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ON A PARTIAL DIFFERENTIAL EQUATION IN 4-DIMENSIONAL EUCLIDEAN SPACE

(submitted by M. M. Arslanov)

ABSTRACT. The objective of this article is to construct in a hyper-rectangular region of the 4-dimensional Euclidean space a solution of the Goursat problem for the following equation:

$$L(u) = \sum_{i_1=0}^{m_1} \sum_{i_2=0}^{m_2} \sum_{i_3=0}^{m_3} \sum_{i_4=0}^{m_4} a_{i_1 i_2 i_3 i_4}(x_1, x_2, x_3, x_4) \frac{\partial^{i_1+i_2+i_3+i_4} u}{\partial x_1^{i_1} \partial x_2^{i_2} \partial x_3^{i_3} \partial x_4^{i_4}}$$
$$= F(x_1, x_2, x_3, x_4).$$

Introduction. Let $D = \{x_{10} < x_1 < x_{11}, x_{20} < x_2 < x_{21}, x_{30} < x_3 < x_{31}, x_{40} < x_4 < x_{41}\}$, where X_1, X_2, X_3, X_4 are the faces of D for $x_i = x_{i0}$ (i=1,2,3,4), respectively. The objective of this article is to construct in D a solution of the Goursat problem for the following equation:

$$L(u) = \sum_{i_1=0}^{m_1} \sum_{i_2=0}^{m_2} \sum_{i_3=0}^{m_3} \sum_{i_4=0}^{m_4} a_{i_1 i_2 i_3 i_4}(x_1, x_2, x_3, x_4) \frac{\partial^{i_1+i_2+i_3+i_4} u}{\partial x_1^{i_1} \partial x_2^{i_2} \partial x_3^{i_3} \partial x_4^{i_4}}$$
$$= F(x_1, x_2, x_3, x_4), \quad (1)$$

where $a_{m_1 m_2 m_3 m_4} \equiv 1$, while the smoothness of remaining coefficients is defined by the inclusions

$$a_{i_1 i_2 i_3 i_4} \in C^{i_1+i_2+i_3+i_4}(\overline{D}), \quad F \in C^{0+0+0+0}(\overline{D}). \quad (2)$$

Here $C^{\alpha_1+\alpha_2+\alpha_3+\alpha_4}$ stands for a class of functions continuous in $\overline{D}$ together with their partial derivatives

$$\partial^{r_1+r_2+r_3+r_4}/\partial x_1^{r_1}\partial x_2^{r_2}\partial x_3^{r_3}\partial x_4^{r_4}$$
$$(r_1 = 0, ..., \alpha_1, r_2 = 0, ..., \alpha_2, r_3 = 0, ..., \alpha_3, r_4 = 0, ..., \alpha_4).$$

The Goursat problem: Determine in D a regular solution of the equation (1), which satisfies the following conditions:

$$
\begin{aligned}
\frac{\partial^{i_1} u}{\partial x_1^{i_1}}\,(x_{10}, x_2, x_3, x_4) &= \varphi_{1i_1}\,(x_2, x_3, x_4)\,, \quad \left(i_1 = \overline{0, m_1 - 1}\right), \\
\frac{\partial^{i_2} u}{\partial x_2^{i_2}}\,(x_1, x_{20}, x_3, x_4) &= \varphi_{2i_2}\,(x_1, x_3, x_4)\,, \quad \left(i_2 = \overline{0, m_2 - 1}\right), \\
\frac{\partial^{i_3} u}{\partial x_3^{i_3}}\,(x_1, x_2, x_{30}, x_4) &= \varphi_{3i_3}\,(x_1, x_2, x_4)\,, \quad \left(i_3 = \overline{0, m_3 - 1}\right), \\
\frac{\partial^{i_4} u}{\partial x_4^{i_4}}\,(x_1, x_2, x_3, x_{40}) &= \varphi_{4i_4}\,(x_1, x_2, x_3)\,, \quad \left(i_4 = \overline{0, m_4 - 1}\right),
\end{aligned}
\tag{3}
$$

where

$$\varphi_{1i_1} \in C^{m_2+m_3+m_4}\left(\overline{X_1}\right), \quad \varphi_{2i_2} \in C^{m_1+m_3+m_4}\left(\overline{X_2}\right),$$
$$\varphi_{3i_3} \in C^{m_1+m_2+m_4}\left(\overline{X_3}\right), \quad \varphi_{4i_4} \in C^{m_1+m_2+m_3}\left(\overline{X_4}\right),$$

and the boundary values in (3) are coordinated on the edges of D as follows:

$$\varphi_{10}\,(x_{20}, x_3, x_4) = \varphi_{20}\,(x_{10}, x_3, x_4)\,, \quad \varphi_{10}\,(x_2, x_{30}, x_4) = \varphi_{30}\,(x_{10}, x_2, x_4)\,,$$
$$\varphi_{10}\,(x_2, x_3, x_{40}) = \varphi_{40}\,(x_{10}, x_2, x_3)\,;$$
$$\varphi_{20}\,(x_1, x_{30}, x_4) = \varphi_{30}\,(x_1, x_{20}, x_4)\,, \quad \varphi_{20}\,(x_1, x_3, x_{40}) = \varphi_{40}\,(x_1, x_{20}, x_3)\,,$$
$$\varphi_{30}\,(x_1, x_2, x_{40}) = \varphi_{40}\,(x_1, x_2, x_{30})\,,$$

and the coordinated values are supposed to be differentiable.

Equation (1) can be treated as an analog of the two-dimensional pseudo-parabolic equation in [1]. However, a version of the Riemann's method to be applied here differs essentially from that used in [1]. One can note that equations considered in this paper have modifications frequently used in applications. One can consider, for example, the Boussinesq-Love equation in the theory of oscillations (see [1], formula (20)), the Aller equation in [2], p.261, applied in the mathematical simulation of moisture absorption by roots of plants.

1. First of all we consider a special case, where the derivatives with respect to the variables x_2, x_3, x_4 are not present:

$$\frac{\partial^m u}{\partial x_1^m} + \sum_{i=0}^{m-1} a_i\,(x_1, y)\, \frac{\partial^i u}{\partial x_1^i} = 0. \qquad (4)$$

Here and on in this section we denote by y the set of coordinates x_2, x_3, x_4. One item of the investigation of (4) will be also used in the general case (see Section 2). In addition, it seems to be of interest to compare the investigation of (4) with the study of the following ordinary differential equation:

$$\frac{d^m u}{dx^m} + \sum_{i=0}^{m-1} a_i\,(x)\, \frac{d^i u}{dx^i} = 0. \qquad (5)$$

Obviously, (4) and (5) possess a similar structure; however, the general theory of the second case (see [3], p.145) depends essentially on the theorem stating that the Wronskian m of its solutions either does not vanish or is identically equal to zero. However, this theorem cannot be generalized to equation (1): one can easily see, for example, that the equation $u_{xx} - 3yu_x + 2y^2 u = 0$ possesses the solutions $exp\,(xy)$, $exp\,(2xy)$. At the same time, its Wronskian $w = y \cdot exp\,(3xy)$ vanishes for $y = 0$ and differs from zero in any other point of a bounded domain. Therefore there arises a problem of construction of a theory for the solutions of (4), which will not use the above property of the Wronskian. In what follows we develop a version of the Riemann method in [4], [5].

The Goursat problem can be transformed to the following well-known *Cauchy problem*: find in D a solution of equation (1) in the class C^{m+0}, which will satisfy the following conditions:

$$\frac{\partial^i u}{\partial x_1^i}\Big|_{x_1=x_{10}} = \varphi_i\,(y) \quad (0 \le i \le m - 1) \qquad (6)$$

$$\varphi_i \in C\,(p), \quad p = [y_0, y_1]\,.$$

We assume that the coefficients a_i are continuous in $\overline{D}$ with respect to the variables x_2, x_3, x_4 and belong to the class $C^i\,(\overline{D})$ on x_1. We define the Riemann function V as a solution of the following Volterra integral equation:

$$V\,(x_1, y) + \sum_{i=0}^{m-1} (-1)^{m-i} \int_{\xi}^{x_1} \frac{(x_1 - t)^{m-i-1}}{(m - i - 1)!} a_i\,(t, y)\, V\,(t, y)\, dt = 1, \qquad (7)$$

which is known to exist and be unique. In addition, from (7) it follows that V is a solution of the equation conjugate to (4):

$$L^{*}\left(V\right) = \sum_{i=0}^{m} \left(-1\right)^{m-i} \frac{\partial^{i}\left(a_{i}V\right)}{\partial x_{1}^{i}} \equiv 0, \quad a_{m} = 1. \tag{8}$$

To indicate that V depends not only on x_{1}, y but also on ξ, we will write this function as $R\left(x_{1}, y, \xi\right)$. The next identities (which will be of use later) easily follow from (7):

$$\sum_{\alpha=i}^{m} \left(-1\right)^{m-\alpha} \frac{\partial^{\alpha-i}\left(a_{\alpha}R\right)}{\partial x_{1}^{\alpha-i}}\left(x_{1}, y, x_{1}\right) \equiv 0, \quad \left(i \leq m-1\right) \tag{9}$$

$$R\left(x_{1}, y, x_{1}\right) \equiv 1.$$

The following identity takes place for any function u from the class $C^{m+0}\left(D\right)$:

$$\frac{\partial^{m}\left[uR\right]}{\partial x_{1}^{m}} \equiv RL\left(u\right) + \sum_{i=1}^{m-1} \left(-1\right)^{m-i+1} \frac{\partial^{i}}{\partial x_{1}^{i}} \left[u \sum_{\alpha=i}^{m} \left(-1\right)^{m-\alpha} \frac{\partial^{\alpha-i}\left[a_{\alpha}R\right]}{\partial x_{1}^{\alpha-i}}\right]$$

$$+ \sum_{i=0}^{m} \sum_{b=0}^{i} K_{ib} \frac{\partial^{b}u}{\partial x_{1}^{b}} \cdot \frac{\partial^{i-b}\left[a_{i}R\right]}{\partial x_{1}^{i-b}}, \tag{10}$$

where a_{i} depend on $\left(x_{1}, y\right)$, while R and its derivatives depend on $\left(x_{1}, y, \xi\right)$. In this situation, $K_{ib} = \sum_{\alpha=b}^{i} \left(-1\right)^{i-\alpha} C_{\alpha}^{b} - M_{ib}$, $M_{ib} = 1$ if $b = i$, and $M_{ib} = 0$ otherwise. To prove (10) rewrite this identity as follows:

$$\frac{\partial^{m}\left[uR\right]}{\partial x_{1}^{m}} = R \cdot \sum_{i=0}^{m} a_{i} \cdot \frac{\partial^{i}u}{\partial x_{1}^{i}} + \sum_{i=1}^{m-1} \left(-1\right)^{i-1} \frac{\partial^{i}}{\partial x_{1}^{i}} \left[u \cdot \sum_{\alpha=i}^{m} \left(-1\right)^{\alpha} \frac{\partial^{\alpha-i}\left[a_{\alpha}R\right]}{\partial x_{1}^{\alpha-i}}\right]$$

$$+ \sum_{i=0}^{m} \sum_{b=0}^{i} \left(-1\right)^{i-b} \frac{\partial^{b}}{\partial x_{1}^{b}} \left[u \cdot \frac{\partial^{i-b}\left(a_{i}R\right)}{\partial x_{1}^{i-b}}\right] - \sum_{i=0}^{m} \frac{\partial^{i}u}{\partial x_{1}^{i}} \cdot a_{i}R.$$

In the right-hand side the first and last addends cancel each other. Transfer the addend from the left-hand side to the right-hand side and add to the second term:

$$\sum_{i=1}^{m} \sum_{\alpha=i}^{m} \left(-1\right)^{i-\alpha+1} \frac{\partial^{i}}{\partial x_{1}^{i}} \left[u \cdot \frac{\partial^{\alpha-i}\left[a_{\alpha}R\right]}{\partial x_{1}^{\alpha-i}}\right]$$

$$+ \sum_{i=0}^{m} \sum_{b=0}^{i} \left(-1\right)^{i-b} \frac{\partial^{b}}{\partial x_{1}^{b}} \left[u \cdot \frac{\partial^{i-b}\left(a_{i}R\right)}{\partial x_{1}^{i-b}}\right] = 0.$$

By using the equality

$$\sum_{b=0}^{i} \sum_{\alpha=b}^{i} C_{\alpha}^{b} \frac{\partial^{b} u}{\partial x_1^{b}} \cdot \frac{\partial^{i-b} (a_i R)}{\partial x_1^{i-b}} = \sum_{\alpha=0}^{i} \sum_{b=0}^{\alpha} C_{b}^{\alpha} \frac{\partial^{b} u}{\partial x_1^{b}} \cdot \frac{\partial^{i-b} (a_i R)}{\partial x_1^{i-b}},$$

we see that identity (10) holds.

