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On Generalized Cullen and Woodall Numbers That are Also Fibonacci Numbers

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Abstract

The m-th Cullen number C_m is a number of the form $m2^m + 1$ and the m-th Woodall number W_m has the form $m2^m - 1$. In 2003, Luca and Stănică proved that the largest Fibonacci number in the Cullen sequence is $F_4 = 3$ and that $F_1 = F_2 = 1$ are the largest Fibonacci numbers in the Woodall sequence. A generalization of these sequences is defined by $C_{m,s} = ms^m + 1$ and $W_{m,s} = ms^m - 1$, for $s > 1$. In this paper, we search for Fibonacci numbers belonging to these generalized Cullen and Woodall sequences.

1 Introduction

A *Cullen number* is a number of the form $m2^m + 1$ (denoted by C_m), where m is a nonnegative integer. The first few terms of this sequence are

$$1, 3, 9, 25, 65, 161, 385, 897, 2049, 4609, 10241, 22529, \ldots$$

which is the OEIS [26] sequence A002064. (This sequence was introduced in 1905 by Father Cullen [6] and it was mentioned in the well-known book of Guy [9, Section **B20**].) These numbers gained great interest in 1976, when Hooley [11] showed that almost all Cullen numbers are composite. However, despite their being very scarce, it is still conjectured that there are infinitely many *Cullen primes*. For instance, $C_{6679881}$ is a prime number with more than 2 millions of digits (PrimeGrid, August 2009).

In a similar way, a *Woodall number* (also called *Cullen number of the second kind*) is a positive integer of the form $m2^m - 1$ (denoted by W_m). The first few terms of this sequence are

$$1, 7, 23, 63, 159, 383, 895, 2047, 4607, 10239, \dots$$

which is the OEIS sequence A003261. In a personal communication to Keller [13, p. 1739], Suyama asserted that Hooley's method can be reformulated to show that it works for any sequence of numbers of the form $m2^{m+a}+b$ where a and b are integers. In particular, Woodall numbers are almost all composites. However, it is conjectured that the set of *Woodall primes* is infinite. We remark that $W_{3752948}$ is a prime number (PrimeGrid, December 2007).

These numbers can be generalized to the *generalized Cullen and Woodall numbers* which are numbers of the form

$$C_{m,s} = ms^m + 1 \text{ and } W_{m,s} = ms^m - 1,$$

where $m \geq 1$ and $s \geq 2$. Clearly, one has that $C_{m,2} = C_m$ and $W_{m,2} = W_m$, for all $m \geq 1$. For simplicity, we call $C_{m,s}$ and $W_{m,s}$ an *s-Cullen number* and an *s-Woodall number*, respectively. Also, an s-Cullen or s-Woodal number is said to be *trivial* if it has the form $s+1$ or $s-1$, respectively, or equivalently when its first index is equal to 1. This family was introduced by Dubner [7] and is one of the main sources for prime number "hunters". A prime of the form $C_{m,s}$ is $C_{139948,151}$ an integer with 304949 digits.

Many authors have searched for special properties of Cullen and Woodall numbers and their generalizations. In regards to these numbers, we refer to [8, 10, 13] for primality results and [17] for their greatest common divisor. The problem of finding Cullen and Woodall numbers belonging to other known sequences has attracted much attention in the last two decades. We cite [18] for pseudoprime Cullen and Woodall numbers, and [1] for Cullen numbers which are both Riesel and Sierpiński numbers.

In 2003, Luca and Stănică [16, Theorem 3] proved that the largest Fibonacci number in the Cullen sequence is $F_4 = 3 = 1 \cdot 2^1 + 1$ and that $F_1 = F_2 = 1 = 1 \cdot 2^1 - 1$ are the largest Fibonacci numbers in the Woodall sequence. Note that these numbers are trivial Cullen and Woodall numbers (in the previous sense, i.e., $m = 1$).

