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WEAKLY PRIME AND PRIME FUZZY IDEALS IN ORDERED SEMIGROUPS

(submitted by M. M. Arslanov)

Abstract. Intra-regular ordered semigroups play an important role in studying the structure, especially the decomposition of ordered semigroups. In this paper we prove that the fuzzy ideals of an ordered semigroup S are weakly prime if and only if they are idempotent and they form a chain, and that they are prime if and only if S is intra-regular and the fuzzy ideals of S form a chain. Moreover we show that a fuzzy ideal of an ordered semigroup is prime if and only if it is both semiprime and weakly prime and that in commutative ordered semigroups the prime and weakly prime fuzzy ideals coincide. Our results extend the corresponding results on semigroups (without order) given by G. Szász in [11] in case of ordered semigroups using fuzzy sets.

1. INTRODUCTION AND PREREQUISITES

Our aim is to promote research and the development of fuzzy technology by studying the fuzzy ordered semigroups. Intra-regular semigroups play an essential role in studying the structure, in particular the decomposition of semigroups (see [2,10]). Intra-regular ordered semigroups play an important role in studying the decomposition of ordered semigroups. As we have seen in [4], an ordered semigroup S is intra-regular if and only if it is a semilattice of simple semigroups, equivalently, if S is a union of simple subsemigroups of S. Moreover, an ordered semigroup

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S is intra-regular and the ideals of S form a chain if and only if S is a chain of simple semigroups [4]. Semigroups in which the ideals are prime or weakly prime have been considered by G. Szász in [11]. G. Szász has shown that the ideals of semigroup S are weakly prime if and only if they are idempotent and they form a chain, and that they are prime if and only if S is intra-regular and the ideals of S form a chain. He also proved that an ideal of a semigroup S is prime if and only if it is both semiprime and weakly prime and that in commutative semigroups the prime and weakly prime ideals coincide. The present paper extends the corresponding results on semigroups (without order) given by G. Szász in [11] in case of ordered semigroups using fuzzy sets. It can be a bridge passing from semigroups or ordered semigroups to fuzzy ordered semigroups. We first prove that a fuzzy ideal of an ordered semigroup is prime if and only if it is both semiprime and weakly prime and that in commutative ordered semigroups the prime and weakly prime fuzzy ideals coincide. Then we prove that the fuzzy ideals of an ordered semigroup S are weakly prime if and only if they are idempotent and they form a chain, and that the fuzzy ideals of an ordered semigroup S are prime if and only if S is intra-regular and the fuzzy ideals of S form a chain.

The important concept of the fuzzy set was first introduced by L.A. Zadeh in [12]. Since then, many papers on fuzzy sets appeared showing the importance of the concept and its applications to logic, set theory, group theory, groupoids, real analysis, measure theory, topology, etc. Many notions of mathematics are extended to such sets, and various properties of these notions in the context of fuzzy sets are established.

Following the terminology given by Zadeh, if S is an ordered semigroup, a fuzzy subset of S (or a fuzzy set in S) is an arbitrary mapping f of S into the real closed interval [0,1] [5]. For $a \in S$, denote by A_a the subset of $S \times S$ defined by:

$$A_a = \{(y,z) \mid a \le yz\}.$$

For two fuzzy subsets f and g of S, the multiplication of f and g is the fuzzy subset of S defined by:

$$f \circ g : \ S \to \ [0,1] \mid \ a \to \begin{cases} \sup\limits_{(y,z)\in A_a} \{\min f(y), g(z)\} & if \ A_a \ne \emptyset \\ 0 & if \ A_a = \emptyset \end{cases}$$

and in the set of all fuzzy subsets of S define the order relation as follows:

$$f \subseteq g \text{ if and only if } f(x) \le g(x) \text{ for all } x \in S.$$