To construct the solution of the problem we represent (10) in another way:

$$\frac{\partial^{m} [uR]}{\partial x_1^{m}} \equiv RL(u) + \sum_{i=1}^{m-1} (-1)^{m-i+1} \frac{\partial^{i}}{\partial x_1^{i}} [u \sum_{\alpha=i}^{m} (-1)^{m-\alpha} \frac{\partial^{\alpha-i} [a_\alpha R]}{\partial x_1^{\alpha-i}}]$$
$$+ \sum_{i=2}^{m} \sum_{b=0}^{i} \frac{\partial^{b+1} u}{\partial x_1^{b+1}} \left\{ C_{i-1-b}^{i-2-2b} \frac{\partial^{i-2-2b} (u)}{\partial x_1^{i-2-2b}} \cdot \frac{\partial^{b+1} (a_i R)}{\partial x_1^{b+1}} \right\}. \quad (11)$$

In order to verify that (11) is valid, one can use the following equality:

$$\sum_{b=0}^{i} \left[\sum_{\alpha=b}^{i} (-1)^{i-\alpha} C_{\alpha}^{b} - M_{ib} \right] \frac{\partial^{b} u}{\partial x_1^{b}} \cdot \frac{\partial^{i-b} (a_i R)}{\partial x_1^{i-b}}$$
$$= \sum_{b=0}^{i} \left[\sum_{k=0}^{b/2} C_{\frac{i}{2}-\left(\mathrm{int}\left(\frac{b}{2}\right)-k\right)}^{2k+1} \cdot C_{\frac{i}{2}+\left(\mathrm{int}\left(\frac{b}{2}\right)-k\right)}^{b-(2k+1)} - M_{ib} \right] \frac{\partial^{b} u}{\partial x_1^{b}} \cdot \frac{\partial^{i-b} (a_i R)}{\partial x_1^{i-b}}. \quad (12)$$

The latter takes place if b is odd and i is even. If both i and b are odd, then the right-hand side of (12) can be replaced with the following expression

$$\sum_{b=0}^{i} \left[\sum_{k=0}^{\mathrm{int}(b/2)} C_{\frac{i+1}{2}-\left(\mathrm{int}\left(\frac{b}{2}\right)-k\right)}^{2k+1} \cdot C_{\frac{i-1}{2}+\left(\mathrm{int}\left(\frac{b}{2}\right)-k\right)}^{b-(2k+1)} - M_{ib} \right] \frac{\partial^{b} u}{\partial x_1^{b}}$$
$$\cdot \frac{\partial^{i-b} (a_i R)}{\partial x_1^{i-b}}. \quad (13)$$

For b even, instead of $2k+1$ one must take $2k$. Relations (12) and (13) can be verified straightforwardly. By changing the summation order in (12), we obtain the last addend in (11). Now, by taking in (11) x_1 equal to ξ and denoting x_1 by ξ, we calculate the integral in $x_{10} < \xi < x_{11}$ and

take into account identities (9). We then arrive at the following:

$$\frac{\partial^{m-1}u}{\partial x_1^{m-1}}(x,y) = \int\limits_{x_{10}}^{x_1} (R \cdot L(u))(\xi,y)\,d\xi -$$

$$- \sum_{i=1}^{m} (-1)^{i-1} \sum_{\alpha=i}^{m} (-1)^{\alpha} \left(\frac{\partial^{i-1}u}{\partial x_1^{i-1}} \cdot \frac{\partial^{\alpha-i}(a_\alpha R)}{\partial x_1^{\alpha-i}} \right)(x_{10},y,x_1). \quad (14)$$

Obtain the required solution by integrating (14) $m-1$ times with respect to x_1 between the limits x_{10} to x_1 and taking into account (6):

$$u(x_1,y) = \varphi_0(y) + \sum_{i=1}^{m-2} \varphi_i(y) \int\limits_{x_{10}}^{x_1} \frac{(x_1-t)^{i-1}}{(i-1)!}dt - \sum_{i=1}^{m} (-1)^{i-1} \varphi_{i-1}(y)$$

$$\cdot \sum_{\alpha=i}^{m} (-1)^{\alpha} \cdot \int\limits_{x_{10}}^{x_1} \frac{(x_1-t)^{m-2}}{(m-2)!} \cdot \frac{\partial^{\alpha-i}(a_\alpha R(x_{10},y,t))}{\partial x_1^{\alpha-i}}dt. \quad (15)$$

As in [6], p. 66, we can assume that for arbitrary $\varphi_i(y)$ the formula (15) supplies the general presentation of the solutions of equation (4).

With $u = u(x)$ and $a_i = a_i(x)$ equation (4) turns into an ordinary differential equation (5). Obviously, formula (15) represents a solution for (5), moreover, here $\varphi_i(y)$ are constants and both the coefficients a_i and the Riemann function do not depend on y. Thus, formula (15) contains the classical result; moreover, it is written there in a closed form which means that the classical theory indicates a method for solving, while such a solution is represented here by a formula.

Now assume $a_i = a_i(y)$, i.e., here we deal with a certain generalization of ordinary differential equations with constant coefficients. We will consider only a particular case with $m = 2$. The formula (15) then takes

the form

$$u(x,y) = \varphi_0(y) \cdot \left\{ 1 + \int_{x_0}^{x} [R_\xi(x_0,y,\xi) - a_1(y) R(x_0,y,\xi)] \, d\xi \right\}$$

$$+ \varphi_1(y) \int_{x_0}^{x} R(x_0,y,\xi) \, d\xi$$

$$= \varphi_0(y) \cdot \left\{ 1 - \int_{x_0}^{x} \int_{x_0}^{t} a_0(y) R(x_0,y,t_1) \, dt_1 dt \right\}$$

$$+ \varphi_1(y) \int_{x_0}^{x} R(x_0,y,\xi) \, d\xi. \quad (16)$$

Find the Riemann function whose equation in this case is as follows:

$$V(x,y) = \sum_{k=0}^{\infty} V_k(x,y),$$

where

$$V_0 \equiv 1, V_1(x,y) = \int_{x_0}^{x} H(x,t,y) \, dt, V_2(x,y) = \int_{x_0}^{x} H(x,t,y) V_1(t,y) \, dt, \ldots,$$

$$V_k(x,y) = \int_{x_0}^{x} H(x,t,y) V_{k-1}(t,y) \, dt, \ldots,$$

and besides $H(x,t,y) = a_1(y) - (x-t) a_0(y)$. We thus have

$$R(x,y,x_0) = V(x,y) = \sum_{n=0}^{\infty} \sum_{k=0}^{n} (-1)^k C_n^k a_1^{n-k}(y) a_0^k(y) \frac{(x-x_0)^{n+k}}{(n+k)!}.$$

Put $a_1(y) = \lambda_1(y) + \lambda_2(y)$, $a_0(y) = \lambda_1(y) \cdot \lambda_2(y)$. For the case with $\lambda_1(y) \neq \lambda_2(y)$, we obtain

$$R(x,y,x_0) = \frac{\lambda_2(y) e^{\lambda_2(y)(x-x_0)} - \lambda_1(y) e^{\lambda_1(y)(x-x_0)}}{\lambda_2(y) - \lambda_1(y)},$$

where $\lambda_{1;2}(y) = \frac{a_1(y) \mp \sqrt{a_1^2(y) - 4a_0(y)}}{2}$. By interchanging x and x_0, we obtain

$$R(x_0,y,x) = \frac{-\lambda_2(y) e^{-\lambda_2(y)(x-x_0)} + \lambda_1(y) e^{-\lambda_1(y)(x-x_0)}}{-\lambda_2(y) + \lambda_1(y)}.$$

Substitute the result into (16). One can easily see that

$$u(x, y) = C_1(y) \cdot e^{-\lambda_1(y)x} + C_2(y) \cdot e^{-\lambda_2(y)x},$$

where

$$C_1(y) = \frac{-\varphi_0(y)\lambda_2(y) - \varphi_1(y)}{(\lambda_1(y) - \lambda_2(y)) e^{-\lambda_1(y)x_0}}, \quad C_2(y) = \frac{-\varphi_0(y)\lambda_1(y) - \varphi_1(y)}{(\lambda_2(y) - \lambda_1(y)) e^{-\lambda_2(y)x_0}}.$$

Now set $\lambda_1(y) = \lambda_2(y)$. Then the Riemann function is

$$R(x, y, x_0) = e^{\lambda_1(y)(x-x_0)} [1 + (x - x_0)\lambda_1(y)].$$

We again interchange x and x_0 and calculate the respective terms. We thus obtain

$$u(x, y) = C_1(y) \cdot e^{-\lambda_1(y)x} + C_2(y) \cdot x \cdot e^{-\lambda_1(y)x},$$

where

$$C_1(y) = (\varphi_0(y)[-x_0 \cdot \lambda_1(y) + 1] - \varphi_1(y)x_0) e^{\lambda_1 x_0},$$
$$C_2(y) = (\varphi_0(y)\lambda_1(y) + \varphi_1(y)) e^{\lambda_1 x_0}.$$

Let us write the solution in the case of conjugate complex roots $\lambda_1(y)$, $\lambda_2(y)$ of the characteristic equation. The roots can be represented as follows

$$\lambda_{1;2} = \frac{\alpha \pm \beta \cdot i}{2},$$

where $\beta \cdot i = \sqrt{a_1^2(y) - 4a_0(y)}$, $\alpha = a_1(y)$.

In this case, by the straightforward calculation yields:

$$u(x, y) = \varphi_0(y) \cdot e^{-\frac{\alpha}{2} \cdot (x-x_0)} \left[\frac{\alpha \sin\left(\frac{\beta}{2}(x - x_0)\right) + \beta \cos\left(\frac{\beta}{2}(x - x_0)\right)}{-\beta} \right]$$

$$+ \varphi_1(y) \cdot e^{-\frac{\alpha}{2} \cdot (x-x_0)} \frac{\sin\left(\frac{\beta}{2}(x - x_0)\right)}{\beta}$$

$$= e^{-\frac{\alpha}{2} \cdot (x-x_0)} \cdot \left[-\sin\left(\frac{\beta}{2}(x - x_0)\right) \left\{ \varphi_0(y)\frac{\alpha}{\beta} - \varphi_1(y) \cdot \frac{1}{\beta} \right\} \right.$$

$$\left. + \cos\left(\frac{\beta}{2}(x - x_0)\right) \cdot \alpha \cdot \varphi_0(y) \right].$$

Consequently, in contrast to the classical technique for solving ordinary differential equations with constant coefficients, the formula obtained contains simultaneously both the case of a non-multiple root of the characteristic equation and that of a multiple root, as well as the case of conjugate roots.

Consider also a non-homogeneous equation

$$\frac{\partial^m u}{\partial x^m} + \sum_{i=0}^{m-1} a_i(x, y) \frac{\partial^i u}{\partial x^i} = f(x, y).$$

Its solution has the form

$$u(x, y) = \varphi_0(y) + \sum_{i=1}^{m-2} \varphi_i(y) \int_{x_0}^{x} \frac{(x-t)^{i-1}}{(i-1)!} dt$$

$$- \sum_{i=1}^{m} (-1)^{i-1} \varphi_{i-1}(y) \cdot \sum_{\alpha=i}^{m} (-1)^{\alpha} \int_{x_0}^{x} \frac{(x-t)^{m-2}}{(m-2)!} \cdot \frac{\partial^{\alpha-i}(a_\alpha R(x_0, y, t))}{\partial x^{\alpha-i}} dt$$

$$+ \int_{x_0}^{x} \frac{(x-t)^{m-1}}{(m-1)!} R(t, y, x) dt.$$

Denote by $K(t, y, x) \frac{(x-t)^{m-1}}{(m-1)!} R(t, y, x)$. One can then easily verify that we get a particular solution obtained in ([7], p. 121), by the Cauchy method. Since $K(t, y, t) = K'_x(t, y, t) = \ldots = K_x^{(m-2)}(t, y, t) = 0$, $K_x^{(m-1)}(t, y, t) = 1$ and $u_*(x, y) = \int_{x_0}^{x} K(t, y, x) f(t, y) dt$ is a particular solution of the equation, satisfying the zero initial conditions $\varphi_0(y) \equiv \varphi_1(y) \equiv \ldots \equiv \varphi_{n-1}(y) \equiv 0$. In addition, $K(t, y, x)$ is written in the explicit form.

2. In this section us consider a general case. For the sake of simplicity we use following notation introduced in [1]: $D_t^k \varphi \equiv \partial^k \varphi / \partial t^k$ for $k = 1, 2, ..$ and $D_{t_0}^k \varphi \equiv \int_{t_0}^{t} \frac{(t-\tau)^{-k-1} \varphi(\tau)}{(-k-1)!} d\tau$ if $k = -1, -2, ..$ and D_t^0 being operator of identical transform.

By the Riemann function $R(x_1, x_2, x_3, x_4; \xi_1, \xi_2, \xi_3, \xi_4)$ we call any solution of the integral equation

$$\sum_{i_1=0}^{m_1} \sum_{i_2=0}^{m_2} \sum_{i_3=0}^{m_3} \sum_{i_4=0}^{m_4} (-1)^{\sum_{\alpha=1}^{4}(m_\alpha - i_\alpha)} \prod_{\alpha=1}^{4} D_{x_\alpha}^{i_\alpha - m_\alpha}(a_{i_1 i_2 i_3 i_4} R) = 1, \quad (17)$$

which both exists and is unique (see [8], p.180). From (17) it follows that, if we consider the first four arguments of the equation, then R is a solution of the following equation conjugate to (1):

$$L^*(V) = \sum_{i_1=0}^{m_1} \sum_{i_2=0}^{m_2} \sum_{i_3=0}^{m_3} \sum_{i_4=0}^{m_4} (-1)^{\sum_{\alpha=1}^{4}(m_\alpha - i_\alpha)} \prod_{\alpha=1}^{4} D_{x_\alpha}^{i_\alpha}(a_{i_1 i_2 i_3 i_4} V) = 0. \quad (18)$$

In what follows we need the next proposition.

For any function u in $C^{\sum\limits_{\alpha=1}^{4} m_\alpha}(D)$, the following identity holds:

$$L(u) + \sum_{i_1=0}^{m_1} \sum_{i_2=0}^{m_2} \sum_{i_3=0}^{m_3} \sum_{\substack{i_4=0 \\ i_1+i_2+i_3+i_4>0}}^{m_4}$$

$$(-1)^{\sum\limits_{k=1}^{4} i_k - 1} \prod_{k=1}^{4} D_{x_k}^{i_k} [u \sum_{\alpha_1=i_1}^{m_1} \sum_{\alpha_2=i_2}^{m_2} \sum_{\alpha_3=i_3}^{m_3} \sum_{\alpha_4=i_4}^{m_4}$$

$$(-1)^{\sum\limits_{k=1}^{4} \alpha_k - 1} \cdot \prod_{k=1}^{4} D_{x_k}^{\alpha_k - i_k} (a_{\alpha_1\alpha_2\alpha_3\alpha_4} R)]$$

$$+ \sum_{i_1=0}^{m_1} \sum_{i_2=0}^{m_2} \sum_{i_3=0}^{m_3} \sum_{i_4=0}^{m_4} \sum_{b_1=0}^{i_1} \sum_{b_2=0}^{i_2} \sum_{b_3=0}^{i_3} \sum_{b_4=0}^{i_4} K_{i_1 i_2 i_3 i_4 b_1 b_2 b_3 b_4}$$

$$\cdot \prod_{k=1}^{4} D_{x_k}^{b_k} (u) \prod_{k=1}^{4} D_{x_k}^{i_k - b_k} (a_{\alpha_1\alpha_2\alpha_3\alpha_4} R) \equiv 0, \quad (19)$$

where

$$K_{i_1 i_2 i_3 i_4 b_1 b_2 b_3 b_4} =$$

$$\sum_{\alpha_1=b_1}^{i_1} (-1)^{i_1-\alpha_1} C_{\alpha_1}^{b_1} \sum_{\alpha_2=b_2}^{i_2} (-1)^{i_2-\alpha_2} C_{\alpha_2}^{b_2} \sum_{\alpha_3=b_3}^{i_3} (-1)^{i_3-\alpha_3} C_{\alpha_3}^{b_3}$$

$$\sum_{\alpha_4=b_4}^{i_4} (-1)^{i_4-\alpha_4} C_{\alpha_4}^{b_4} - M_{b_1 b_2 b_3 b_4},$$

and $M_{b_1 b_2 b_3 b_4} = 1$ if $b_1 = i_1$, $b_2 = i_2$, $b_3 = i_3$, $b_4 = i_4$, otherwise $M_{b_1 b_2 b_3 b_4} = 0$. Here $a_{i_1 i_2 i_3 i_4}$ depend on (x_1, x_2, x_3, x_4), while R and its derivatives on $(x_1, x_2, x_3, x_4, \xi_1, \xi_2, \xi_3, \xi_4)$.