In this paper, we search for Fibonacci numbers among s-Cullen numbers and s-Woodall numbers, for $s > 1$. Our main result is the following

Theorem 1. *Let s be a positive integer. If (m, n, ℓ) is an integer solution of*

$$F_n = ms^m + \ell, \tag{1}$$

where $\ell \in \{-1, 1\}$ and $m, n > 0$, then

$$m < (6.2 + 1.9P(s)) \log(3.1 + P(s)), \tag{2}$$

and

$$n < \frac{\log((6.2 + 1.9P(s)) \log(3.1 + P(s)) s^{(6.2+1.9P(s)) \log(3.1+P(s))} + 1)}{\log \alpha} + 2,$$

where $P(s)$ denotes the largest prime factor of s and $\alpha = (1 + \sqrt{5})/2$.

In particular, the above theorem ensures that for any given $s \geq 2$, there are only finitely many Fibonacci numbers which are also s-Cullen numbers or s-Woodall numbers and they are effectively computable.

We should recall that $\nu_p(r)$ denotes the p-adic order (or valuation) of r which is the exponent of the highest power of a prime p which divides r. Also, the *order (or rank) of appearance* of n in the Fibonacci sequence, denoted by $z(n)$, is defined to be the smallest positive integer k, such that $n \mid F_k$ (some authors call it *order of apparition*, or *Fibonacci entry point*). We refer the reader to [19, 20, 21, 22, 23] for some results about this function. Let p be a prime number and set $e(p) := \nu_p(F_{z(p)})$. By evaluating $e(p)$, for primes $p < 30$, one can see that $e(p) = 1$. In fact, $e(p) = 1$ for all primes $p < 2.8 \cdot 10^{16}$ (PrimeGrid, March 2014). Moreover, the assertion $e(p) = 1$ for all prime p is equivalent to $z(p) \neq z(p^2)$, for all primes p (this is related to Wall's question [28]). This question raised interest in 1992, when Sun and Sun [27] proved (in an equivalent form) that $e(p) = 1$ for all primes p, implies the first case of Fermat's "last theorem".

In view of the previous discussion, it seems reasonable to consider problems involving primes with $e(p) = 1$ (because of their abundance). Our next result deals with this kind of primes.

Theorem 2. *There is no integer solution (m, n, s, ℓ) for Eq. (1) with $n > 0$, $m > 1$, $\ell \in \{-1, 1\}$ and $s > 1$ such that $e(p) = 1$ for all prime factor p of s.*

In particular, the only solutions of Eq. (1), with the previous conditions, occur when $m = 1$ and have the form

$$(m, n, s, \ell) = (1, n, F_n - \ell, \ell).$$

An immediate consequence of Theorem 2 and the fact that $e(p) = 1$ for all primes $p < 2.8 \cdot 10^{16}$ is the following

Corollary 3. *There is no Fibonacci number that is also a nontrivial s-Cullen number or s-Woodall number when the set of prime divisors of s is contained in*

$$\{2, 3, 5, 7, 11, \ldots, 27999999999999971, 27999999999999991\}.$$

This is the set of the first 759997990476073 *prime numbers.*

Here is an outline of the paper. In Section 2, we recall some facts which will be useful in the proofs of our theorems, such as the result concerning the p-adic order of F_n and factorizations of the form $F_n \pm 1 = F_a L_b$, with $|a - b| \leq 2$. In the last section, we combine these mentioned tools, the fact that a common divisor of F_a and L_b is small and a lower bound for Fibonacci and Lucas numbers, to get an inequality of the form $m < C \log m$ which gives an upper bound for m in terms of C which in turn depends on the factorization of s. With this bound, we reduce the analysis of Eq. (1) for a finite number of cases which can be settled by using an approach used in a recent paper by Bugeaud, Luca, Mignotte and Siksek. By using these ingredients (with some more technicalities) we deal with the proofs of Theorems 1 and 2.

2 Auxiliary results

We cannot go very far into the lore of Fibonacci numbers without encountering its companion Lucas sequence $(L_n)_{n\geq 0}$ which follows the same recursive pattern as the Fibonacci numbers, but with initial values $L_0 = 2$ and $L_1 = 1$. First, we recall some classical and helpful facts which will be essential ingredients to prove Theorems 1 and 2.

Lemma 4. *We have*

(a) If $d = \gcd(m, n)$, *then*

$$\gcd(F_m, L_n) = \begin{cases} L_d, & \text{if } m/d \text{ is even and } n/d \text{ is odd;} \\ 1 \text{ or } 2, & \text{otherwise.} \end{cases}$$

(b) If $d = \gcd(m, n)$, *then* $\gcd(F_m, F_n) = F_d$.