Denote by 1 the fuzzy subset of S defined by

$$1 : S \to [0,1] \mid x \to 1(x) := 1.$$

If $F(S)$ is the set of all fuzzy subsets of S, it is clear that the fuzzy subset 1 of S is the greatest element of the ordered set $(F(S), \subseteq)$. As we have seen in [6], if S is an ordered semigroup, then the set $F(S)$ with the multiplication " $\circ$ " and the order " $\subseteq$ " above is an ordered semigroup as well. Moreover, as we have seen in [7], $F(S)$ is a lattice ordered semigroup (l-semigroup) [1,3] with the operations of supremum and infimum on $F(S)$ defined as follows:

$$f \cup g : S \to [0,1] \mid x \to \max\{f(x), g(x)\},$$

$$f \cap g : S \to [0,1] \mid x \to \min\{f(x), g(x)\}.$$

That is, $(F(S), \cap, \cup)$ is a lattice, and if f, g, h are fuzzy subsets of S, then

$$(f \cup g) \circ h = (f \circ h) \cup (g \circ h),$$

$$h \circ (f \cup g) = (h \circ f) \cup (f \circ g).$$

Clearly, $(f \cap g) \circ h \subseteq (f \circ h) \cap (g \circ h)$ and $h \circ (f \cap g) \subseteq (h \circ f) \cap (f \circ g)$.

For a fuzzy subset f of S, denote by $\overline{f}$ the fuzzy ideal of S generated by f (i.e. the least –with respect to the inclusion relation– fuzzy ideal of S containing f). One can easily prove the following:

$$\overline{f} = f \cup (1 \circ f) \cup (f \circ 1) \cup (1 \circ f \circ 1).$$

The following Kuratowski's closure axioms are satisfied:
(1) $f \subseteq \overline{f}$
(2) If $f \subseteq g$, then $\overline{f} \subseteq \overline{g}$
(3) $\overline{\overline{f}} = \overline{f}$.

An ordered semigroup $(S, ., \leq)$ is called intra-regular if for each $a \in S$ there exist $x, y \in S$ such that $a \leq xa^2y$ [4]. This concept extents the concept of intra-regular semigroups (without order) [2] in case of ordered semigroups.

Definition 1.1. [5,9] Let $(S, ., \leq)$ be an ordered semigroup. A fuzzy subset f of S is called a *fuzzy right ideal* of S if (1) $f(xy) \geq f(x)$ for every $x, y \in S$ and (2) $x \leq y$ implies $f(x) \geq f(y)$. Equivalent Definition: (1) $f \circ 1 \subseteq f$ and (2) $x \leq y$ implies $f(x) \geq f(y)$.
It is called a *fuzzy left ideal* of S if (1) $f(xy) \geq f(y)$ for every $x, y \in S$ and (2) $x \leq y$ implies $f(x) \geq f(y)$. Equivalent Definition: (1) $1 \circ f \subseteq f$ and (2) $x \leq y$ implies $f(x) \geq f(y)$. A fuzzy subset of S which is both a fuzzy right and a fuzzy left ideal of S is called a *fuzzy ideal* of S.

2. MAIN RESULTS

Let S be an ordered semigroup. A fuzzy subset h of S is called *weakly prime* if for any pair f, g of fuzzy ideals of S such that $f \circ g \subseteq h$, we have $f \subseteq h$ or $g \subseteq h$. It is called *prime* if for all fuzzy subsets f, g of S such that $f \circ g \subseteq h$, we have $f \subseteq h$ or $g \subseteq h$. A fuzzy subset h of S is called *semiprime* if for all fuzzy subsets f of S such that $f^2 \subseteq h$, we have $f \subseteq h$.

Proposition 2.1. *Let S be an ordered semigroup and h a weakly prime fuzzy ideal of S. Let f, g be fuzzy subsets of S such that $f \circ 1 \circ g \subseteq h$. Then we have $f \subseteq h$ or $g \subseteq h$.*

Proof. As we can easily see, the fuzzy sets $1 \circ f \circ 1$, $1 \circ g \circ 1$ are fuzzy ideals of S and

$$(1 \circ f \circ 1) \circ (1 \circ g \circ 1) \subseteq 1 \circ (f \circ 1 \circ g) \circ 1 \subseteq 1 \circ h \circ 1 \subseteq h.$$