In order to prove this rewrite the last addend in (19) and note that R is a solution of equation (18). Next, present this addend as follows:

$$\sum_{i_1=2}^{m_1} \sum_{i_2=0}^{m_2} \sum_{i_3=0}^{m_3} \sum_{i_4=0}^{m_4} \sum_{b_1=0}^{\frac{i_1}{2}-1}$$

$$D_{x_1}^{b_1+1} \left\{ C_{i_1-1-b_1}^{i_1-2-2b_1} D_{x_1}^{i_1-2-2b_1} \prod_{\alpha=2}^{4} D_{x_\alpha}^{i_\alpha} u \cdot D_{x_1}^{b_1+1} (a_{i_1 i_2 i_3 i_4} R) \right\}$$

$$(20)$$

$$+ \sum_{i_1=0}^{m_1} \sum_{i_2=2}^{m_2} \sum_{i_3=0}^{m_3} \sum_{i_4=0}^{m_4} \sum_{b_2=0}^{\frac{i_2}{2}-1}$$

$$D_{x_2}^{b_2+1} \left\{ C_{i_2-1-b_2}^{i_2-2-2b_2} D_{x_2}^{i_2-2-2b_2} \prod_{\alpha=1,\alpha\neq2}^{4} D_{x_\alpha}^{i_\alpha} u \cdot D_{x_2}^{b_2+1} \left(a_{i_1 i_2 i_3 i_4} R \right) \right\}$$

$$+ \sum_{i_1=0}^{m_1} \sum_{i_2=0}^{m_2} \sum_{i_3=2}^{m_3} \sum_{i_4=0}^{m_4} \sum_{b_3=0}^{\frac{i_3}{2}-1}$$

$$D_{x_3}^{b_3+1} \left\{ C_{i_3-1-b_3}^{i_3-2-2b_3} D_{x_3}^{i_3-2-2b_3} \prod_{\alpha=1,\alpha\neq3}^{4} D_{x_\alpha}^{i_\alpha} u \cdot D_{x_3}^{b_3+1} \left(a_{i_1 i_2 i_3 i_4} R \right) \right\}$$

$$+ \sum_{i_1=0}^{m_1} \sum_{i_2=0}^{m_2} \sum_{i_3=0}^{m_3} \sum_{i_4=2}^{m_4} \sum_{b_4=0}^{\frac{i_4}{2}-1}$$

$$D_{x_4}^{b_4+1} \left\{ C_{i_4-1-b_4}^{i_4-2-2b_4} D_{x_4}^{i_4-2-2b_4} \prod_{\alpha=1}^{3} D_{x_\alpha}^{i_\alpha} u \cdot D_{x_4}^{b_4+1} \left(a_{i_1 i_2 i_3 i_4} R \right) \right\}$$

$$+ \sum_{i_1=2}^{m_1} \sum_{i_2=2}^{m_2} \sum_{i_3=0}^{m_3} \sum_{i_4=0}^{m_4} \sum_{b_1=0}^{\frac{i_1}{2}-1} \sum_{b_2=0}^{\frac{i_2}{2}-1} D_{x_1}^{b_1+1} D_{x_2}^{b_2+1}$$

$$+ \sum_{i_1=2}^{m_1} \sum_{i_2=0}^{m_2} \sum_{i_3=2}^{m_3} \sum_{i_4=0}^{m_4} \sum_{b_1=0}^{\frac{i_1}{2}-1} \sum_{b_3=0}^{\frac{i_3}{2}-1} D_{x_1}^{b_1+1} D_{x_3}^{b_3+1}$$

$$\cdot \left\{ C_{i_1-1-b_1}^{i_1-2-2b_1} C_{i_3-1-b_3}^{i_3-2-2b_3} D_{x_1}^{i_1-2-2b_1} D_{x_2}^{i_2} D_{x_3}^{i_3-2-2b_3} D_{x_4}^{i_4} u \cdot D_{x_1}^{b_1+1} D_{x_3}^{b_3+1} \left(a_{i_1 i_2 i_3 i_4} R \right) \right\}$$

$$+ \sum_{i_1=2}^{m_1} \sum_{i_2=0}^{m_2} \sum_{i_3=0}^{m_3} \sum_{i_4=2}^{m_4} \sum_{b_1=0}^{\frac{i_1}{2}-1} \sum_{b_4=0}^{\frac{i_4}{2}-1} D_{x_1}^{b_1+1} D_{x_4}^{b_4+1}$$

$$\cdot \left\{ C_{i_1-1-b_1}^{i_1-2-2b_1} C_{i_4-1-b_4}^{i_4-2-2b_4} D_{x_1}^{i_1-2-2b_1} D_{x_2}^{i_2} D_{x_3}^{i_3} D_{x_4}^{i_4-2-2b_4} u \right.$$

$$\cdot \left. D_{x_1}^{b_1+1} D_{x_4}^{b_4+1} \left(a_{i_1 i_2 i_3 nnnn i_4} R \right) \right\}$$

$$+ \sum_{i_1=0}^{m_1} \sum_{i_2=2}^{m_2} \sum_{i_3=2}^{m_3} \sum_{i_4=0}^{m_4} \sum_{b_2=0}^{\frac{i_2}{2}-1} \sum_{b_3=0}^{\frac{i_3}{2}-1} D_{x_2}^{b_2+1} D_{x_3}^{b_3+1}$$

$$\cdot \left\{ C_{i_2-1-b_2}^{i_2-2-2b_2} C_{i_3-1-b_3}^{i_3-2-2b_3} D_{x_1}^{i_1} D_{x_2}^{i_2-2-2b_2} D_{x_3}^{i_3-2-2b_3} D_{x_4}^{i_4} u \cdot D_{x_2}^{b_2+1} D_{x_3}^{b_3+1} \left(a_{i_1 i_2 i_3 i_4} R \right) \right\}$$

$$+ \sum_{i_1=0}^{m_1} \sum_{i_2=2}^{m_2} \sum_{i_3=0}^{m_3} \sum_{i_4=2}^{m_4} \sum_{b_2=0}^{\frac{i_2}{2}-1} \sum_{b_4=0}^{\frac{i_4}{2}-1} D_{x_2}^{b_2+1} D_{x_4}^{b_4+1}$$

$$\cdot \left\{ C_{i_2-1-b_2}^{i_2-2-2b_2} C_{i_4-1-b_4}^{i_4-2-2b_4} D_{x_1}^{i_1} D_{x_2}^{i_2-2-2b_2} D_{x_3}^{i_3} D_{x_4}^{i_4-2-2b_4} u \cdot D_{x_2}^{b_2+1} D_{x_4}^{b_4+1} \left(a_{i_1 i_2 i_3 i_4} R \right) \right\}$$

$$+ \sum_{i_1=0}^{m_1} \sum_{i_2=0}^{m_2} \sum_{i_3=2}^{m_3} \sum_{i_4=2}^{m_4} \sum_{b_3=0}^{\frac{i_3}{2}-1} \sum_{b_4=0}^{\frac{i_4}{2}-1} D_{x_3}^{b_3+1} D_{x_4}^{b_4+1}$$

$$\cdot \left\{ C_{i_3-1-b_3}^{i_3-2-2b_3} C_{i_4-1-b_4}^{i_4-2-2b_4} D_{x_1}^{i_1} D_{x_2}^{i_2} D_{x_3}^{i_3-2-2b_3} D_{x_4}^{i_4-2-2b_4} u \cdot D_{x_3}^{b_3+1} D_{x_4}^{b_4+1} \left(a_{i_1 i_2 i_3 i_4} R \right) \right\}$$

$$+ \sum_{i_1=2}^{m_1} \sum_{i_2=2}^{m_2} \sum_{i_3=2}^{m_3} \sum_{i_4=0}^{m_4} \sum_{b_1=0}^{\frac{i_1}{2}-1} \sum_{b_2=0}^{\frac{i_2}{2}-1} \sum_{b_3=0}^{\frac{i_3}{2}-1} D_{x_1}^{b_1+1} D_{x_2}^{b_2+1} D_{x_3}^{b_3+1}$$

$$\cdot \left\{ C_{i_1-1-b_1}^{i_1-2-2b_1} C_{i_2-1-b_2}^{i_2-2-2b_2} C_{i_3-1-b_3}^{i_3-2-2b_3} D_{x_1}^{i_1-2-2b_1} D_{x_2}^{i_2-2-2b_2} D_{x_3}^{i_3-2-2b_3} D_{x_4}^{i_4} u \right.$$

$$\left. \cdot \prod_{k=1}^{3} D_{x_k}^{b_k+1} \left(a_{i_1 i_2 i_3 i_4} R \right) \right\}$$

$$+ \sum_{i_1=2}^{m_1} \sum_{i_2=2}^{m_2} \sum_{i_3=0}^{m_3} \sum_{i_4=2}^{m_4} \sum_{b_1=0}^{\frac{i_1}{2}-1} \sum_{b_2=0}^{\frac{i_2}{2}-1} \sum_{b_4=0}^{\frac{i_4}{2}-1} D_{x_1}^{b_1+1} D_{x_2}^{b_2+1} D_{x_4}^{b_4+1}$$

$$\cdot \left\{ C_{i_1-1-b_1}^{i_1-2-2b_1} C_{i_2-1-b_2}^{i_2-2-2b_2} C_{i_4-1-b_4}^{i_4-2-2b_4} D_{x_1}^{i_1-2-2b_1} D_{x_2}^{i_2-2-2b_2} D_{x_3}^{i_3} D_{x_4}^{i_4-2-2b_4} u \right.$$

$$\left. \cdot \prod_{k=1,k\neq 3}^{4} D_{x_k}^{b_k+1} \left(a_{i_1 i_2 i_3 i_4} R \right) \right\}$$

$$+ \sum_{i_1=2}^{m_1} \sum_{i_2=0}^{m_2} \sum_{i_3=2}^{m_3} \sum_{i_4=2}^{m_4} \sum_{b_1=0}^{\frac{i_1}{2}-1} \sum_{b_3=0}^{\frac{i_3}{2}-1} \sum_{b_4=0}^{\frac{i_4}{2}-1} D_{x_1}^{b_1+1} D_{x_3}^{b_3+1} D_{x_4}^{b_4+1}$$

$$\cdot \left\{ C_{i_1-1-b_1}^{i_1-2-2b_1} C_{i_3-1-b_3}^{i_3-2-2b_3} C_{i_4-1-b_4}^{i_4-2-2b_4} D_{x_1}^{i_1-2-2b_1} D_{x_2}^{i_2} D_{x_3}^{i_3-2-2b_3} D_{x_4}^{i_4-2-2b_4} u \right.$$

$$\left. \cdot \prod_{k=1,k\neq 2}^{4} D_{x_k}^{b_k+1} \left(a_{i_1 i_2 i_3 i_4} R \right) \right\}$$

$$+ \sum_{i_1=0}^{m_1} \sum_{i_2=2}^{m_2} \sum_{i_3=2}^{m_3} \sum_{i_4=2}^{m_4} \sum_{b_2=0}^{\frac{i_2}{2}-1} \sum_{b_3=0}^{\frac{i_3}{2}-1} \sum_{b_4=0}^{\frac{i_4}{2}-1} D_{x_2}^{b_2+1} D_{x_3}^{b_3+1} D_{x_4}^{b_4+1}$$

$$\cdot \left\{ C_{i_2-1-b_2}^{i_2-2-2b_2} C_{i_3-1-b_3}^{i_3-2-2b_3} C_{i_4-1-b_4}^{i_4-2-2b_4} D_{x_1}^{i_1} D_{x_2}^{i_2-2-2b_2} D_{x_3}^{i_3-2-2b_3} D_{x_4}^{i_4-2-2b_4} u \right.$$

$$\left. \cdot \prod_{k=2}^{4} D_{x_k}^{b_k+1} \left(a_{i_1 i_2 i_3 i_4} R \right) \right\}$$

$$+ \sum_{i_1=2}^{m_1} \sum_{i_2=2}^{m_2} \sum_{i_3=2}^{m_3} \sum_{i_4=2}^{m_4} \sum_{b_1=0}^{\frac{i_1}{2}-1} \sum_{b_2=0}^{\frac{i_2}{2}-1} \sum_{b_3=0}^{\frac{i_3}{2}-1} \sum_{b_4=0}^{\frac{i_4}{2}-1} D_{x_1}^{b_1+1} D_{x_2}^{b_2+1} D_{x_3}^{b_3+1} D_{x_4}^{b_4+1}$$

$$\cdot \left\{ \prod_{k=1}^{4} C_{i_k-1-b_k}^{i_k-2-2b_k} D_{x_k}^{i_k-2-2b_k} u \cdot \prod_{k=1}^{4} D_{x_k}^{b_k+1} \left(a_{i_1 i_2 i_3 i_4} R \right) \right\}.$$

To prove the above representation one can use formula (11). Add to and subtract from each factor of the form $\sum_{\alpha_1=b_1}^{i_1} (-1)^{i_1-\alpha_1} C_{\alpha_1}^{b_1}$ the corresponding addend $M_{i_1 b_1}$. Then we have

$$
K_{i_1 i_2 i_3 i_4 b_1 b_2 b_3 b_4} = \sum_{\alpha_1=b_1}^{i_1} (-1)^{i_1-\alpha_1} C_{\alpha_1}^{b_1} \sum_{\alpha_2=b_2}^{i_2} (-1)^{i_2-\alpha_2} C_{\alpha_2}^{b_2}
$$

$$
\sum_{\alpha_3=b_3}^{i_3} (-1)^{i_3-\alpha_3} C_{\alpha_3}^{b_3} \sum_{\alpha_4=\beta_4}^{i_4} (-1)^{i_4-\alpha_4} C_{\alpha_4}^{b_4} - M_{b_1 b_2 b_3 b_4} =
$$

$$
= \left[\sum_{\alpha_1=b_1}^{i_1} (-1)^{i_1-\alpha_1} C_{\alpha_1}^{b_1} \pm M_{i_1 b_1} \right] \cdot \left[\sum_{\alpha_2=b_2}^{i_2} (-1)^{i_2-\alpha_2} C_{\alpha_2}^{b_2} \pm M_{i_2 b_2} \right] \cdot
$$

$$
\cdot \left[\sum_{\alpha_3=b_3}^{i_3} (-1)^{i_3-\alpha_3} C_{\alpha_3}^{b_3} \pm M_{i_3 b_3} \right] \cdot \left[\sum_{\alpha_4=b_4}^{i_4} (-1)^{i_4-\alpha_4} C_{\alpha_4}^{b_4} \pm M_{i_4 b_4} \right]
$$

$$
- M_{b_1 b_2 b_3 b_4}.
$$

Remove the parenthesis and apply formula (11).