(c) (Binet's formulae) If $\alpha = (1 + \sqrt{5})/2$ *and* $\beta = (1 - \sqrt{5})/2$, *then*

$$F_n = \frac{\alpha^n - \beta^n}{\alpha - \beta} \quad \text{and} \quad L_n = \alpha^n + \beta^n.$$

(d) $\alpha^{n-2} \leq F_n \leq \alpha^{n-1}$ *and* $\alpha^{n-1} < L_n < \alpha^{n+1}$ *for all* $n \geq 1$.

(e) $z(p) \leq p + 1$, *for all prime numbers* p.

Proofs of these assertions can be found in [14]. We refer the reader to [2, 12, 24] for more details and additional bibliography.

The equation $F_n + 1 = y^2$ and more generally $F_n \pm 1 = y^\ell$ with integer y and $\ell \geq 2$ have been solved in [25] and [5], respectively. The solution for the last equation makes appeal to Fibonacci and Lucas numbers with negative indices which are defined as follows: let $F_n = F_{n+2} - F_{n+1}$ and $L_n = L_{n+2} - L_{n+1}$. Thus, for example, $F_{-1} = 1, F_{-2} = -1$, and so on. Bugeaud et al. [5, Section 5] used these numbers to give factorizations for $F_n \pm 1$. Let us sketch their method for the convenience of the reader.

Since the Binet's formulae remain valid for Fibonacci and Lucas numbers with negative indices, one can deduce the following result.

Lemma 5. *For any integers* a, b, *we have*

$$F_a L_b = F_{a+b} + (-1)^b F_{a-b}.$$

Proof. The identity $\alpha = (-\beta)^{-1}$ together with Lemma 4 (c) leads to

$$F_a L_b = \frac{\alpha^a - \beta^a}{\alpha - \beta}(\alpha^b + \beta^b) = F_{a+b} + \frac{\alpha^a \beta^b - \beta^a \alpha^b}{\alpha - \beta} = F_{a+b} + (-1)^b F_{a-b}.$$

□

Lemma 5 gives immediate factorizations for $F_n \pm 1$, depending on the class of n modulo 4. For example, if $n \equiv 0 \pmod 4$, then $F_n + 1 = F_{(n/2)-1}L_{(n/2)+1}$. In general, we have $F_n \pm 1 = F_a L_b$, where $2a, 2b \in \{n \pm 2, n \pm 1\}$.

We remark that the p-adic order of Fibonacci and Lucas numbers has been completely characterized. For instance, from the main results of Lengyel [15], we extract the following results.

Lemma 6. *For $n \geq 1$,*

$$\nu_2(F_n) = \begin{cases} 0, & \text{if } n \equiv 1, 2 \pmod 3; \\ 1, & \text{if } n \equiv 3 \pmod 6; \\ 3, & \text{if } n \equiv 6 \pmod{12}; \\ \nu_2(n) + 2, & \text{if } n \equiv 0 \pmod{12}, \end{cases}.$$

$\nu_5(F_n) = \nu_5(n)$, *and if p is prime $\neq 2$ or 5, then*

$$\nu_p(F_n) = \begin{cases} \nu_p(n) + e(p), & \text{if } n \equiv 0 \pmod{z(p)}; \\ 0, & \text{otherwise,} \end{cases}$$

where $e(p) := \nu_p(F_{z(p)})$.

Lemma 7. *Let $k(p)$ be the period modulo p of the Fibonacci sequence. For all primes $p \neq 5$, we have*

$$\nu_2(L_n) = \begin{cases} 0, & \text{if } n \equiv 1, 2 \pmod 3; \\ 2, & \text{if } n \equiv 3 \pmod 6; \\ 1, & \text{if } n \equiv 0 \pmod 6) \end{cases}$$

and

$$\nu_p(L_n) = \begin{cases} \nu_p(n) + e(p), & \text{if } k(p) \neq 4z(p) \text{ and } n \equiv \frac{z(p)}{2} \pmod{z(p)}; \\ 0, & \text{otherwise.} \end{cases}$$

Observe that the relation $L_n^2 = 5F_n^2 + 4(-1)^n$ implies that $\nu_5(L_n) = 0$, for all $n \geq 1$.