Since h is weakly prime, we have $1 \circ f \circ 1 \subseteq h$ or $1 \circ g \circ 1 \subseteq h$. If $1 \circ f \circ 1 \subseteq h$, we get

$$\begin{aligned}(\overline{f})^3 &= (f \cup (1 \circ f) \cup (f \circ 1) \cup (1 \circ f \circ 1))^2 \circ (f \cup (1 \circ f) \cup (f \circ 1) \cup (1 \circ f \circ 1)) \\ &\subseteq ((1 \circ f) \cup (1 \circ f \circ 1)) \circ (f \cup (1 \circ f) \cup (f \circ 1) \cup (1 \circ f \circ 1)) \\ &\qquad \subseteq 1 \circ f \circ 1 \subseteq h.\end{aligned}$$

Then, since h is weakly prime and $\overline{f}$ a fuzzy ideal of S, we have $f \subseteq \overline{f} \subseteq h$. If $1 \circ g \circ 1 \subseteq h$, in a similar way we obtain $g \subseteq h$. □

Proposition 2.2. *Let S be an ordered semigroup and h a fuzzy ideal of S satisfying the condition:*

For all fuzzy subsets f, g of S such that $f \circ 1 \circ g \subseteq h$, we have $f \subseteq h$ or $g \subseteq h$.

Then

(1) *For each fuzzy right ideal f and each fuzzy subset g of S, $f \circ g \subseteq h$ implies $f \subseteq h$ or $g \subseteq h$.*
(2) *For each fuzzy subset f and each fuzzy left ideal g of S, $f \circ g \subseteq h$ implies $f \subseteq h$ or $g \subseteq h$.*

Proof (1) Let f be a fuzzy right ideal and g a fuzzy subset of S such that $f \circ g \subseteq h$. Since $(f \circ 1) \circ g \subseteq f \circ g \subseteq h$, by hypothesis, we have $f \subseteq h$ or $g \subseteq h$.
The proof of (2) is similar. □
By Propositions 2.1 and 2.2, we have the following:

Corollary 2.3. *Let S be an ordered semigroup and h a fuzzy ideal of S. Then h is weakly prime if and only if for all fuzzy subsets f, g of S such that $f \circ 1 \circ g \subseteq h$, we have $f \subseteq h$ or $g \subseteq h$.*

Proposition 2.4. *Let S be an ordered semigroup and h a fuzzy ideal of S. Then h is prime if and only if it is both semiprime and weakly prime.*

Proof. $\Longrightarrow$. It is clear.
$\Longleftarrow$. Let f, g be fuzzy subsets of S such that $f \circ g \subseteq h$. Then

$$(g \circ 1 \circ f)^2 = (g \circ 1 \circ f) \circ (g \circ 1 \circ f) \subseteq 1 \circ (f \circ g) \circ 1 \subseteq 1 \circ h \circ 1.$$

Since h is a fuzzy ideal of S, we have $1 \circ h \circ 1 \subseteq h$, thus $(g \circ 1 \circ f)^2 \subseteq h$. Since $g \circ 1 \circ f$ is a fuzzy subset of S and h is semiprime, we have $g \circ 1 \circ f \subseteq h$. Since h is weakly prime, by Proposition 2.1, we get $f \subseteq h$ or $g \subseteq h$. Thus h is prime. $\square$

Proposition 2.5. *Let S be a commutative ordered semigroup and h a fuzzy ideal of S. Then h is prime if and only if it is weakly prime.*

Proof. $\Longrightarrow$. It is clear.
$\Longleftarrow$. Let f, g be fuzzy subsets of S such that $f \circ g \subseteq h$. Since S is commutative, we have $\overline{f} \circ \overline{g} = \overline{f \circ g} \subseteq \overline{h} = h$. Since h is weakly prime, we get $f \subseteq \overline{f} \subseteq h$ or $g \subseteq \overline{g} \subseteq h$. $\square$

Lemma 2.6. *Let S be an ordered semigroup. The fuzzy ideals of S are idempotent if and only if for all fuzzy ideals f, g of S we have $f \cap g = f \circ g$.*

Proof. $\Longrightarrow$. Let f, g be fuzzy ideals of S. Since $f \cap g$ is a fuzzy ideal of S, by hypothesis, we have $f \cap g = (f \cap g) \circ (f \cap g) \subseteq f \circ g$. On the other hand, $f \circ g \subseteq f \circ 1 \subseteq f$ and $f \circ g \subseteq 1 \circ g \subseteq g$, thus $f \circ g \subseteq f \cap g$. Therefore $f \cap g = f \circ g$.
$\Longleftarrow$. It is clear. $\square$