Moreover, from (17) one can see the next formulas:

$$
\sum_{\alpha_1=i_1}^{m_1} \sum_{\alpha_2=0}^{m_2} \sum_{\alpha_3=0}^{m_3} \sum_{\alpha_4=0}^{m_4} (-1)^{\sum_{k=1}^{4}(m_k-\alpha_k)} D_{x_1}^{\alpha_1-i_1}
$$

$$
\prod_{k=2}^{4} D_{x_k}^{\alpha_k} (a_{\alpha_1 \alpha_2 \alpha_3 \alpha_4} R)(x_1, \xi_2, \xi_3, \xi_4) \equiv 0.
$$

Let us integrate (19) with regard for (20) and the above identities. To this end in (19) we set $x_1 = \xi_1$, $x_2 = \xi_2$, $x_3 = \xi_3$, $x_4 = \xi_4$ and calculate fourfold integrals of the left- and right-hand sides of the identity within the limits $x_{10} < \xi_1 < x_1$, $x_{20} < \xi_2 < x_2$, $x_{30} < \xi_3 < x_3$, $x_{40} < \xi_4 < x_4$:

$$
\prod_{k=1}^{4} D_{x_k}^{m_k-1} (u)(x_1, x_2, x_3, x_4)
$$

$$
= \prod_{k=1}^{4} D_{x_{k0}}^{-1} (RL(u)) + \prod_{k=1}^{4} D_{x_k}^{m_k-1} [u] \cdot R(x_{10}, x_2, x_3, x_4) \tag{21}
$$

$$
+ \prod_{k=1}^{4} D_{x_k}^{m_k-1} [u] \cdot R(x_1, x_{20}, x_3, x_4) + \prod_{k=1}^{4} D_{x_k}^{m_k-1} [u] \cdot R(x_1, x_2, x_{30}, x_4)
$$

$$+ \prod_{k=1}^{4} D_{x_k}^{m_k-1} [u] \cdot R (x_1, x_2, x_3, x_{40}) - \prod_{k=1}^{4} D_{x_k}^{m_k-1} [u] \cdot R (x_{10}, x_{20}, x_3, x_4)$$

$$- \prod_{k=1}^{4} D_{x_k}^{m_k-1} [u] \cdot R (x_{10}, x_2, x_{30}, x_4) - \prod_{k=1}^{4} D_{x_k}^{m_k-1} [u] \cdot R (x_{10}, x_2, x_3, x_{40})$$

$$- \prod_{k=1}^{4} D_{x_k}^{m_k-1} [u] \cdot R (x_1, x_{20}, x_{30}, x_4) - \prod_{k=1}^{4} D_{x_k}^{m_k-1} [u] \cdot R (x_1, x_{20}, x_3, x_{40})$$

$$- \prod_{k=1}^{4} D_{x_k}^{m_k-1} [u] \cdot R (x_1, x_2, x_{30}, x_{40}) + \prod_{k=1}^{4} D_{x_k}^{m_k-1} [u] \cdot R (x_{10}, x_{20}, x_{30}, x_4)$$

$$+ \prod_{k=1}^{4} D_{x_k}^{m_k-1} [u] \cdot R (x_{10}, x_{20}, x_3, x_{40}) + \prod_{k=1}^{4} D_{x_k}^{m_k-1} [u] \cdot R (x_{10}, x_2, x_{30}, x_{40})$$

$$+ \prod_{k=1}^{4} D_{x_k}^{m_k-1} [u] \cdot R (x_1, x_{20}, x_{30}, x_{40}) - \prod_{k=1}^{4} D_{x_k}^{m_k-1} [u] \cdot R (x_{10}, x_{20}, x_{30}, x_{40})$$

$$- \sum_{i_1=1}^{m_1} \sum_{i_2=1}^{m_2} \sum_{i_3=1}^{m_3} \sum_{i_4=1}^{m_4} (-1)^{i_1+i_2+i_3+i_4-1} \prod_{k=1}^{4} D_{x_k}^{i_k-1} (u (x_{10}, x_2, x_3, x_4))$$

$$\cdot \sum_{\alpha_1=i_1}^{m_1} \sum_{\alpha_2=i_2}^{m_2} \sum_{\alpha_3=i_3}^{m_3} \sum_{\alpha_4=i_4}^{m_4} (-1)^{\alpha_1+\alpha_2+\alpha_3+\alpha_4} \prod_{k=1}^{4} D_{x_k}^{\alpha_k} (a_{\alpha_1\alpha_2\alpha_3\alpha_4} (x_{10}, x_2, x_3, x_4)$$

$$\cdot R (x_{10}, x_2, x_3, x_4))$$

$$- \sum_{i_1=1}^{m_1} \sum_{i_2=1}^{m_2} \sum_{i_3=1}^{m_3} \sum_{i_4=1}^{m_4} (-1)^{i_1+i_2+i_3+i_4-1} \prod_{k=1}^{4} D_{x_k}^{i_k-1} (u (x_1, x_{20}, x_3, x_4))$$

$$\cdot \sum_{\alpha_1=i_1}^{m_1} \sum_{\alpha_2=i_2}^{m_2} \sum_{\alpha_3=i_3}^{m_3} \sum_{\alpha_4=i_4}^{m_4} (-1)^{\alpha_1+\alpha_2+\alpha_3+\alpha_4} \prod_{k=1}^{4} D_{x_k}^{\alpha_k} (a_{\alpha_1\alpha_2\alpha_3\alpha_4} (x_1, x_{20}, x_3, x_4)$$

$$\cdot R (x_1, x_2, x_3, x_4)) -$$

$$- \sum_{i_1=1}^{m_1} \sum_{i_2=1}^{m_2} \sum_{i_3=1}^{m_3} \sum_{i_4=1}^{m_4} (-1)^{i_1+i_2+i_3+i_4-1} \prod_{k=1}^{4} D_{x_k}^{i_k-1} (u (x_1, x_2, x_{30}, x_4)) \cdot$$

$$\cdot \sum_{\alpha_1=i_1}^{m_1} \sum_{\alpha_2=i_2}^{m_2} \sum_{\alpha_3=i_3}^{m_3} \sum_{\alpha_4=i_4}^{m_4} (-1)^{\alpha_1+\alpha_2+\alpha_3+\alpha_4} \prod_{k=1}^{4} D_{x_k}^{\alpha_k} (a_{\alpha_1\alpha_2\alpha_3\alpha_4} (x_1, x_2, x_{30}, x_4)$$

$$\cdot R (x_1, x_2, x_{30}, x_4))$$

$$- \sum_{i_1=1}^{m_1} \sum_{i_2=1}^{m_2} \sum_{i_3=1}^{m_3} \sum_{i_4=1}^{m_4} (-1)^{i_1+i_2+i_3+i_4-1} \prod_{k=1}^{4} D_{x_k}^{i_k-1} (u (x_1, x_2, x_3, x_{40}))$$

$$\cdot \sum_{\alpha_1=i_1}^{m_1} \sum_{\alpha_2=i_2}^{m_2} \sum_{\alpha_3=i_3}^{m_3} \sum_{\alpha_4=i_4}^{m_4} (-1)^{\alpha_1+\alpha_2+\alpha_3+\alpha_4} \prod_{k=1}^{4} D_{x_k}^{\alpha_k} \left(a_{\alpha_1\alpha_2\alpha_3\alpha_4}\left(x_1, x_2, x_3, x_{40}\right)\right.$$

$$\left.\cdot R\left(x_1, x_2, x_3, x_{40}\right)\right)$$

$$+ \sum_{i_1=1}^{m_1} \sum_{i_2=1}^{m_2} \sum_{i_3=1}^{m_3} \sum_{i_4=1}^{m_4} (-1)^{i_1+i_2+i_3+i_4-1} \prod_{k=1}^{4} D_{x_k}^{i_k-1} \left(u\left(x_{10}, x_{20}, x_3, x_4\right)\right)$$

$$\cdot \sum_{\alpha_1=i_1}^{m_1} \sum_{\alpha_2=i_2}^{m_2} \sum_{\alpha_3=i_3}^{m_3} \sum_{\alpha_4=i_4}^{m_4} (-1)^{\alpha_1+\alpha_2+\alpha_3+\alpha_4} \prod_{k=1}^{4} D_{x_k}^{\alpha_k} \left(a_{\alpha_1\alpha_2\alpha_3\alpha_4}\left(x_{10}, x_{20}, x_3, x_4\right)\right.$$

$$\left.\cdot R\left(x_{10}, x_{20}, x_3, x_4\right)\right)$$

$$+ \sum_{i_1=1}^{m_1} \sum_{i_2=1}^{m_2} \sum_{i_3=1}^{m_3} \sum_{i_4=1}^{m_4} (-1)^{i_1+i_2+i_3+i_4-1} \prod_{k=1}^{4} D_{x_k}^{i_k-1} \left(u\left(x_{10}, x_2, x_{30}, x_4\right)\right)$$

$$\cdot \sum_{\alpha_1=i_1}^{m_1} \sum_{\alpha_2=i_2}^{m_2} \sum_{\alpha_3=i_3}^{m_3} \sum_{\alpha_4=i_4}^{m_4} (-1)^{\alpha_1+\alpha_2+\alpha_3+\alpha_4} \prod_{k=1}^{4} D_{x_k}^{\alpha_k} \left(a_{\alpha_1\alpha_2\alpha_3\alpha_4}\left(x_{10}, x_2, x_{30}, x_4\right)\right.$$

$$\left.\cdot R\left(x_{10}, x_2, x_{30}, x_4\right)\right)$$

$$+ \sum_{i_1=1}^{m_1} \sum_{i_2=1}^{m_2} \sum_{i_3=1}^{m_3} \sum_{i_4=1}^{m_4} (-1)^{i_1+i_2+i_3+i_4-1} \prod_{k=1}^{4} D_{x_k}^{i_k-1} \left(u\left(x_{10}, x_2, x_3, x_{40}\right)\right)$$

$$\cdot \sum_{\alpha_1=i_1}^{m_1} \sum_{\alpha_2=i_2}^{m_2} \sum_{\alpha_3=i_3}^{m_3} \sum_{\alpha_4=i_4}^{m_4} (-1)^{\alpha_1+\alpha_2+\alpha_3+\alpha_4} \prod_{k=1}^{4} D_{x_k}^{\alpha_k} \left(a_{\alpha_1\alpha_2\alpha_3\alpha_4}\left(x_{10}, x_2, x_3, x_{40}\right)\right.$$

$$\left.\cdot R\left(x_{10}, x_2, x_3, x_{40}\right)\right)$$

$$+ \sum_{i_1=1}^{m_1} \sum_{i_2=1}^{m_2} \sum_{i_3=1}^{m_3} \sum_{i_4=1}^{m_4} (-1)^{i_1+i_2+i_3+i_4-1} \prod_{k=1}^{4} D_{x_k}^{i_k-1} \left(u\left(x_1, x_{20}, x_{30}, x_4\right)\right)$$

$$\cdot \sum_{\alpha_1=i_1}^{m_1} \sum_{\alpha_2=i_2}^{m_2} \sum_{\alpha_3=i_3}^{m_3} \sum_{\alpha_4=i_4}^{m_4} (-1)^{\alpha_1+\alpha_2+\alpha_3+\alpha_4} \prod_{k=1}^{4} D_{x_k}^{\alpha_k} \left(a_{\alpha_1\alpha_2\alpha_3\alpha_4}\left(x_1, x_{20}, x_{30}, x_4\right)\right.$$

$$\left.\cdot R\left(x_1, x_{20}, x_{30}, x_4\right)\right)$$

$$+ \sum_{i_1=1}^{m_1} \sum_{i_2=1}^{m_2} \sum_{i_3=1}^{m_3} \sum_{i_4=1}^{m_4} (-1)^{i_1+i_2+i_3+i_4-1} \prod_{k=1}^{4} D_{x_k}^{i_k-1} \left(u\left(x_1, x_{20}, x_3, x_{40}\right)\right)$$

$$\cdot \sum_{\alpha_1=i_1}^{m_1} \sum_{\alpha_2=i_2}^{m_2} \sum_{\alpha_3=i_3}^{m_3} \sum_{\alpha_4=i_4}^{m_4} (-1)^{\alpha_1+\alpha_2+\alpha_3+\alpha_4} \prod_{k=1}^{4} D_{x_k}^{\alpha_k} \left(a_{\alpha_1\alpha_2\alpha_3\alpha_4}\left(x_1, x_{20}, x_3, x_{40}\right)\right.$$

$$\left.\cdot R\left(x_1, x_{20}, x_3, x_{40}\right)\right)$$

$$+ \sum_{i_1=1}^{m_1} \sum_{i_2=1}^{m_2} \sum_{i_3=1}^{m_3} \sum_{i_4=1}^{m_4} (-1)^{i_1+i_2+i_3+i_4-1} \prod_{k=1}^{4} D_{x_k}^{i_k-1} \left(u\left(x_1, x_2, x_{30}, x_{40}\right)\right)$$

$$\cdot \sum_{\alpha_1=i_1}^{m_1} \sum_{\alpha_2=i_2}^{m_2} \sum_{\alpha_3=i_3}^{m_3} \sum_{\alpha_4=i_4}^{m_4} (-1)^{\alpha_1+\alpha_2+\alpha_3+\alpha_4} \prod_{k=1}^{4} D_{x_k}^{\alpha_k} \left(a_{\alpha_1\alpha_2\alpha_3\alpha_4} \left(x_1, x_2, x_{30}, x_{40}\right)\right.$$

$$\left. \cdot R\left(x_1, x_2, x_{30}, x_{40}\right)\right)$$

$$- \sum_{i_1=1}^{m_1} \sum_{i_2=1}^{m_2} \sum_{i_3=1}^{m_3} \sum_{i_4=1}^{m_4} (-1)^{i_1+i_2+i_3+i_4-1} \prod_{k=1}^{4} D_{x_k}^{i_k-1} \left(u\left(x_{10}, x_{20}, x_{30}, x_4\right)\right)$$

