)}{f(z)} + \frac{z f'(z)}{f(z)}$ lie in the circle centered at $w = 1$ whose radius is $1 - \beta$. Hence $f(z)$ is in the class $\overline{H}(\alpha, \beta)$.

To prove the converse, assume that $f(z)$ defined by (1.2) is in the class $\overline{H}(\alpha, \beta)$. Then

$$(2.2) \quad \begin{aligned} \operatorname{Re} \left(\frac{\alpha z^2 f''(z)}{f(z)} + \frac{z f'(z)}{f(z)} \right) \\ = \operatorname{Re} \left(\frac{1 - \sum_{k=2}^{\infty} [\alpha k(k-1) + k] a_k z^{k-1}}{1 - \sum_{k=2}^{\infty} a_k z^{k-1}} \right) > \beta \end{aligned}$$

for $z \in U$. Choose values of z on the real axis so that $\frac{\alpha z^2 f''(z)}{f(z)} + \frac{z f'(z)}{f(z)}$ is real. Upon



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clearing the denominator in (2.2) and letting $z \rightarrow 1^-$ through real values, we have

$$\beta \left(1 - \sum_{k=2}^{\infty} a_k \right) \leq 1 - \sum_{k=2}^{\infty} [\alpha k(k-1) + k] a_k,$$

which obviously is the required result (2.1).

Finally, we note that the assertion (2.1) of Theorem 2.1 is sharp, with the extremal function being

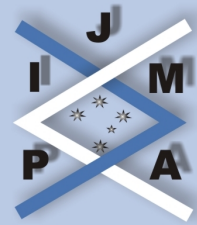
$$(2.3) \quad f(z) = z - \frac{1 - \beta}{[(k-1)(\alpha k + 1) + (1 - \beta)]} z^k \quad (k \geq 2).$$

□

Corollary 2.2. *Let $f(z) \in T$ be in the class $\overline{H}(\alpha, \beta)$. Then we have*

$$(2.4) \quad a_k \leq \frac{1 - \beta}{[(k-1)(\alpha k + 1) + (1 - \beta)]} \quad (k \geq 2).$$

Equality in (2.4) holds true for the function $f(z)$ given by (2.3).



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3. Some Properties of the Class $\overline{H}(\alpha, \beta)$

Theorem 3.1. Let $0 \leq \alpha_1 < \alpha_2$ and $0 \leq \beta < 1$. Then $\overline{H}(\alpha_2, \beta) \subset \overline{H}(\alpha_1, \beta)$.

Proof. It follows from Theorem 2.1. That

$$\sum_{k=2}^{\infty} [(k-1)(\alpha_1 k + 1) + (1-\beta)] a_k < \sum_{k=2}^{\infty} [(k-1)(\alpha_2 k + 1) + (1-\beta)] a_k \leq 1 - \beta$$

for $f(z) \in \overline{H}(\alpha_2, \beta)$. Hence $f(z) \in \overline{H}(\alpha_1, \beta)$. □

Corollary 3.2. $\overline{H}(\alpha, \beta) \subseteq T^*(\beta)$.

The proof is now immediate because $\alpha \geq 0$.

4. Neighborhood Results

Following the earlier investigations of Goodman [1] and Ruscheweyh [9], we define the δ -neighborhood of function $f(z) \in T$ by:

$$N_\delta(f) = \left\{ g \in T : g(z) = z - \sum_{k=2}^{\infty} b_k z^k, \sum_{k=2}^{\infty} k |a_k - b_k| \leq \delta \right\}.$$

In particular, for the identity function

$$e(z) = z,$$

we immediately have

$$(4.1) \quad N_\delta(e) = \left\{ g \in T : g(z) = z - \sum_{k=2}^{\infty} b_k z^k, \sum_{k=2}^{\infty} k |b_k| \leq \delta \right\}.$$

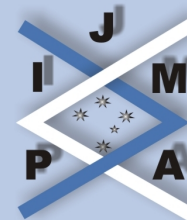
Theorem 4.1. $\overline{H}(\alpha, \beta) \subseteq N_\delta(e)$, where $\delta = \frac{2(1-\beta)}{(2\alpha+2-\beta)}$.