Now we are ready to deal with the proofs of our results.

3 The proofs

3.1 The proof of Theorem 1

In order to simplify our presentation, we use the familiar notation $[a, b] = \{a, a + 1, \ldots, b\}$, for integers $a < b$.

The smallest value of $(6.2 + 1.9P(s)) \log(3.1 + P(s))$ is $16.292\ldots$ (it happens when s is a power of 2). Thus, we may suppose $m > 16$ (however, for our purpose it suffices to consider $m > 4$).

We rewrite Eq. (1) as $F_n - \ell = ms^m$. Since $\ell \in \{-1, 1\}$, then Lemma 5 gives $F_a L_b = ms^m$, where $2a, 2b \in \{n \pm 2, n \pm 1\}$ and $|a - b| \in \{1, 2\}$. Observe that in this case $\gcd(F_a, L_b) = 1$ or 3 (Lemma 4 (a)).

Case 1. If $s \not\equiv 0 \pmod 3$. Since $\gcd(F_a, L_b) = 1$ or 3 and $3 \nmid s$, then, without loss of any generality, we can write $s = p_1^{a_1} \cdots p_k^{a_k}$, with $a_i \geq 0$, such that $p_1^{a_1} \cdots p_t^{a_t}$ divides F_a and $p_{t+1}^{a_{t+1}} \cdots p_k^{a_k}$ divides L_b, where $p_1, \ldots, p_k$ are distinct primes. Thus $\nu_{p_i}(F_a) \geq a_i m$, for $i \in [1, t]$ and $\nu_{p_j}(L_b) \geq a_j m$, for $j \in [t+1, k]$, and on the other hand, Lemmas 6 and 7 imply

$$\nu_{p_i}(F_a) \leq \nu_{p_i}(a) + (1 - \delta_{p_i,5} + \delta_{p_i,2}) e(p_i)$$

and

$$\nu_{p_j}(L_b) \leq \max\{\nu_{p_j}(b) + e(p_j), 2\},$$

where $\delta_{r,s}$ denotes the usual Kronecker delta. Since $1 - \delta_{p_i,5} + \delta_{p_i,2} \leq 2$ and $m > 2$, then

$$\nu_{p_i}(F_a) \leq \nu_{p_i}(a) + 2e(p_i) \text{ and } \nu_{p_j}(L_b) \leq \nu_{p_j}(b) + e(p_j).$$

Thus one obtains that $\nu_{p_i}(a) \geq a_i m - 2e(p_i)$, for all $i \in [1, t]$ and $\nu_{p_j}(b) \geq a_j m - e(p_j)$, for all $j \in [t+1, k]$. Since $p_1, \ldots, p_k$ are pairwise coprime, we have $a \geq p_1^{a_1 m - 2e(p_1)} \cdots p_k^{a_k m - 2e(p_t)}$ and $b \geq p_{t+1}^{a_{t+1} m - e(p_{t+1})} \cdots p_k^{a_k m - e(p_k)}$. Hence $F_a L_b = ms^m$ together with the estimates in Lemma 4 (d) yields

$$\begin{aligned} m p_1^{m a_1} \cdots p_k^{m a_k} &\geq \alpha^{a+b-3} \\ &> \alpha^{\prod_{i=1}^{t} p_i^{m a_i - 2e(p_i)} + \prod_{i=t+1}^{k} p_i^{m a_i - 2e(p_i)} - 3}, \end{aligned}$$

where we used the inequality $m a_i - e(p_i) > m a_i - 2e(p_i)$. Note that we may suppose that $m a_i > 2e(p_i)$, for all $i \in [1, k]$ (otherwise, we would have $m \leq 2e(p_i)$, for some i and Theorem 1 is proved). Also $p_i \geq 2$, for all $i \in [1, k]$ and then

$$\prod_{i=1}^{t} p_i^{m a_i - 2e(p_i)} + \prod_{i=t+1}^{k} p_i^{m a_i - 2e(p_i)} \geq \sum_{i=1}^{k} p_i^{m a_i - 2e(p_i)}.$$