$$\cdot \sum_{\alpha_1=i_1}^{m_1} \sum_{\alpha_2=i_2}^{m_2} \sum_{\alpha_3=i_3}^{m_3} \sum_{\alpha_4=i_4}^{m_4} (-1)^{\alpha_1+\alpha_2+\alpha_3+\alpha_4} \prod_{k=1}^{4} D_{x_k}^{\alpha_k} \left(a_{\alpha_1\alpha_2\alpha_3\alpha_4} \left(x_{10}, x_{20}, x_{30}, x_4\right)\right.$$

$$\left. \cdot R\left(x_{10}, x_{20}, x_{30}, x_4\right)\right)$$

$$- \sum_{i_1=1}^{m_1} \sum_{i_2=1}^{m_2} \sum_{i_3=1}^{m_3} \sum_{i_4=1}^{m_4} (-1)^{i_1+i_2+i_3+i_4-1} \prod_{k=1}^{4} D_{x_k}^{i_k-1} \left(u\left(x_{10}, x_{20}, x_3, x_{40}\right)\right)$$

$$\cdot \sum_{\alpha_1=i_1}^{m_1} \sum_{\alpha_2=i_2}^{m_2} \sum_{\alpha_3=i_3}^{m_3} \sum_{\alpha_4=i_4}^{m_4} (-1)^{\alpha_1+\alpha_2+\alpha_3+\alpha_4} \prod_{k=1}^{4} D_{x_k}^{\alpha_k} \left(a_{\alpha_1\alpha_2\alpha_3\alpha_4} \left(x_{10}, x_{20}, x_3, x_{40}\right)\right.$$

$$\left. \cdot R\left(x_{10}, x_{20}, x_3, x_{40}\right)\right)$$

$$- \sum_{i_1=1}^{m_1} \sum_{i_2=1}^{m_2} \sum_{i_3=1}^{m_3} \sum_{i_4=1}^{m_4} (-1)^{i_1+i_2+i_3+i_4-1} \prod_{k=1}^{4} D_{x_k}^{i_k-1} \left(u\left(x_{10}, x_2, x_{30}, x_{40}\right)\right)$$

$$\cdot \sum_{\alpha_1=i_1}^{m_1} \sum_{\alpha_2=i_2}^{m_2} \sum_{\alpha_3=i_3}^{m_3} \sum_{\alpha_4=i_4}^{m_4} (-1)^{\alpha_1+\alpha_2+\alpha_3+\alpha_4} \prod_{k=1}^{4} D_{x_k}^{\alpha_k} \left(a_{\alpha_1\alpha_2\alpha_3\alpha_4} \left(x_{10}, x_2, x_{30}, x_{40}\right)\right.$$

$$\left. \cdot R\left(x_{10}, x_2, x_{30}, x_{40}\right)\right)$$

$$- \sum_{i_1=1}^{m_1} \sum_{i_2=1}^{m_2} \sum_{i_3=1}^{m_3} \sum_{i_4=1}^{m_4} (-1)^{i_1+i_2+i_3+i_4-1} \prod_{k=1}^{4} D_{x_k}^{i_k-1} \left(u\left(x_1, x_{20}, x_{30}, x_{40}\right)\right)$$

$$\cdot \sum_{\alpha_1=i_1}^{m_1} \sum_{\alpha_2=i_2}^{m_2} \sum_{\alpha_3=i_3}^{m_3} \sum_{\alpha_4=i_4}^{m_4} (-1)^{\alpha_1+\alpha_2+\alpha_3+\alpha_4} \prod_{k=1}^{4} D_{x_k}^{\alpha_k} \left(a_{\alpha_1\alpha_2\alpha_3\alpha_4} \left(x_1, x_{20}, x_{30}, x_{40}\right)\right.$$

$$\left. \cdot R\left(x_1, x_{20}, x_{30}, x_{40}\right)\right)$$

$$+ \sum_{i_1=1}^{m_1} \sum_{i_2=1}^{m_2} \sum_{i_3=1}^{m_3} \sum_{i_4=1}^{m_4} (-1)^{i_1+i_2+i_3+i_4-1} \prod_{k=1}^{4} D_{x_k}^{i_k-1} \left(u\left(x_{10}, x_{20}, x_{30}, x_{40}\right)\right)$$

$$\cdot \sum_{\alpha_1=i_1}^{m_1} \sum_{\alpha_2=i_2}^{m_2} \sum_{\alpha_3=i_3}^{m_3} \sum_{\alpha_4=i_4}^{m_4} (-1)^{\alpha_1+\alpha_2+\alpha_3+\alpha_4} \prod_{k=1}^{4} D_{x_k}^{\alpha_k} \left(a_{\alpha_1\alpha_2\alpha_3\alpha_4} \left(x_{10}, x_{20}, x_{30}, x_{40}\right)\right.$$

$$\left. \cdot R\left(x_{10}, x_{20}, x_{30}, x_{40}\right)\right)$$

$$-\int_{x_{10}}^{x_1}\left\{\sum_{i_2=1}^{m_2}\sum_{i_3=1}^{m_3}\sum_{i_4=1}^{m_4}(-1)^{i_2+i_3+i_4-1}\left\{D_{x_2}^{i_2-1}D_{x_3}^{i_3-1}D_{x_4}^{i_4-1}u\left(\xi_1,x_{20},x_3,x_4\right)\right.\right.$$

$$\cdot\sum_{\alpha_1=0}^{m_1}\sum_{\alpha_2=i_2}^{m_2}\sum_{\alpha_3=i_3}^{m_3}\sum_{\alpha_4=i_4}^{m_4}(-1)^{\alpha_1+\alpha_2+\alpha_3+\alpha_4}$$

$$D_{x_1}^{\alpha_1}D_{x_2}^{\alpha_2-i_2}D_{x_3}^{\alpha_3-i_3}D_{x_4}^{\alpha_4-i_4}\left(a_{\alpha_1\alpha_2\alpha_3\alpha_4}\left(\xi_1,x_{20},x_3,x_4\right)\cdot R\left(\xi_1,x_{20},x_3,x_4\right)\right)$$

$$+\,D_{x_2}^{i_2-1}D_{x_3}^{i_3-1}D_{x_4}^{i_4-1}u\left(\xi_1,x_2,x_{30},x_4\right)$$

$$\sum_{\alpha_1=0}^{m_1}\sum_{\alpha_2=i_2}^{m_2}\sum_{\alpha_3=i_3}^{m_3}\sum_{\alpha_4=i_4}^{m_4}(-1)^{\alpha_1+\alpha_2+\alpha_3+\alpha_4}$$

$$D_{x_1}^{\alpha_1}D_{x_2}^{\alpha_2-i_2}D_{x_3}^{\alpha_3-i_3}D_{x_4}^{\alpha_4-i_4}\left(a_{\alpha_1\alpha_2\alpha_3\alpha_4}\left(\xi_1,x_2,x_{30},x_4\right)\cdot R\left(\xi_1,x_2,x_{30},x_4\right)\right)$$

$$+\,D_{x_2}^{i_2-1}D_{x_3}^{i_3-1}D_{x_4}^{i_4-1}u\left(\xi_1,x_2,x_3,x_{40}\right)$$

$$\cdot\sum_{\alpha_1=0}^{m_1}\sum_{\alpha_2=i_2}^{m_2}\sum_{\alpha_3=i_3}^{m_3}\sum_{\alpha_4=i_4}^{m_4}(-1)^{\alpha_1+\alpha_2+\alpha_3+\alpha_4}$$

$$D_{x_1}^{\alpha_1}D_{x_2}^{\alpha_2-i_2}D_{x_3}^{\alpha_3-i_3}D_{x_4}^{\alpha_4-i_4}\left(a_{\alpha_1\alpha_2\alpha_3\alpha_4}\left(\xi_1,x_2,x_3,x_{40}\right)\right.$$

$$\cdot\,R\left(\xi_1,x_2,x_3,x_{40}\right))-(-1)^{i_2+i_3+i_4-1}\,D_{x_2}^{i_2-1}D_{x_3}^{i_3-1}D_{x_4}^{i_4-1}u\left(\xi_1,x_{20},x_{30},x_4\right)$$

$$\cdot\sum_{\alpha_1=0}^{m_1}\sum_{\alpha_2=i_2}^{m_2}\sum_{\alpha_3=i_3}^{m_3}\sum_{\alpha_4=i_4}^{m_4}(-1)^{\alpha_1+\alpha_2+\alpha_3+\alpha_4}$$

$$D_{x_1}^{\alpha_1}D_{x_2}^{\alpha_2-i_2}D_{x_3}^{\alpha_3-i_3}D_{x_4}^{\alpha_4-i_4}\left(a_{\alpha_1\alpha_2\alpha_3\alpha_4}\left(\xi_1,x_{20},x_{30},x_4\right)\right.$$

$$\cdot\sum_{\alpha_1=0}^{m_1}\sum_{\alpha_2=i_2}^{m_2}\sum_{\alpha_3=i_3}^{m_3}\sum_{\alpha_4=i_4}^{m_4}(-1)^{\alpha_1+\alpha_2+\alpha_3+\alpha_4}$$

$$D_{x_1}^{\alpha_1}D_{x_2}^{\alpha_2-i_2}D_{x_3}^{\alpha_3-i_3}D_{x_4}^{\alpha_4-i_4}\left(a_{\alpha_1\alpha_2\alpha_3\alpha_4}\left(\xi_1,x_2,x_{30},x_{40}\right)\cdot R\left(\xi_1,x_2,x_{30},x_{40}\right)\right)$$

$$+\,D_{x_2}^{i_2-1}D_{x_3}^{i_3-1}D_{x_4}^{i_4-1}u\left(\xi_1,x_{20},x_{30},x_{40}\right)$$

$$\cdot\sum_{\alpha_1=0}^{m_1}\sum_{\alpha_2=i_2}^{m_2}\sum_{\alpha_3=i_3}^{m_3}\sum_{\alpha_4=i_4}^{m_4}(-1)^{\alpha_1+\alpha_2+\alpha_3+\alpha_4}D_{x_1}^{\alpha_1}D_{x_2}^{\alpha_2-i_2}D_{x_3}^{\alpha_3-i_3}D_{x_4}^{\alpha_4-i_4}$$

$$\left.\left.\left(a_{\alpha_1\alpha_2\alpha_3\alpha_4}\left(\xi_1,x_{20},x_{30},x_{40}\right)\cdot R\left(\xi_1,x_{20},x_{30},x_{40}\right)\right)\right\}\,d\xi_1\right\}-$$

$$-\int_{x_{20}}^{x_2}\left\{\sum_{i_1=1}^{m_1}\sum_{i_3=1}^{m_3}\sum_{i_4=1}^{m_4}(-1)^{i_1+i_3+i_4-1}\left\{D_{x_1}^{i_1-1}D_{x_3}^{i_3-1}D_{x_4}^{i_4-1}u\left(x_{10},\xi_2,x_3,x_4\right)\right.\right.$$

$$\cdot\sum_{\alpha_1=i_1}^{m_1}\sum_{\alpha_2=0}^{m_2}\sum_{\alpha_3=i_3}^{m_3}\sum_{\alpha_4=i_4}^{m_4}(-1)^{\alpha_1+\alpha_2+\alpha_3+\alpha_4}$$

$$D_{x_1}^{\alpha_1-i_1}D_{x_2}^{\alpha_2}D_{x_3}^{\alpha_3-i_3}D_{x_4}^{\alpha_4-i_4}\left(a_{\alpha_1\alpha_2\alpha_3\alpha_4}\left(x_{10},\xi_2,x_3,x_4\right)\right.$$

$$\cdot R\left(x_{10},\xi_2,x_3,x_4\right))$$

$$+ D_{x_1}^{i_1-1} D_{x_3}^{i_3-1} D_{x_4}^{i_4-1} u\left(x_1, \xi_2, x_{30}, x_4\right)$$

$$\cdot \sum_{\alpha_1=i_1}^{m_1} \sum_{\alpha_2=0}^{m_2} \sum_{\alpha_3=i_3}^{m_3} \sum_{\alpha_4=i_4}^{m_4} (-1)^{\alpha_1+\alpha_2+\alpha_3+\alpha_4}$$

$$D_{x_1}^{\alpha_1-i_1} D_{x_2}^{\alpha_2} D_{x_3}^{\alpha_3-i_3} D_{x_4}^{\alpha_4-i_4} \left(a_{\alpha_1\alpha_2\alpha_3\alpha_4}\left(x_1, \xi_2, x_{30}, x_4\right)\cdot R\left(x_1, \xi_2, x_{30}, x_4\right)\right)$$

$$+ D_{x_1}^{i_1-1} D_{x_3}^{i_3-1} D_{x_4}^{i_4-1} u\left(x_1, \xi_2, x_3, x_{40}\right)$$

$$\cdot \sum_{\alpha_1=i_1}^{m_1} \sum_{\alpha_2=0}^{m_2} \sum_{\alpha_3=i_3}^{m_3} \sum_{\alpha_4=i_4}^{m_4} (-1)^{\alpha_1+\alpha_2+\alpha_3+\alpha_4}$$

$$D_{x_1}^{\alpha_1-i_1} D_{x_2}^{\alpha_2} D_{x_3}^{\alpha_3-i_3} D_{x_4}^{\alpha_4-i_4} \left(a_{\alpha_1\alpha_2\alpha_3\alpha_4}\left(x_1, \xi_2, x_3, x_{40}\right) \cdot R\left(x_1, \xi_2, x_3, x_{40}\right)\right)$$

$$- D_{x_1}^{i_1-1} D_{x_3}^{i_3-1} D_{x_4}^{i_4-1} u\left(x_{10}, \xi_2, x_{30}, x_4\right) \cdot \sum_{\alpha_1=i_1}^{m_1} \sum_{\alpha_2=0}^{m_2} \sum_{\alpha_3=i_3}^{m_3} \sum_{\alpha_4=i_4}^{m_4} (-1)^{\alpha_1+\alpha_2+\alpha_3+\alpha_4}$$

$$D_{x_1}^{\alpha_1-i_1} D_{x_2}^{\alpha_2} D_{x_3}^{\alpha_3-i_3} D_{x_4}^{\alpha_4-i_4} \left(a_{\alpha_1\alpha_2\alpha_3\alpha_4}\left(x_{10}, \xi_2, x_{30}, x_4\right) \cdot R\left(x_{10}, \xi_2, x_{30}, x_4\right)\right)$$

$$- D_{x_1}^{i_1-1} D_{x_3}^{i_3-1} D_{x_4}^{i_4-1} u\left(x_{10}, \xi_2, x_3, x_{40}\right) \cdot \sum_{\alpha_1=i_1}^{m_1} \sum_{\alpha_2=0}^{m_2} \sum_{\alpha_3=i_3}^{m_3} \sum_{\alpha_4=i_4}^{m_4} (-1)^{\alpha_1+\alpha_2+\alpha_3+\alpha_4}$$