Theorem 2.7. *Let S be an ordered semigroup. The fuzzy ideals of S are weakly prime if and only if they are idempotent and they form a chain.*

Proof. $\Longrightarrow$. Let f, g be fuzzy ideals of S. Then $f \cap g = f \circ g$. Indeed: Since $f \circ g$ and $f \cap g$ are fuzzy ideals of S and $(f \cap g) \circ (f \cap g) \subseteq f \circ g$, by hypothesis, we have $f \cap g \subseteq f \circ g$. Since $f \circ g \subseteq f \circ 1$ and $f \circ g \subseteq 1 \circ g$, we get $f \circ g \subseteq (f \circ 1) \cap (1 \circ g) \subseteq f \cap g$. Hence $f \cap g = f \circ g$. By Lemma 2.6, the fuzzy ideals of S are idempotent.
Let now f, g be fuzzy ideals of S. Since $f \circ g$ is a fuzzy ideal of S, by hypothesis, we have $f \subseteq f \circ g$ or $g \subseteq f \circ g$. If $f \subseteq f \circ g$, then $f \subseteq f \circ g \subseteq 1 \circ g \subseteq g$. If $g \subseteq f \circ g$, then $g \subseteq f \circ g \subseteq f \circ 1 \subseteq f$.
$\Longleftarrow$. Let f, g, h be fuzzy ideals of S such that $f \circ g \subseteq h$. Since the fuzzy ideals of S are idempotent, by Lemma 2.6, we have $f \cap g = f \circ g$. By

hypothesis, $f \subseteq g$ or $g \subseteq f$. If $f \subseteq g$, then $f = f \cap g = f \circ g \subseteq h$. If $g \subseteq f$, then $g = f \cap g = f \circ g \subseteq h$. So h is weakly prime. □

Lemma 2.8. [8] *An ordered semigroup S is intra-regular if and only if for each fuzzy subset f of S, we have $f \subseteq 1 \circ f^2 \circ 1$.*

Lemma 2.9. *An ordered semigroup S is intra-regular if and only if for each fuzzy subset f of S, we have $f \subseteq \overline{f^2}$.*

Proof. $\Longrightarrow$. Let f be a fuzzy subset of S. Since S is intra-regular, by Lemma 2.8, we have $f \subseteq 1 \circ f^2 \circ 1$. Then we have

$$\begin{aligned} f &\subseteq \overline{f} = f \cup (1 \circ f) \cup (f \circ 1) \cup (1 \circ f \circ 1) \\ &\subseteq (1 \circ f^2 \circ 1) \cup (1^2 \circ f^2 \circ 1) \cup (1 \circ f^2 \circ 1^2) \cup (1^2 \circ f^2 \circ 1^2) \\ &= 1 \circ f^2 \circ 1 \subseteq \overline{f^2}. \end{aligned}$$

$\Longleftarrow$. For each fuzzy subset f of S, by hypothesis, we have

$$f \subseteq \overline{f^2} = f^2 \cup (1 \circ f^2) \cup (f^2 \circ 1) \cup (1 \circ f^2 \circ 1).$$

Then we have

$$\begin{aligned} f^2 \subseteq (f^2 \cup (1 \circ f^2) \cup (f^2 \cup 1) \cup (1 \circ f^2 \circ 1)) \\ \circ (f^2 \cup (1 \circ f^2) \cup (f^2 \cup 1) \cup (1 \circ f^2 \circ 1)) \\ \subseteq (1 \circ f^2 \circ 1) \cup (1 \circ f^2). \end{aligned}$$

Then $f^2 \circ 1 \subseteq (1 \circ f^2 \circ 1^2) \cup (1 \circ f^2 \circ 1) = 1 \circ f^2 \circ 1$. Hence $f \subseteq (1 \circ f^2) \cup (1 \circ f^2 \circ 1)$, and $f^2 \subseteq (1 \circ f^3) \cup (1 \circ f^2 \circ 1 \circ f) \subseteq 1 \circ f^2 \circ 1$. So $1 \circ f^2 \subseteq 1^2 \circ f^2 \circ 1 \subseteq 1 \circ f^2 \circ 1$. Finally, $f \subseteq 1 \circ f^2 \circ 1$ and, by Lemma 2.8, S is intra-regular. □