Proof. Let $f(z) \in \overline{H}(\alpha, \beta)$. Then, in view of Theorem 2.1, since $[(k-1)(\alpha k+1) + (1-\beta)]$ is an increasing function of k ($k \geq 2$), we have

$$(2\alpha + 2 - \beta) \sum_{k=2}^{\infty} a_k \leq \sum_{k=2}^{\infty} [(k-1)(\alpha k+1) + (1-\beta)] a_k \leq 1 - \beta,$$

which immediately yields

$$(4.2) \quad \sum_{k=2}^{\infty} a_k \leq \frac{1 - \beta}{(2\alpha + 2 - \beta)}.$$



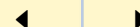
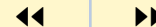
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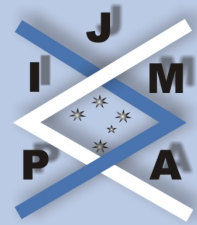
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On the other hand, we also find from (2.1)

$$\begin{aligned}(\alpha + 1) \sum_{k=2}^{\infty} k a_k - \beta \sum_{k=2}^{\infty} a_k &\leq \sum_{k=2}^{\infty} [(\alpha(k-1) + 1)k - \beta] a_k \\(4.3) \qquad \qquad \qquad &= \sum_{k=2}^{\infty} [(k-1)(\alpha k + 1) + (1 - \beta)] a_k \leq 1 - \beta.\end{aligned}$$

From (4.3) and (4.2), we have

$$\begin{aligned}(\alpha + 1) \sum_{k=2}^{\infty} k a_k &\leq (1 - \beta) + \beta \sum_{k=2}^{\infty} a_k \\&\leq (1 - \beta) + \beta \frac{1 - \beta}{(2\alpha + 2 - \beta)} \\&\leq \frac{2(\alpha + 1)(1 - \beta)}{(2\alpha + 2 - \beta)},\end{aligned}$$

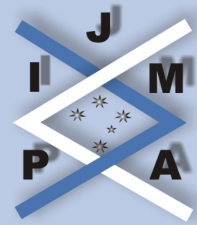
that is,

$$(4.4) \qquad \sum_{k=2}^{\infty} k a_k \leq \frac{2(1 - \beta)}{(2\alpha + 2 - \beta)} = \delta,$$

which in view of the definition (4.1), prove Theorem 4.1. □

Letting $\alpha = 0$, in the above theorem, we have:

Corollary 4.2. $T^*(\beta) \subseteq N_\delta(e)$, where $\delta = \frac{2(1-\beta)}{(2-\beta)}$.



5. Integral Means Inequalities

We need the following lemma.

Lemma 5.1 ([3]). *If f and g are analytic in U with $f \prec g$, then*

$$\int_0^{2\pi} |g(re^{i\theta})|^\delta d\theta \leq \int_0^{2\pi} |f(re^{i\theta})|^\delta d\theta,$$

where $\delta > 0$, $z = re^{i\theta}$ and $0 < r < 1$.

Applying Lemma 5.1, and (2.1), we prove the following theorem.

Theorem 5.2. *Let $\delta > 0$. If $f(z) \in \overline{H}(\alpha, \beta)$, then for $z = re^{i\theta}$, $0 < r < 1$, we have*

$$\int_0^{2\pi} |f(re^{i\theta})|^\delta d\theta \leq \int_0^{2\pi} |f_2(re^{i\theta})|^\delta d\theta,$$

where

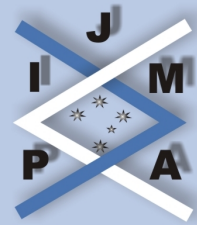
$$(5.1) \quad f_2(z) = z - \frac{(1-\beta)}{(2\alpha+2-\beta)}z^2.$$

Proof. Let $f(z)$ defined by (1.2) and $f_2(z)$ be given by (5.1). We must show that

$$\int_0^{2\pi} \left| 1 - \sum_{k=2}^{\infty} a_k z^{k-1} \right|^\delta d\theta \leq \int_0^{2\pi} \left| 1 - \frac{(1-\beta)}{(2\alpha+2-\beta)}z \right|^\delta d\theta.$$

By Lemma 5.1, it suffices to show that

$$1 - \sum_{k=2}^{\infty} a_k z^{k-1} \prec 1 - \frac{(1-\beta)}{(2\alpha+2-\beta)}z.$$



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Setting

$$(5.2) \quad 1 - \sum_{k=2}^{\infty} a_k z^{k-1} = 1 - \frac{(1-\beta)}{(2\alpha+2-\beta)} w(z).$$

From (5.2) and (2.1), we obtain

$$\begin{aligned} |w(z)| &= \left| \sum_{k=2}^{\infty} \frac{(2\alpha+2-\beta)}{(1-\beta)} a_k z^{k-1} \right| \\ &\leq |z| \sum_{k=2}^{\infty} \frac{[(k-1)(\alpha k+1) + (1-\beta)] a_k}{1-\beta} \leq |z|. \end{aligned}$$