$$D_{x_1}^{\alpha_1-i_1} D_{x_2}^{\alpha_2} D_{x_3}^{\alpha_3-i_3} D_{x_4}^{\alpha_4-i_4} \left(a_{\alpha_1\alpha_2\alpha_3\alpha_4}\left(x_{10}, \xi_2, x_3, x_{40}\right) \cdot R\left(x_{10}, \xi_2, x_3, x_{40}\right)\right)$$

$$- D_{x_1}^{i_1-1} D_{x_3}^{i_3-1} D_{x_4}^{i_4-1} u\left(x_1, \xi_2, x_{30}, x_{40}\right) \cdot \sum_{\alpha_1=i_1}^{m_1} \sum_{\alpha_2=0}^{m_2} \sum_{\alpha_3=i_3}^{m_3} \sum_{\alpha_4=i_4}^{m_4} (-1)^{\alpha_1+\alpha_2+\alpha_3+\alpha_4}$$

$$D_{x_1}^{\alpha_1-i_1} D_{x_2}^{\alpha_2} D_{x_3}^{\alpha_3-i_3} D_{x_4}^{\alpha_4-i_4} \left(a_{\alpha_1\alpha_2\alpha_3\alpha_4}\left(x_1, \xi_2, x_{30}, x_{40}\right)\cdot R\left(x_1, \xi_2, x_{30}, x_{40}\right)\right)$$

$$+ D_{x_1}^{i_1-1} D_{x_3}^{i_3-1} D_{x_4}^{i_4-1} u\left(x_{10}, \xi_2, x_{30}, x_{40}\right) \cdot \sum_{\alpha_1=i_1}^{m_1} \sum_{\alpha_2=0}^{m_2} \sum_{\alpha_3=i_3}^{m_3} \sum_{\alpha_4=i_4}^{m_4} (-1)^{\alpha_1+\alpha_2+\alpha_3+\alpha_4}$$

$$D_{x_1}^{\alpha_1-i_1} D_{x_2}^{\alpha_2} D_{x_3}^{\alpha_3-i_3} D_{x_4}^{\alpha_4-i_4} \left(a_{\alpha_1\alpha_2\alpha_3\alpha_4}\left(x_{10}, \xi_2, x_{30}, x_{40}\right)\cdot R\left(x_{10}, \xi_2, x_{30}, x_{40}\right)\right)\} \, d\xi_2\}$$

$$- \int\limits_{x_{30}}^{x_3} \left\{ \sum_{i_1=1}^{m_1} \sum_{i_2=1}^{m_2} \sum_{i_4=1}^{m_4} (-1)^{i_1+i_2+i_4-1} \left\{ D_{x_1}^{i_1-1} D_{x_2}^{i_2-1} D_{x_4}^{i_4-1} u\left(x_{10}, x_2, \xi_3, x_4\right) \right.\right.$$

$$\cdot \sum_{\alpha_1=i_1}^{m_1} \sum_{\alpha_2=i_2}^{m_2} \sum_{\alpha_3=0}^{m_3} \sum_{\alpha_4=i_4}^{m_4} (-1)^{\alpha_1+\alpha_2+\alpha_3+\alpha_4}$$

$$D_{x_1}^{\alpha_1-i_1} D_{x_2}^{\alpha_2-i_2} D_{x_3}^{\alpha_3} D_{x_4}^{\alpha_4-i_4} \left(a_{\alpha_1\alpha_2\alpha_3\alpha_4}\left(x_{10}, x_2, \xi_3, x_4\right)\cdot R\left(x_{10}, x_2, \xi_3, x_4\right)\right)$$

$$+ D_{x_1}^{i_1-1} D_{x_2}^{i_2-1} D_{x_4}^{i_4-1} u\left(x_1, x_{20}, \xi_3, x_4\right) \cdot \sum_{\alpha_1=i_1}^{m_1} \sum_{\alpha_2=i_2}^{m_2} \sum_{\alpha_3=0}^{m_3} \sum_{\alpha_4=i_4}^{m_4} (-1)^{\alpha_1+\alpha_2+\alpha_3+\alpha_4}$$

$$D_{x_1}^{\alpha_1-i_1} D_{x_2}^{\alpha_2-i_2} D_{x_3}^{\alpha_3} D_{x_4}^{\alpha_4-i_4} \left(a_{\alpha_1\alpha_2\alpha_3\alpha_4}\left(x_1, x_{20}, \xi_3, x_4\right)\right.$$

$$\left. \cdot R\left(x_1, x_{20}, \xi_3, x_4\right)\right) + D_{x_1}^{i_1-1} D_{x_2}^{i_2-1} D_{x_4}^{i_4-1} u\left(x_1, x_2, \xi_3, x_{40}\right) \cdot$$

$$\cdot \sum_{\alpha_1=i_1}^{m_1} \sum_{\alpha_2=i_2}^{m_2} \sum_{\alpha_3=0}^{m_3} \sum_{\alpha_4=i_4}^{m_4} (-1)^{\alpha_1+\alpha_2+\alpha_3+\alpha_4}$$

$$D_{x_1}^{\alpha_1-i_1} D_{x_2}^{\alpha_2-i_2} D_{x_3}^{\alpha_3} D_{x_4}^{\alpha_4-i_4} \left(a_{\alpha_1\alpha_2\alpha_3\alpha_4}\left(x_1,x_2,\xi_3,x_{40}\right) \cdot\cdot R\left(x_1,x_2,\xi_3,x_{40}\right)\right)$$

$$- D_{x_1}^{i_1-1} D_{x_2}^{i_2-1} D_{x_4}^{i_4-1} u\left(x_{10},x_{20},\xi_3,x_4\right) \cdot \sum_{\alpha_1=i_1}^{m_1} \sum_{\alpha_2=i_2}^{m_2} \sum_{\alpha_3=0}^{m_3} \sum_{\alpha_4=i_4}^{m_4} (-1)^{\alpha_1+\alpha_2+\alpha_3+\alpha_4}$$

$$D_{x_1}^{\alpha_1-i_1} D_{x_2}^{\alpha_2-i_2} D_{x_3}^{\alpha_3} D_{x_4}^{\alpha_4-i_4} \left(a_{\alpha_1\alpha_2\alpha_3\alpha_4}\left(x_{10},x_{20},\xi_3,x_4\right) \cdot R\left(x_{10},x_{20},\xi_3,x_4\right)\right)$$

$$- D_{x_1}^{i_1-1} D_{x_2}^{i_2-1} D_{x_4}^{i_4-1} u\left(x_{10},x_2,\xi_3,x_{40}\right) \cdot \sum_{\alpha_1=i_1}^{m_1} \sum_{\alpha_2=i_2}^{m_2} \sum_{\alpha_3=0}^{m_3} \sum_{\alpha_4=i_4}^{m_4} (-1)^{\alpha_1+\alpha_2+\alpha_3+\alpha_4}$$

$$D_{x_1}^{\alpha_1-i_1} D_{x_2}^{\alpha_2-i_2} D_{x_3}^{\alpha_3} D_{x_4}^{\alpha_4-i_4} \left(a_{\alpha_1\alpha_2\alpha_3\alpha_4}\left(x_{10},x_2,\xi_3,x_{40}\right) \cdot R\left(x_{10},x_2,\xi_3,x_{40}\right)\right)$$

$$- D_{x_1}^{i_1-1} D_{x_2}^{i_2-1} D_{x_4}^{i_4-1} u\left(x_1,x_{20},\xi_3,x_{40}\right) \cdot \sum_{\alpha_1=i_1}^{m_1} \sum_{\alpha_2=i_2}^{m_2} \sum_{\alpha_3=0}^{m_3} \sum_{\alpha_4=i_4}^{m_4} (-1)^{\alpha_1+\alpha_2+\alpha_3+\alpha_4}$$

$$D_{x_1}^{\alpha_1-i_1} D_{x_2}^{\alpha_2-i_2} D_{x_3}^{\alpha_3} D_{x_4}^{\alpha_4-i_4} \left(a_{\alpha_1\alpha_2\alpha_3\alpha_4}\left(x_1,x_{20},\xi_3,x_{40}\right) \cdot R\left(x_1,x_{20},\xi_3,x_{40}\right)\right)$$

$$+ D_{x_1}^{i_1-1} D_{x_2}^{i_2-1} D_{x_4}^{i_4} - u\left(x_{10},x_{20},\xi_3,x_{40}\right) \cdot \sum_{\alpha_1=i_1}^{m_1} \sum_{\alpha_2=i_2}^{m_2} \sum_{\alpha_3=0}^{m_3} \sum_{\alpha_4=i_4}^{m_4} (-1)^{\alpha_1+\alpha_2+\alpha_3+\alpha_4}$$

$$D_{x_1}^{\alpha_1-i_1} D_{x_2}^{\alpha_2-i_2} D_{x_3}^{\alpha_3} D_{x_4}^{\alpha_4-i_4} \left(a_{\alpha_1\alpha_2\alpha_3\alpha_4}\left(x_{10},x_{20},\xi_3,x_{40}\right) \cdot R\left(x_{10},x_{20},\xi_3,x_{40}\right)\right)\} \, d\xi_3\}$$

$$- \int_{x_{40}}^{x_4} \left\{ \sum_{i_1=1}^{m_1} \sum_{i_2=1}^{m_2} \sum_{i_3=1}^{m_3} (-1)^{i_1+i_2+i_3-1} \left\{ D_{x_1}^{i_1-1} D_{x_2}^{i_2-1} D_{x_3}^{i_3-1} u\left(x_{10},x_2,x_3,\xi_4\right) \right. \right.$$

$$\cdot \sum_{\alpha_1=i_1}^{m_1} \sum_{\alpha_2=i_2}^{m_2} \sum_{\alpha_3=i_3}^{m_3} \sum_{\alpha_4=0}^{m_4} (-1)^{\alpha_1+\alpha_2+\alpha_3+\alpha_4}$$

$$D_{x_1}^{\alpha_1-i_1} D_{x_2}^{\alpha_2-i_2} D_{x_3}^{\alpha_3-i_3} D_{x_4}^{\alpha_4} \left(a_{\alpha_1\alpha_2\alpha_3\alpha_4}\left(x_{10},x_2,x_3,\xi_4\right) \cdot R\left(x_{10},x_2,x_3,\xi_4\right)\right)$$

$$+ D_{x_1}^{i_1-1} D_{x_2}^{i_2-1} D_{x_3}^{i_3-1} u\left(x_1,x_{20},x_3,\xi_4\right) \cdot \sum_{\alpha_1=i_1}^{m_1} \sum_{\alpha_2=i_2}^{m_2} \sum_{\alpha_3=i_3}^{m_3} \sum_{\alpha_4=0}^{m_4} (-1)^{\alpha_1+\alpha_2+\alpha_3+\alpha_4}$$

$$D_{x_1}^{\alpha_1-i_1} D_{x_2}^{\alpha_2-i_2} D_{x_3}^{\alpha_3-i_3} D_{x_4}^{\alpha_4} \left(a_{\alpha_1\alpha_2\alpha_3\alpha_4}\left(x_1,x_{20},x_3,\xi_4\right) \cdot R\left(x_1,x_{20},x_3,\xi_4\right)\right)$$

$$+ D_{x_1}^{i_1-1} D_{x_2}^{i_2-1} D_{x_3}^{i_3-1} u\left(x_1,x_2,x_{30},\xi_4\right) \cdot \sum_{\alpha_1=i_1}^{m_1} \sum_{\alpha_2=i_2}^{m_2} \sum_{\alpha_3=i_3}^{m_3} \sum_{\alpha_4=0}^{m_4} (-1)^{\alpha_1+\alpha_2+\alpha_3+\alpha_4}$$

$$D_{x_1}^{\alpha_1-i_1} D_{x_2}^{\alpha_2-i_2} D_{x_3}^{\alpha_3-i_3} D_{x_4}^{\alpha_4} \left(a_{\alpha_1\alpha_2\alpha_3\alpha_4}\left(x_1,x_2,x_{30},\xi_4\right) \cdot R\left(x_1,x_2,x_{30},\xi_4\right)\right)$$

$$- D_{x_1}^{i_1-1} D_{x_2}^{i_2-1} D_{x_3}^{i_3-1} u\left(x_{10},x_{20},x_3,\xi_4\right) \cdot \sum_{\alpha_1=i_1}^{m_1} \sum_{\alpha_2=i_2}^{m_2} \sum_{\alpha_3=i_3}^{m_3} \sum_{\alpha_4=0}^{m_4} (-1)^{\alpha_1+\alpha_2+\alpha_3+\alpha_4}$$

$$D_{x_1}^{\alpha_1-i_1} D_{x_2}^{\alpha_2-i_2} D_{x_3}^{\alpha_3-i_3} D_{x_4}^{\alpha_4} \left(a_{\alpha_1\alpha_2\alpha_3\alpha_4}\left(x_{10},x_{20},x_3,\xi_4\right) \cdot R\left(x_{10},x_{20},x_3,\xi_4\right)\right)$$

$$- D_{x_1}^{i_1-1} D_{x_2}^{i_2-1} D_{x_3}^{i_3-1} u\left(x_{10}, x_2, x_{30}, \xi_4\right) \cdot \sum_{\alpha_1=i_1}^{m_1} \sum_{\alpha_2=i_2}^{m_2} \sum_{\alpha_3=i_3}^{m_3} \sum_{\alpha_4=0}^{m_4} (-1)^{\alpha_1+\alpha_2+\alpha_3+\alpha_4}$$

$$D_{x_1}^{\alpha_1-i_1} D_{x_2}^{\alpha_2-i_2} D_{x_3}^{\alpha_3-i_3} D_{x_4}^{\alpha_4}\left(a_{\alpha_1\alpha_2\alpha_3\alpha_4}\left(x_{10}, x_2, x_{30}, \xi_4\right) \cdot R\left(x_{10}, x_2, x_{30}, \xi_4\right)\right)$$

$$- D_{x_1}^{i_1-1} D_{x_2}^{i_2-1} D_{x_3}^{i_3-1} u\left(x_1, x_{20}, x_{30}, \xi_4\right) \cdot \sum_{\alpha_1=i_1}^{m_1} \sum_{\alpha_2=i_2}^{m_2} \sum_{\alpha_3=i_3}^{m_3} \sum_{\alpha_4=0}^{m_4} (-1)^{\alpha_1+\alpha_2+\alpha_3+\alpha_4}$$