Thus we have

$$4.3 m p_1^{m a_1} \cdots p_k^{m a_k} > \alpha^{p_1^{m a_1 - 2e(p_1)}} \cdots \alpha^{p_k^{m a_k - 2e(p_k)}},$$

where we have used that $\alpha^3 < 4.3$. But for $m > 4$, it holds that $4.3m < 2^m \leq p_1^m$ and so

$$p_1^{m(a_1+1)} \cdots p_k^{m a_k} > \alpha^{p_1^{m a_1 - 2e(p_1)}} \cdots \alpha^{p_k^{m a_k - 2e(p_k)}}.$$

If the inequality $p_i^{m a_i - 2e(p_i)} > m(a_i + 1)(\log p_i)/\log \alpha$ holds for all $i \in [1, k]$, we arrive at the following absurdity

$$p_1^{m(a_1+1)} \cdots p_k^{m a_k} > \alpha^{p_1^{m a_1 - 2e(p_1)}} \cdots \alpha^{p_k^{m a_k - 2e(p_k)}} > p_1^{m(a_1+1)} \cdots p_k^{m(a_k+1)}.$$

Thus, there exists $i \in [1,k]$, such that $p_i^{ma_i - 2e(p_i)} \le m(a_i+1)(\log p_i)/\log\alpha$. Now, by applying the log function in the previous inequality together with a straightforward calculation, we obtain

$$\frac{m}{\log m} \le \frac{1}{\log p_i} + \frac{\log(a_i+1)}{a_i \log m \log p_i} + \frac{\log\left(\frac{\log p_i}{\log \alpha}\right)}{a_i \log m \log p_i} + \frac{2e(p_i)}{a_i \log m}.$$

Note that $a_i \ge 1$, $\log p_i \ge \log 2 > 0.69$ and $\log m \ge \log 3 > 1.09$. Also, the functions $x \mapsto (\log(x+1))/x$ and $x \mapsto (\log(\log x/\log\alpha))/(\log x)$ are nonincreasing for $x \ge 1$ and $x \ge 4$, respectively, then $(\log(a_i+1))/a_i \le \log 2$ and $(\log(\log p_i/\log\alpha))/(\log p_i) < 0.77$. Therefore,

$$\frac{m}{\log m} < 3.1 + 1.9e(p_i).$$

Since the function $x \mapsto x/\log x$ is increasing for $x > e$, it is a simple matter to prove that, for $A > e$,

$$\frac{x}{\log x} < A \ \text{ implies that } \ x < 2A\log A. \tag{3}$$

A proof for that can be found in [3, p. 74].

By using (3) for $x := m$ and $A := 3.1 + 1.9e(p_i) > e$, we have that

$$m < (6.2 + 3.8e(p_i))\log(3.1 + 1.9e(p_i)). \tag{4}$$

Since $p_i \le P(s)$, then in order to get the desired inequality in (2), it suffices to prove that $e(p) \le p/2$ for all primes p. Clearly, the inequality holds for $p = 2$, so we may suppose $p \ge 3$. Since $p^{e(p)} \mid F_{z(p)}$, we have $p^{e(p)} \le F_{z(p)}$. Suppose, towards a contradiction, that $e(p) > p/2$, then we use Lemma 4 (d) and (e) to obtain

$$p^{p/2} < p^{e(p)} \le F_{z(p)} \le \alpha^{z(p)-1} \le \alpha^p.$$

This yields that $p < \alpha^2 < 2.619$ which is impossible, since $p \ge 3$. Thus $e(p) \le p/2$ *. Thus

$$m < (6.2 + 1.9P(s))\log(3.1 + P(s)),$$

where we used that $P(s) \ge p_i$, for all $i \in [1,k]$.

Now, we use Lemma 4 (d) to get $\alpha^{n-2} \le F_n \le ms^m + 1$ and a straightforward calculation yields

$$n < \frac{\log((6.2+1.9P(s))\log(3.1+P(s))s^{(6.2+1.9P(s))\log(3.1+P(s))} + 1)}{\log\alpha} + 2,$$

where we used the upper bound for m in terms of s.