This completes the proof of the theorem. \square

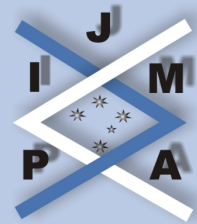
Letting $\alpha = 0$ in the above theorem, we have:

Corollary 5.3. *Let $\delta > 0$. If $f(z) \in T^*(\beta)$, then for $z = re^{i\theta}$, $0 < r < 1$, we have*

$$\int_0^{2\pi} |f(re^{i\theta})|^\delta d\theta \leq \int_0^{2\pi} |f_2(re^{i\theta})|^\delta d\theta,$$

where

$$f_2(z) = z - \frac{(1-\beta)}{(2-\beta)} z^2.$$



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6. Partial Sums

In this section we will examine the ratio of a function of the form (1.2) to its sequence of partial sums defined by $f_1(z) = z$ and $f_n(z) = z - \sum_{k=2}^n a_k z^k$ when the coefficients of f are sufficiently small to satisfy the condition (2.1). We will determine sharp lower bounds for $\operatorname{Re} \left(\frac{f(z)}{f_n(z)} \right)$, $\operatorname{Re} \left(\frac{f_n(z)}{f(z)} \right)$, $\operatorname{Re} \left(\frac{f'(z)}{f'_n(z)} \right)$ and $\operatorname{Re} \left(\frac{f'_n(z)}{f'(z)} \right)$.

In what follows, we will use the well known result that

$$\operatorname{Re} \frac{1 - w(z)}{1 + w(z)} > 0, \quad z \in U,$$

if and only if

$$w(z) = \sum_{k=1}^{\infty} c_k z^k$$

satisfies the inequality $|w(z)| \leq |z|$.

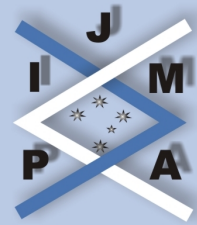
Theorem 6.1. *If $f(z) \in \overline{H}(\alpha, \beta)$, then*

$$(6.1) \quad \operatorname{Re} \frac{f(z)}{f_n(z)} \geq 1 - \frac{1}{c_{n+1}} \quad (z \in U, n \in N)$$

and

$$(6.2) \quad \operatorname{Re} \left(\frac{f_n(z)}{f(z)} \right) \geq \frac{c_{n+1}}{1 + c_{n+1}} \quad (z \in U, n \in N),$$

where $\left(c_k =: \frac{[(k-1)(\alpha k+1)+(1-\beta)]}{1-\beta} \right)$. The estimates in (6.1) and (6.2) are sharp.



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Proof. We employ the same technique used by Silverman [12]. The function $f(z) \in \overline{H}(\alpha, \beta)$, if and only if $\sum_{k=2}^{\infty} c_k a_k \leq 1$. It is easy to verify that $c_{k+1} > c_k > 1$. Thus,

$$(6.3) \quad \sum_{k=2}^n a_k + c_{n+1} \sum_{k=n+1}^{\infty} a_k \leq \sum_{k=2}^{\infty} c_k a_k \leq 1.$$

We may write

$$c_{n+1} \left\{ \frac{f(z)}{f_n(z)} - \left(1 - \frac{1}{c_{n+1}} \right) \right\} = \frac{1 - \sum_{k=2}^n a_k z^{k-1} - c_{n+1} \sum_{k=n+1}^{\infty} a_k z^{k-1}}{1 - \sum_{k=2}^n a_k z^{k-1}}$$

$$= \frac{1 + D(z)}{1 + E(z)}.$$

Set

$$\frac{1 + D(z)}{1 + E(z)} = \frac{1 - w(z)}{1 + w(z)},$$

so that

$$w(z) = \frac{E(z) - D(z)}{2 + D(z) + E(z)}.$$

Then

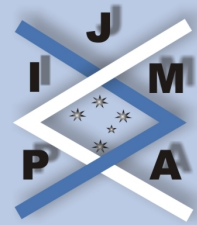
$$w(z) = \frac{c_{n+1} \sum_{k=n+1}^{\infty} a_k z^{k-1}}{2 - 2 \sum_{k=2}^n a_k z^{k-1} - c_{n+1} \sum_{k=n+1}^{\infty} a_k z^{k-1}}$$

and

$$|w(z)| \leq \frac{c_{n+1} \sum_{k=n+1}^{\infty} a_k}{2 - 2 \sum_{k=2}^n a_k - c_{n+1} \sum_{k=n+1}^{\infty} a_k}.$$

Now $|w(z)| \leq 1$ if and only if

$$\sum_{k=2}^n a_k + c_{n+1} \sum_{k=n+1}^{\infty} a_k \leq 1,$$



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which is true by (6.3). This readily yields the assertion (6.1) of Theorem 6.1.