$$D_{x_1}^{\alpha_1-i_1} D_{x_2}^{\alpha_2-i_2} D_{x_3}^{\alpha_3-i_3} D_{x_4}^{\alpha_4}\left(a_{\alpha_1\alpha_2\alpha_3\alpha_4}\left(x_1, x_{20}, x_{30}, \xi_4\right) \cdot \cdot R\left(x_1, x_{20}, x_{30}, \xi_4\right)\right)$$

$$+ D_{x_1}^{i_1-1} D_{x_2}^{i_2-1} D_{x_3}^{i_3-1} u\left(x_{10}, x_{20}, x_{30}, \xi_4\right) \cdot \sum_{\alpha_1=i_1}^{m_1} \sum_{\alpha_2=i_2}^{m_2} \sum_{\alpha_3=i_3}^{m_3} \sum_{\alpha_4=0}^{m_4} (-1)^{\alpha_1+\alpha_2+\alpha_3+\alpha_4}$$

$$D_{x_1}^{\alpha_1-i_1} D_{x_2}^{\alpha_2-i_2} D_{x_3}^{\alpha_3-i_3} D_{x_4}^{\alpha_4}\left(a_{\alpha_1\alpha_2\alpha_3\alpha_4}\left(x_{10}, x_{20}, x_{30}, \xi_4\right) \cdot \cdot R\left(x_{10}, x_{20}, x_{30}, \xi_4\right)\right)\} \, d\xi_4\}$$

$$- \int_{x_{10}}^{x_1} \int_{x_{20}}^{x_2} \{ \sum_{i_3=1}^{m_3} \sum_{i_4=1}^{m_4} (-1)^{i_3+i_4-1} \{ D_{x_3}^{i_3-1} D_{x_4}^{i_4-1} u\left(\xi_1, \xi_2, x_{30}, x_4\right)$$

$$\cdot \sum_{\alpha_1=0}^{m_1} \sum_{\alpha_2=0}^{m_2} \sum_{\alpha_3=i_3}^{m_3} \sum_{\alpha_4=i_4}^{m_4} (-1)^{\alpha_1+\alpha_2+\alpha_3+\alpha_4}$$

$$D_{x_1}^{\alpha_1} D_{x_2}^{\alpha_2} D_{x_3}^{\alpha_3-i_3} D_{x_4}^{\alpha_4-i_4}\left(a_{\alpha_1\alpha_2\alpha_3\alpha_4}\left(\xi_1, \xi_2, x_{30}, x_4\right) \cdot \cdot R\left(\xi_1, \xi_2, x_{30}, x_4\right)\right)$$

$$+ D_{x_3}^{i_3-1} D_{x_4}^{i_4-1} u\left(\xi_1, \xi_2, x_3, x_{40}\right) \cdot \sum_{\alpha_1=0}^{m_1} \sum_{\alpha_2=0}^{m_2} \sum_{\alpha_3=i_3}^{m_3} \sum_{\alpha_4=i_4}^{m_4} (-1)^{\alpha_1+\alpha_2+\alpha_3+\alpha_4}$$

$$D_{x_1}^{\alpha_1} D_{x_2}^{\alpha_2} D_{x_3}^{\alpha_3-i_3} D_{x_4}^{\alpha_4-i_4}\left(a_{\alpha_1\alpha_2\alpha_3\alpha_4}\left(\xi_1, \xi_2, x_3, x_{40}\right) \cdot R\left(\xi_1, \xi_2, x_3, x_{40}\right)\right)$$

$$- D_{x_3}^{i_3-1} D_{x_4}^{i_4-1} u\left(\xi_1, \xi_2, x_{30}, x_{40}\right) \cdot \sum_{\alpha_1=0}^{m_1} \sum_{\alpha_2=0}^{m_2} \sum_{\alpha_3=i_3}^{m_3} \sum_{\alpha_4=i_4}^{m_4} (-1)^{\alpha_1+\alpha_2+\alpha_3+\alpha_4}$$

$$D_{x_1}^{\alpha_1} D_{x_2}^{\alpha_2} D_{x_3}^{\alpha_3-i_3} D_{x_4}^{\alpha_4-i_4}\left(a_{\alpha_1\alpha_2\alpha_3\alpha_4}\left(\xi_1, \xi_2, x_{30}, x_{40}\right) \cdot R\left(\xi_1, \xi_2, x_{30}, x_{40}\right)\right)\} \, d\xi_2 d\xi_1\}$$

$$- \int_{x_{10}}^{x_1} \int_{x_{30}}^{x_3} \{ \sum_{i_2=1}^{m_2} \sum_{i_4=1}^{m_4} (-1)^{i_2+i_4-1} \{ D_{x_2}^{i_2-1} D_{x_4}^{i_4-1} u\left(\xi_1, x_{20}, \xi_3, x_4\right) \cdot$$

$$\sum_{\alpha_1=0}^{m_1} \sum_{\alpha_2=i_2}^{m_2} \sum_{\alpha_3=0}^{m_3} \sum_{\alpha_4=i_4}^{m_4} (-1)^{\alpha_1+\alpha_2+\alpha_3+\alpha_4}$$

$$D_{x_1}^{\alpha_1} D_{x_2}^{\alpha_2-i_2} D_{x_3}^{\alpha_3} D_{x_4}^{\alpha_4-i_4}\left(a_{\alpha_1\alpha_2\alpha_3\alpha_4}\left(\xi_1, x_{20}, \xi_3, x_4\right) \cdot R\left(\xi_1, x_{20}, \xi_3, x_4\right)\right)$$

$$+ D_{x_2}^{i_2-1} D_{x_4}^{i_4-1} u\left(\xi_1, x_2, \xi_3, x_{40}\right)$$

$$\cdot \sum_{\alpha_1=0}^{m_1} \sum_{\alpha_2=i_2}^{m_2} \sum_{\alpha_3=0}^{m_3} \sum_{\alpha_4=i_4}^{m_4} (-1)^{\alpha_1+\alpha_2+\alpha_3+\alpha_4}$$

$$D_{x_1}^{\alpha_1} D_{x_2}^{\alpha_2-i_2} D_{x_3}^{\alpha_3} D_{x_4}^{\alpha_4-i_4}\left(a_{\alpha_1\alpha_2\alpha_3\alpha_4}\left(\xi_1, x_2, \xi_3, x_{40}\right) \cdot \cdot R\left(\xi_1, x_2, \xi_3, x_{40}\right)\right)$$

$$-\int_{x_{10}}^{x_1}\int_{x_{20}}^{x_2}\left\{\sum_{i_3=1}^{m_3}\sum_{i_4=1}^{m_4}(-1)^{i_3+i_4-1}\left\{D_{x_3}^{i_3-1}D_{x_4}^{i_4-1}u\left(\xi_1,\xi_2,x_{30},x_4\right)\right.\right.$$

$$\cdot\sum_{\alpha_1=0}^{m_1}\sum_{\alpha_2=0}^{m_2}\sum_{\alpha_3=i_3}^{m_3}\sum_{\alpha_4=i_4}^{m_4}(-1)^{\alpha_1+\alpha_2+\alpha_3+\alpha_4}$$

$$D_{x_1}^{\alpha_1}D_{x_2}^{\alpha_2}D_{x_3}^{\alpha_3-i_3}D_{x_4}^{\alpha_4-i_4}\left(a_{\alpha_1\alpha_2\alpha_3\alpha_4}\left(\xi_1,\xi_2,x_{30},x_4\right)\right.$$

$$-\int_{x_{10}}^{x_1}\int_{x_{40}}^{x_4}\left\{\sum_{i_2=1}^{m_2}\sum_{i_3=1}^{m_3}(-1)^{i_2+i_3-1}\left\{D_{x_2}^{i_2-1}D_{x_3}^{i_3-1}u\left(\xi_1,x_{20},x_3,\xi_4\right)\right.\right.$$

$$\cdot\sum_{\alpha_1=0}^{m_1}\sum_{\alpha_2=i_2}^{m_2}\sum_{\alpha_3=i_3}^{m_3}\sum_{\alpha_4=0}^{m_4}(-1)^{\alpha_1+\alpha_2+\alpha_3+\alpha_4}$$

$$D_{x_1}^{\alpha_1}D_{x_2}^{\alpha_2-i_2}D_{x_3}^{\alpha_3-i_3}D_{x_4}^{\alpha_4}\left(a_{\alpha_1\alpha_2\alpha_3\alpha_4}\left(\xi_1,x_{20},x_3,\xi_4\right)\cdot\cdot R\left(\xi_1,x_{20},x_3,\xi_4\right)\right)$$

$$+D_{x_2}^{i_2-1}D_{x_3}^{i_3-1}u\left(\xi_1,x_2,x_{30},\xi_4\right)$$

$$\cdot\sum_{\alpha_1=0}^{m_1}\sum_{\alpha_2=i_2}^{m_2}\sum_{\alpha_3=i_3}^{m_3}\sum_{\alpha_4=0}^{m_4}(-1)^{\alpha_1+\alpha_2+\alpha_3+\alpha_4}$$

$$D_{x_1}^{\alpha_1}D_{x_2}^{\alpha_2-i_2}D_{x_3}^{\alpha_3-i_3}D_{x_4}^{\alpha_4}\left(a_{\alpha_1\alpha_2\alpha_3\alpha_4}\left(\xi_1,x_2,x_{30},\xi_4\right)\cdot\right.$$

$$\cdot R\left(\xi_1,x_2,x_{30},\xi_4\right))-D_{x_2}^{i_2-1}D_{x_3}^{i_3-1}u\left(\xi_1,x_{20},x_{30},\xi_4\right)$$

$$\cdot\sum_{\alpha_1=0}^{m_1}\sum_{\alpha_2=i_2}^{m_2}\sum_{\alpha_3=i_3}^{m_3}\sum_{\alpha_4=0}^{m_4}(-1)^{\alpha_1+\alpha_2+\alpha_3+\alpha_4}$$

$$D_{x_1}^{\alpha_1}D_{x_2}^{\alpha_2-i_2}D_{x_3}^{\alpha_3-i_3}D_{x_4}^{\alpha_4}\left(a_{\alpha_1\alpha_2\alpha_3\alpha_4}\left(\xi_1,x_{20},x_{30},\xi_4\right)\right.$$

$$\cdot R\left(\xi_1,x_{20},x_{30},\xi_4\right))\}d\xi_4d\xi_1\}$$

$$-\int_{x_{20}}^{x_2}\int_{x_{30}}^{x_3}\left\{\sum_{i_1=1}^{m_1}\sum_{i_4=1}^{m_4}(-1)^{i_1+i_4-1}\left\{D_{x_1}^{i_1-1}D_{x_4}^{i_4-1}u\left(x_{10},\xi_2,\xi_3,x_4\right)\right.\right.$$

$$\cdot\sum_{\alpha_1=i_1}^{m_1}\sum_{\alpha_2=0}^{m_2}\sum_{\alpha_3=0}^{m_3}\sum_{\alpha_4=i_4}^{m_4}(-1)^{\alpha_1+\alpha_2+\alpha_3+\alpha_4}$$

$$D_{x_1}^{\alpha_1-i_1}D_{x_2}^{\alpha_2}D_{x_3}^{\alpha_3}D_{x_4}^{\alpha_4-i_4}\left(a_{\alpha_1\alpha_2\alpha_3\alpha_4}\left(x_{10},\xi_2,\xi_3,x_4\right)\cdot R\left(x_{10},\xi_2,\xi_3,x_4\right)\right)$$

$$+D_{x_1}^{i_1-1}D_{x_4}^{i_4-1}u\left(x_1,\xi_2,\xi_3,x_{40}\right)\cdot\sum_{\alpha_1=i_1}^{m_1}\sum_{\alpha_2=0}^{m_2}\sum_{\alpha_3=0}^{m_3}\sum_{\alpha_4=i_4}^{m_4}(-1)^{\alpha_1+\alpha_2+\alpha_3+\alpha_4}$$

$$D_{x_1}^{\alpha_1-i_1}D_{x_2}^{\alpha_2}D_{x_3}^{\alpha_3}D_{x_4}^{\alpha_4-i_4}\left(a_{\alpha_1\alpha_2\alpha_3\alpha_4}\left(x_1,\xi_2,\xi_3,x_{40}\right)\cdot R\left(x_1,\xi_2,\xi_3,x_{40}\right)\right)$$

$$-D_{x_1}^{i_1-1}D_{x_4}^{i_4-1}u\left(x_{10},\xi_2,\xi_3,x_{40}\right)\cdot\sum_{\alpha_1=i_1}^{m_1}\sum_{\alpha_2=0}^{m_2}\sum_{\alpha_3=0}^{m_3}\sum_{\alpha_4=i_4}^{m_4}(-1)^{\alpha_1+\alpha_2+\alpha_3+\alpha_4}$$

$$D_{x_1}^{\alpha_1-i_1} D_{x_2}^{\alpha_2} D_{x_3}^{\alpha_3} D_{x_4}^{\alpha_4-i_4} \left(a_{\alpha_1\alpha_2\alpha_3\alpha_4} \left(x_{10}, \xi_2, \xi_3, x_{40} \right) \right.$$
$$\left. \cdot R \left(x_{10}, \xi_2, \xi_3, x_{40} \right) \right) \} \, d\xi_3 d\xi_2 \}$$

$$- \int\limits_{x_{20}}^{x_2} \int\limits_{x_{40}}^{x_4} \{ \sum_{i_1=1}^{m_1} \sum_{i_3=1}^{m_3} (-1)^{i_1+i_3-1} \{ D_{x_1}^{i_1-1} D_{x_3}^{i_3-1} u \left(x_{10}, \xi_2, x_3, \xi_4 \right)$$

$$\cdot \sum_{\alpha_1=i_1}^{m_1} \sum_{\alpha_2=0}^{m_2} \sum_{\alpha_3=i_3}^{m_3} \sum_{\alpha_4=0}^{m_4} (-1)^{\alpha_1+\alpha_2+\alpha_3+\alpha_4}$$

$$D_{x_1}^{\alpha_1-i_1} D_{x_2}^{\alpha_2} D_{x_3}^{\alpha_3-i_3} D_{x_4}^{\alpha_4} \left(a_{\alpha_1\alpha_2\alpha_3\alpha_4} \left(x_{10}, \xi_2, x_3, \xi_4 \right) \cdot R \left(x_{10}, \xi_2, x_3, \xi_4 \right) \right)$$