Lemma 2.10. *An ordered semigroup S is intra-regular if and only if for each fuzzy subset f of S, we have $\overline{f} = \overline{f^2}$.*

Proof. $\Longrightarrow$. Let f be a fuzzy subset of S. Since S is intra-regular, by Lemma 2.9, we have $f \subseteq \overline{f^2}$. Then $\overline{f} \subseteq \overline{\overline{f^2}} = \overline{f^2}$. On the other hand,

$$\overline{f^2} = f^2 \cup (1 \circ f^2) \cup (f^2 \circ 1) \cup (1 \circ f^2 \circ 1) \subseteq (1 \circ f) \cup (1 \circ f \circ 1) \subseteq \overline{f},$$

so $\overline{f} = \overline{f^2}$.
$\Longleftarrow$. If $\overline{f} = \overline{f^2}$, then $f \subseteq \overline{f^2}$ and, by Lemma 2.9, S is intra-regular. □

Lemma 2.11. *Let S be an intra-regular ordered semigroup. Then, for each fuzzy subset f of S, we have $\overline{f} = 1 \circ f \circ 1$.*

Proof. Let f be a fuzzy subset of S. Clearly, $f^4 \subseteq 1 \circ f \circ 1 \subseteq \overline{f}$. Since S is intra-regular and f, f^2 are fuzzy subsets of S, by Lemma 2.10, we have

$$\overline{f} = \overline{f^2} = \overline{f^4} \subseteq \overline{1 \circ f \circ 1} \subseteq \overline{\overline{f}} = \overline{f}.$$

Then we have

$$\begin{aligned} \overline{f} &= \overline{1 \circ f \circ 1} \\ &= (1 \circ f \circ 1) \cup (1 \circ (1 \circ f \circ 1)) \cup ((1 \circ f \circ 1) \circ 1) \cup (1 \circ (1 \circ f \circ 1) \circ 1) \\ &= 1 \circ f \circ 1. \end{aligned}$$

□

Lemma 2.12. *If S is an intra-regular ordered semigroup, then $1 = 1^2$.*

Proof. By Lemma 2.11, we have $1 \subseteq \overline{1} = 1 \circ 1 \circ 1 \subseteq 1 \circ 1 \subseteq 1$, so $1 = 1^2$.

Lemma 2.13. *Let S be an intra-regular ordered semigroup. Then for all fuzzy subsets f, g of S, we have $\overline{f \circ g} = \overline{g \circ f}$.*

Proof. By Lemma 2.10 and Lemma 2.11, we get

$$\overline{f \circ g} = \overline{(f \circ g)^2} = 1 \circ (f \circ g) \circ (f \circ g) \circ 1 \subseteq 1 \circ g \circ f \circ 1 = \overline{g \circ f}.$$

By symmetry, $\overline{g \circ f} \subseteq \overline{f \circ g}$, thus $\overline{f \circ g} = \overline{g \circ f}$. □

Lemma 2.14. *Let S be an intra-regular ordered semigroup and f, g fuzzy subsets of S. Then $\overline{f} \circ \overline{g} = \overline{f \circ g}$.*

Proof. By Lemmas 2.11, 2.12 and 2.13, we have

$$\begin{aligned} \overline{f} \circ \overline{g} &= 1 \circ f \circ 1 \circ 1 \circ g \circ 1 = 1 \circ f \circ (1 \circ g) \circ 1 = \overline{f \circ (1 \circ g)} \\ &= \overline{(1 \circ g) \circ f} = 1 \circ 1 \circ g \circ f \circ 1 = 1 \circ (g \circ f) \circ 1 \\ &= \overline{g \circ f} = \overline{f \circ g}. \quad \square \end{aligned}$$

Lemma 2.15. *Let S be an ordered semigroup. If the fuzzy ideals of S are semiprime, then S is intra-regular.*