To see that

$$(6.4) \quad f(z) = z - \frac{z^{n+1}}{c_{n+1}}$$

gives sharp results, we observe that

$$\frac{f(z)}{f_n(z)} = 1 - \frac{z^n}{c_{n+1}}.$$

Letting $z \rightarrow 1^-$, we have

$$\frac{f(z)}{f_n(z)} = 1 - \frac{1}{c_{n+1}},$$

which shows that the bounds in (6.1) are the best possible for each $n \in N$. Similarly, we take

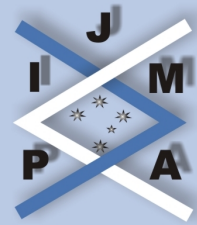
$$\begin{aligned} (1 + c_{n+1}) \left(\frac{f_n(z)}{f(z)} - \frac{c_{n+1}}{1 + c_{n+1}} \right) &= \frac{1 - \sum_{k=2}^n a_k z^{k-1} + c_{n+1} \sum_{k=n+1}^{\infty} a_k z^{k-1}}{1 - \sum_{k=2}^{\infty} a_k z^{k-1}} \\ &:= \frac{1 - w(z)}{1 + w(z)}, \end{aligned}$$

where

$$|w(z)| \leq \frac{(1 + c_{n+1}) \sum_{k=n+1}^{\infty} a_k}{2 - 2 \sum_{k=2}^n a_k + (1 - c_{n+1}) \sum_{k=n+1}^{\infty} a_k}.$$

Now $|w(z)| \leq 1$ if and only if

$$\sum_{k=2}^n a_k + c_{n+1} \sum_{k=n+1}^{\infty} a_k \leq 1,$$



which is true by (6.3). This immediately leads to the assertion (6.2) of Theorem 6.1.

The estimate in (6.2) is sharp with the extremal function $f(z)$ given by (6.4). This completes the proof of Theorem 6.1. \square

Letting $\alpha = 0$ in the above theorem, we have:

Corollary 6.2. *If $f(z) \in T^*(\beta)$, then*

$$\operatorname{Re} \frac{f(z)}{f_n(z)} \geq \frac{n}{(n+1-\beta)}, \quad (z \in U)$$

and

$$\operatorname{Re} \frac{f_n(z)}{f(z)} \geq \frac{n+1-\beta}{(n+2-2\beta)}, \quad (z \in U).$$

The result is sharp for every n , with the extremal function

$$(6.5) \quad f(z) = z - \frac{1-\beta}{(n+1-\beta)} z^{n+1}.$$

We now turn to the ratios involving derivatives. The proof of Theorem 6.3 below follows the pattern of that in Theorem 6.1, and so the details may be omitted.

Theorem 6.3. *If $f(z) \in \overline{H}(\alpha, \beta)$, then*

$$(6.6) \quad \operatorname{Re} \frac{f'(z)}{f'_n(z)} \geq 1 - \frac{n+1}{c_{n+1}} \quad (z \in U),$$

and

$$(6.7) \quad \operatorname{Re} \left(\frac{f'_n(z)}{f'(z)} \right) \geq \frac{c_{n+1}}{n+1+c_{n+1}} \quad (z \in U, n \in \mathbb{N}).$$

The estimates in (6.6) and (6.7) are sharp with the extremal function given by (6.4).



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Letting $\alpha = 0$ in the above theorem, we have:

Corollary 6.4. *If $f(z) \in T^*(\beta)$, then*

$$\operatorname{Re} \frac{f'(z)}{f'_n(z)} \geq \frac{\beta n}{(n+1-\beta)}, \quad (z \in U),$$

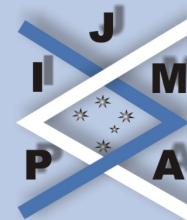
and

$$\operatorname{Re} \frac{f'_n(z)}{f'(z)} \geq \frac{n+1-\beta}{n+(1-\beta)(n+2)}, \quad (z \in U).$$

The result is sharp for every n , with the extremal function given by (6.5).

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