Case 2. If $s \equiv 0 \pmod 3$. We can proceed as before unless $\gcd(F_a, L_b) = 3$. In this case, for some suitable choice of $\epsilon_1, \epsilon_2 \in \{0,1\}$, with $\epsilon_1 + \epsilon_2 = 1$, we have

$$\frac{F_a}{3^{\epsilon_1}} \cdot \frac{L_b}{3^{\epsilon_2}} = \frac{ms^m}{3}$$

and $\gcd(F_a/3^{\epsilon_1}, L_b/3^{\epsilon_2}) = 1$. From this point on the proof proceeds along the same lines as the proof of previous case. □

*In fact, the same proof gives the sharper bound $e(p) \le (p\log\alpha)/\log p$. This bound together with the squeeze theorem gives $\lim_{p\to\infty} e(p)/p = 0$.

3.2 The proof of Theorem 2

Let (m, n, s, ℓ) be a solution of Eq. (1) satisfying the conditions in the statement of Theorem 2 and suppose that $m \leq 16$. Note that $F_n = 4s^4 \pm 1$ can be rewritten as $F_n = (2s^2)^2 \pm 1$, but Bugeaud et al. [5, Theorem 2] listed all solutions of the Diophantine equation $F_n \pm 1 = y^t$. A quick inspection in their list gives $y = 1, 2$ or 3, but none of these values have the form $2s^2$, for $s > 1$. So, there is no solution for Eq. (1) when $m = 4$.

So, we wish to solve the equation $F_n = ms^m \pm 1$, when $m \in [2, 16] \backslash \{4\}$. As previously done, let us rewrite it as $F_a L_b = ms^m$. Note that m has at most 2 distinct prime factors and they belong to $\{2, 3, 5, 7, 11, 13\}$. Since $\gcd(F_a, L_b) = 1$ or 3, we can deduce that

$$F_a = p_1^{\alpha_1} p_2^{\alpha_2} p_3^{\alpha_3} (s_1^r)^p, \tag{5}$$

where p_1, p_2, p_3, p are primes less than 17, $\alpha_1, \alpha_2, \alpha_3 \in \{0, 1, 2, 3, 4\}$, $s_1 \mid s$ and $m = pr$.

However, in 2007, by combining some deep tools in number theory, Bugeaud, Luca, Mignotte and Siksek [4, Theorem 4 for $m = 1$], found (in particular) all solutions of the Diophantine equation

$$F_t = 2^{x_1} \cdot 3^{x_2} \cdots 541^{x_{100}} y^p,$$

where $x_i \geq 0$ and p is a prime number. More precisely, they proved that in this case, one has

$$t \in [1, 16] \cup [19, 22] \cup \{24, 26, 27, 28, 30, 36, 42, 44\}. \tag{6}$$

In particular, $t \leq 44$.

Note that Eq. (5) is a particular case already treated by Theorem 4 of [4]. However, for convenience of the reader, we describe in a few words how these calculations can be performed. First, if $m = 2$, then we have the equation $F_a = 2^{\alpha_1} \cdot 3^{\alpha_2} s_1^2$ and after a quick inspection in (6), one can see that possible values for a do not give any solution for Eq. (1). In the case of $m \geq 3$, we use that $a \leq 44$ together with $a \geq (n-2)/2$ to get $n \leq 90$ and so

$$3s^3 - 1 \leq ms^m + \ell = F_n \leq F_{90} = 2880067194370816120.$$

Therefore $s \leq 986492$. Now by using *Mathematica* [29], one deduces that the Diophantine equation $F_n = ms^m + \ell$, for $2 \leq n \leq 90$, $3 \leq m \leq 16$, $\ell \in \{-1, 1\}$ and $2 \leq s \leq 986492$, has no any solution.

Let us suppose that $m > 16$. We remark that in order to get the inequality (4) in the proof of Theorem 1, we only assume that $m > 4$. Thus, we have

$$m < (6.2 + 3.8e(p_i)) \log(3.1 + 1.9e(p_i)),$$

where p_i is a prime factor of s. However, by hypothesis, all prime factors p of s satisfy $e(p) = 1$. In particular, $e(p_i) = 1$ and so we have the following absurdity: $16 < m < 10 \log 5 = 16.094\ldots$. This completes the proof of the theorem. $\square$

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