Proof. Let f be a fuzzy subset of S. Since $f^2 \subseteq \overline{f^2}$ and $\overline{f^2}$ is a semiprime fuzzy ideal of S, we have $f \subseteq \overline{f^2}$. Then, by Lemma 2.9, S is intra-regular. □

Lemma 2.16. *Let S be an intra-regular ordered semigroup and f, g fuzzy subsets of S. Then $\overline{f} \cap \overline{g} = \overline{f \circ g}$.*

Proof. By Lemma 2.11, we have
$\overline{f \circ g} = 1 \circ f \circ g \circ 1 \subseteq 1 \circ f \circ 1 = \overline{f}$ and $\overline{f \circ g} = 1 \circ f \circ g \circ 1 \subseteq 1 \circ g \circ 1 = \overline{g}$
i.e. $\overline{f \circ g}$ is a lower bound of $\overline{f}$ and $\overline{g}$. Let now h be a fuzzy subset of S

such that $h \subseteq \overline{f}$ and $h \subseteq \overline{g}$. Then, by Lemmas 2.10, 2.11, 2.12 and 2.14, we obtain

$$\begin{aligned} h &\subseteq \overline{h} = \overline{h^2} = 1 \circ h^2 \circ 1 \subseteq 1 \circ \overline{f} \circ \overline{g} \circ 1 \\ &= 1 \circ (1 \circ f \circ 1) \circ (1 \circ g \circ 1) \circ 1 \\ &= 1 \circ f \circ 1 \circ 1 \circ g \circ 1 = \overline{f} \circ \overline{g} = \overline{f \circ g}. \end{aligned}$$

Therefore $\overline{f \circ g} = \overline{f} \cap \overline{g}$. □

Theorem 2.17. *Let S be an ordered semigroup. The fuzzy ideals of S are prime if and only if S is intra-regular and the fuzzy ideals of S form a chain.*

Proof. $\Longrightarrow$. Suppose the fuzzy ideals of S are prime. Since they are semiprime, by Lemma 2.15, S is intra-regular. Since they are weakly prime, by Theorem 2.7, they form a chain.
$\Longleftarrow$. Let h be a fuzzy ideal of S and f, g fuzzy subsets of S such that $f \circ g \subseteq h$. Since S is intra-regular, by Lemma 2.16, we have $\overline{f} \cap \overline{g} = \overline{f \circ g}$. By hypothesis, $\overline{f} \subseteq \overline{g}$ or $\overline{g} \subseteq \overline{f}$. If $\overline{f} \subseteq \overline{g}$, then

$$f \subseteq \overline{f} = \overline{f} \cap \overline{g} = \overline{f \circ g} \subseteq \overline{h} = h.$$

If $\overline{g} \subseteq \overline{f}$, then

$$g \subseteq \overline{g} = \overline{f} \cap \overline{g} = \overline{f \circ g} \subseteq \overline{h} = h.$$

So h is prime. □

Remark 2.18. A fuzzy ideal h of an ordered semigroup S is called *maximal* if $h \neq 1$ and there is no fuzzy ideal f of S such that $f \neq 1$ and $f \supset h$. If S is an ordered semigroup such that $1^2 = 1$, then the maximal fuzzy ideals of S are weakly prime. Indeed: Let h be a maximal fuzzy ideal of S and f, g fuzzy ideals of S such that $f \circ g \subseteq h$. Suppose $f \not\subseteq h$ and $g \not\subseteq h$. Then $h \cup f$ and $h \cup g$ are fuzzy ideals of S such that $h \cup f \supset h$ and $h \cup g \supset g$. Since h is maximal, we have $h \cup f = 1$ and $h \cup g = 1$. Then we have

$$\begin{aligned} 1 = 1^2 &= (h \cup f) \circ (h \cup g) = h^2 \cup (f \circ h) \cup (h \circ g) \cup (f \circ g) \\ &\subseteq (h \circ 1) \cup (1 \circ h) \cup h = h \subseteq 1, \end{aligned}$$

so we have $h = 1$ which is impossible.
By Lemma 2.12, in intra-regular ordered semigroups, we have $1 = 1^2$. So in intra-regular ordered semigroups the maximal fuzzy ideals are weakly prime.

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