$$+ D_{x_1}^{i_1-1} D_{x_3}^{i_3-1} u \left(x_1, \xi_2, x_{30}, \xi_4 \right)$$

$$\cdot \sum_{\alpha_1=i_1}^{m_1} \sum_{\alpha_2=0}^{m_2} \sum_{\alpha_3=i_3}^{m_3} \sum_{\alpha_4=0}^{m_4} (-1)^{\alpha_1+\alpha_2+\alpha_3+\alpha_4}$$

$$D_{x_1}^{\alpha_1-i_1} D_{x_2}^{\alpha_2} D_{x_3}^{\alpha_3-i_3} D_{x_4}^{\alpha_4} \left(a_{\alpha_1\alpha_2\alpha_3\alpha_4} \left(x_1, \xi_2, x_{30}, \xi_4 \right) \cdot R \left(x_1, \xi_2, x_{30}, \xi_4 \right) \right)$$

$$- D_{x_1}^{i_1-1} D_{x_3}^{i_3-1} u \left(x_{10}, \xi_2, x_{30}, \xi_4 \right)$$

$$\cdot \sum_{\alpha_1=i_1}^{m_1} \sum_{\alpha_2=0}^{m_2} \sum_{\alpha_3=i_3}^{m_3} \sum_{\alpha_4=0}^{m_4} (-1)^{\alpha_1+\alpha_2+\alpha_3+\alpha_4}$$

$$D_{x_1}^{\alpha_1-i_1} D_{x_2}^{\alpha_2} D_{x_3}^{\alpha_3-i_3} D_{x_4}^{\alpha_4} \left(a_{\alpha_1\alpha_2\alpha_3\alpha_4} \left(x_{10}, \xi_2, x_{30}, \xi_4 \right) \cdot R \left(x_{10}, \xi_2, x_{30}, \xi_4 \right) \right) \} \, d\xi_4 d\xi_2 \}$$

$$- \int\limits_{x_{30}}^{x_3} \int\limits_{x_{40}}^{x_4} \{ \sum_{i_1=1}^{m_1} \sum_{i_2=1}^{m_2} (-1)^{i_1+i_2-1} \{ D_{x_1}^{i_1-1} D_{x_2}^{i_2-1} u \left(x_{10}, x_2, \xi_3, \xi_4 \right)$$

$$\cdot \sum_{\alpha_1=i_1}^{m_1} \sum_{\alpha_2=i_2}^{m_2} \sum_{\alpha_3=0}^{m_3} \sum_{\alpha_4=0}^{m_4} (-1)^{\alpha_1+\alpha_2+\alpha_3+\alpha_4}$$

$$D_{x_1}^{\alpha_1-i_1} D_{x_2}^{\alpha_2-i_2} D_{x_3}^{\alpha_3} D_{x_4}^{\alpha_4} \left(a_{\alpha_1\alpha_2\alpha_3\alpha_4} \left(x_{10}, x_2, \xi_3, \xi_4 \right) \cdot R \left(x_{10}, x_2, \xi_3, \xi_4 \right) \right)$$

$$+ D_{x_1}^{i_1-1} D_{x_2}^{i_2-1} u \left(x_1, x_{20}, \xi_3, \xi_4 \right)$$

$$\cdot \sum_{\alpha_1=i_1}^{m_1} \sum_{\alpha_2=i_2}^{m_2} \sum_{\alpha_3=0}^{m_3} \sum_{\alpha_4=0}^{m_4} (-1)^{\alpha_1+\alpha_2+\alpha_3+\alpha_4}$$

$$D_{x_1}^{\alpha_1-i_1} D_{x_2}^{\alpha_2-i_2} D_{x_3}^{\alpha_3} D_{x_4}^{\alpha_4} \left(a_{\alpha_1\alpha_2\alpha_3\alpha_4} \left(x_1, x_{20}, \xi_3, \xi_4 \right) \cdot R \left(x_1, x_{20}, \xi_3, \xi_4 \right) \right)$$

$$- D_{x_1}^{i_1-1} D_{x_2}^{i_2-1} u \left(x_{10}, x_{20}, \xi_3, \xi_4 \right)$$

$$\cdot \sum_{\alpha_1=i_1}^{m_1} \sum_{\alpha_2=i_2}^{m_2} \sum_{\alpha_3=0}^{m_3} \sum_{\alpha_4=0}^{m_4} (-1)^{\alpha_1+\alpha_2+\alpha_3+\alpha_4}$$

$$D_{x_1}^{\alpha_1-i_1} D_{x_2}^{\alpha_2-i_2} D_{x_3}^{\alpha_3} D_{x_4}^{\alpha_4} \left(a_{\alpha_1\alpha_2\alpha_3\alpha_4} \left(x_{10}, x_{20}, \xi_3, \xi_4 \right) \cdot R \left(x_{10}, x_{20}, \xi_3, \xi_4 \right) \right) \} \, d\xi_4 d\xi_3 \}$$

$$- \int\limits_{x_{10}}^{x_1} \int\limits_{x_{20}}^{x_2} \int\limits_{x_{30}}^{x_3} \{ \sum_{i_4=1}^{m_4} (-1)^{i_4-1} \{ D_{x_4}^{i_4-1} u \left(\xi_1, \xi_2, \xi_3, x_{40} \right)$$

$$\cdot \sum_{\alpha_1=0}^{m_1} \sum_{\alpha_2=0}^{m_2} \sum_{\alpha_3=0}^{m_3} \sum_{\alpha_4=i_4}^{m_4} (-1)^{\alpha_1+\alpha_2+\alpha_3+\alpha_4}$$

$$D_{x_1}^{\alpha_1} D_{x_2}^{\alpha_2} D_{x_3}^{\alpha_3} D_{x_4}^{\alpha_4-i_4} \left(a_{\alpha_1\alpha_2\alpha_3\alpha_4} \left(\xi_1, \xi_2, \xi_3, x_{40} \right) \cdot R \left(\xi_1, \xi_2, \xi_3, x_{40} \right) \right) \} \, d\xi_3 d\xi_2 d\xi_1 \}$$

$$- \int_{x_{10}}^{x_1} \int_{x_{20}}^{x_2} \int_{x_{40}}^{x_4} \{ \sum_{i_3=1}^{m_3} (-1)^{i_3-1} \{ D_{x_3}^{i_3-1} u \left(\xi_1, \xi_2, x_{30}, \xi_4 \right) $$

$$\cdot \sum_{\alpha_1=0}^{m_1} \sum_{\alpha_2=0}^{m_2} \sum_{\alpha_3=i_3}^{m_3} \sum_{\alpha_4=0}^{m_4} (-1)^{\alpha_1+\alpha_2+\alpha_3+\alpha_4}$$

$$D_{x_1}^{\alpha_1} D_{x_2}^{\alpha_2} D_{x_3}^{\alpha_3-i_3} D_{x_4}^{\alpha_4} \left(a_{\alpha_1\alpha_2\alpha_3\alpha_4} \left(\xi_1, \xi_2, x_{30}, \xi_4 \right) \cdot R \left(\xi_1, \xi_2, x_{30}, \xi_4 \right) \right) \} \, d\xi_4 d\xi_2 d\xi_1 \}$$

$$- \int_{x_{10}}^{x_1} \int_{x_{30}}^{x_3} \int_{x_{40}}^{x_4} \{ \sum_{i_2=1}^{m_2} (-1)^{i_2-1} \{ D_{x_2}^{i_2-1} u \left(\xi_1, x_{20}, \xi_3, \xi_4 \right) $$

$$\cdot \sum_{\alpha_1=0}^{m_1} \sum_{\alpha_2=i_2}^{m_2} \sum_{\alpha_3=0}^{m_3} \sum_{\alpha_4=0}^{m_4} (-1)^{\alpha_1+\alpha_2+\alpha_3+\alpha_4}$$

$$D_{x_1}^{\alpha_1} D_{x_2}^{\alpha_2-i_2} D_{x_3}^{\alpha_3} D_{x_4}^{\alpha_4} \left(a_{\alpha_1\alpha_2\alpha_3\alpha_4} \left(\xi_1, x_{20}, \xi_3, \xi_4 \right) \cdot R \left(\xi_1, x_{20}, \xi_3, \xi_4 \right) \right) \} \, d\xi_4 d\xi_3 d\xi_1 \}$$

$$- \int_{x_{20}}^{x_2} \int_{x_{30}}^{x_3} \int_{x_{40}}^{x_4} \{ \sum_{i_1=1}^{m_1} (-1)^{i_1-1} \{ D_{x_1}^{i_1-1} u \left(x_{10}, \xi_2, \xi_3, \xi_4 \right) $$

$$\cdot \sum_{\alpha_1=i_1}^{m_1} \sum_{\alpha_2=0}^{m_2} \sum_{\alpha_3=0}^{m_3} \sum_{\alpha_4=0}^{m_4} (-1)^{\alpha_1+\alpha_2+\alpha_3+\alpha_4}$$

$$D_{x_1}^{\alpha_1-i_1} D_{x_2}^{\alpha_2} D_{x_3}^{\alpha_3} D_{x_4}^{\alpha_4} \left(a_{\alpha_1\alpha_2\alpha_3\alpha_4} \left(x_{10}, \xi_2, \xi_3, \xi_4 \right) \cdot R \left(x_{10}, \xi_2, \xi_3, \xi_4 \right) \right) \} \, d\xi_4 d\xi_3 d\xi_2 \}$$

Now it remains to integrate (21) with respect to x_1, x_2, x_3, and x_4 $m_1 - 1$, $m_2 - 1$, $m_3 - 1$, and $m_4 - 1$ times, respectively. Denote by $H\left(x_1, x_2, x_3, x_4, x_1, x_2, x_3, x_4\right)$ the right-hand side of (21) without the first addend. Then

$$u\left(x_1, x_2, x_3, x_4\right) = \prod_{k=1}^{4} D_{x_k}^{-m_k} \left[RF \right] + \prod_{k=1}^{n} D_{x_k}^{-m_k+1} \left[H \right] + F_1, \qquad (22)$$

where

$$F_1 = \sum_{i_1=0}^{m_1-2} D_{x_1}^{-i_1} \left(\varphi_{i_1} \left(x_2, x_3, x_4 \right) \right) + \sum_{i_2=0}^{m_2-2} D_{x_2}^{-i_2} \left(\varphi_{i_2} \left(x_1, x_3, x_4 \right) \right)$$

$$+ \sum_{i_3=0}^{m_3-2} D_{x_3}^{-i_3} \left(\varphi_{i_3} \left(x_1, x_2, x_4 \right) \right) + \sum_{i_4=0}^{m_4-2} D_{x_4}^{-i_4} \left(\varphi_{i_4} \left(x_1, x_2, x_3 \right) \right)$$

$$-\sum_{i_1=0}^{m_1-2}\sum_{i_2=0}^{m_2-2} D_{x_1}^{-i_1} D_{x_2}^{-i_2}\left(D_{x_2}^{i_2}\varphi_{i_1}\left(x_{20},x_3,x_4\right)\right)$$

$$-\sum_{i_1=0}^{m_1-2}\sum_{i_3=0}^{m_3-2} D_{x_1}^{-i_1} D_{x_3}^{-i_3}\left(D_{x_3}^{i_3}\varphi_{i_1}\left(x_2,x_{30},x_4\right)\right)$$

$$-\sum_{i_1=0}^{m_1-2}\sum_{i_4=0}^{m_4-2} D_{x_1}^{-i_1} D_{x_4}^{-i_4}\left(D_{x_4}^{i_4}\varphi_{i_1}\left(x_2,x_3,x_{40}\right)\right)$$

$$-\sum_{i_2=0}^{m_2-2}\sum_{i_3=0}^{m_3-2} D_{x_2}^{-i_2} D_{x_3}^{-i_3}\left(D_{x_3}^{i_3}\varphi_{i_2}\left(x_1,x_{30},x_4\right)\right)$$

$$-\sum_{i_2=0}^{m_2-2}\sum_{i_4=0}^{m_4-2} D_{x_2}^{-i_2} D_{x_4}^{-i_4}\left(D_{x_4}^{i_4}\varphi_{i_2}\left(x_1,x_3,x_{40}\right)\right)-$$

$$-\sum_{i_3=0}^{m_3-2}\sum_{i_4=0}^{m_4-2} D_{x_3}^{-i_3} D_{x_4}^{-i_4}\left(D_{x_4}^{i_4}\varphi_{i_3}\left(x_1,x_2,x_{40}\right)\right)$$

$$+\sum_{i_1=0}^{m_1-2}\sum_{i_2=0}^{m_2-2}\sum_{i_3=0}^{m_3-2} D_{x_1}^{-i_1} D_{x_2}^{-i_2} D_{x_3}^{-i_3}\left(D_{x_2}^{i_2} D_{x_3}^{i_3}\varphi_{i_1}\left(x_{20},x_{30},x_4\right)\right)$$

$$+\sum_{i_1=0}^{m_1-2}\sum_{i_2=0}^{m_2-2}\sum_{i_4=0}^{m_4-2} D_{x_1}^{-i_1} D_{x_2}^{-i_2} D_{x_4}^{-i_4}\left(D_{x_2}^{i_2} D_{x_4}^{i_4}\varphi_{i_1}\left(x_{20},x_3,x_{40}\right)\right)$$

$$+\sum_{i_1=0}^{m_1-2}\sum_{i_3=0}^{m_3-2}\sum_{i_4=0}^{m_4-2} D_{x_1}^{-i_1} D_{x_3}^{-i_3} D_{x_4}^{-i_4}\left(D_{x_3}^{i_3} D_{x_4}^{i_4}\varphi_{i_1}\left(x_2,x_{30},x_{40}\right)\right)$$

$$+\sum_{i_2=0}^{m_2-2}\sum_{i_3=0}^{m_3-2}\sum_{i_4=0}^{m_4-2} D_{x_2}^{-i_2} D_{x_3}^{-i_3} D_{x_4}^{-i_4}\left(D_{x_3}^{i_3} D_{x_4}^{i_4}\varphi_{i_2}\left(x_1,x_{30},x_{40}\right)\right)$$

$$-\sum_{i_1=0}^{m_1-2}\sum_{i_2=0}^{m_2-2}\sum_{i_3=0}^{m_3-2}\sum_{i_4=0}^{m_4-2} D_{x_1}^{-i_1} D_{x_2}^{-i_2} D_{x_3}^{-i_3} D_{x_4}^{-i_4}\left(D_{x_2}^{i_2} D_{x_3}^{i_3} D_{x_4}^{i_4}\varphi_{i_1}\left(x_{20},x_{30},x_{40}\right)\right).$$

If one assumes that the functions $\varphi_{pi_p}\left(i_p=\overline{0,m_p}\quad,p=1,2,3,4\right)$ are arbitrary, then (22) can be treated as the general representation of the solution of equation (1), as this was done in [6], p.66.

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