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ON SEQUENCE-COVERING π-s-IMAGES OF LOCALLY SEPARABLE METRIC SPACES

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Abstract. We introduce the notion of double *cs*-cover and give a characterization on sequence-covering π-s-images of locally separable metric spaces by means of double *cs*-covers having π-property of $\aleph_0$-spaces.

1. Introduction

To determine what spaces are the images of "nice" spaces under "nice" mappings is one of the central questions of general topology [2]. In the past, many noteworthy results on images of metric spaces have been obtained. For a survey in this field, see [15], for example. Recently, π-images of metric spaces cause attention once again [6, 9, 10, 16]. It is known that a space is a sequence-covering π-s-image of a metric space if and only if it has a point-star network consisting of point-countable *cs*-covers [10]. In a personal communication, the first author of [16] informs that it seems to be difficult to obtain "nice" characterizations of π-images of locally separable metric spaces (instead of metric). Related to these characterizations, we are interested in the following question.

QUESTION 1.1. How are sequence-covering π-s-images of locally separable metric spaces characterized?

In this paper, we introduce the notion of double *cs*-cover and establish the characterization of locally separable metric spaces under sequence-covering π-s-mappings by means of double *cs*-covers having π-property of $\aleph_0$-spaces.

Throughout this paper, all spaces are assumed to be regular and T_1, all mappings are assumed continuous and onto, a convergent sequence includes its limit point, $\mathbf{N}$ denotes the set of all natural numbers, and $\omega = \mathbf{N} \cup \{0\}$. Let $f\colon X \longrightarrow Y$

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be a mapping, $x \in X$, and $\mathcal{P}$ be a collection of subsets of X, we denote

$$\mathcal{P}_x = \{P \in \mathcal{P} : x \in P\}, \quad \bigcup \mathcal{P} = \bigcup \{P : P \in \mathcal{P}\},$$
$$st(x, \mathcal{P}) = \bigcup \mathcal{P}_x, \quad f(\mathcal{P}) = \{f(P) : P \in \mathcal{P}\}.$$

We say that a convergent sequence $\{x_n : n \in \mathbf{N}\} \cup \{x\}$ converging to x is *eventually in* A if $\{x_n : n \geq n_0\} \cup \{x\} \subset A$ for some $n_0 \in \mathbf{N}$.

Let $\mathcal{P}$ be a collection of subsets of a space X. For each $x \in X$, $\mathcal{P}$ is a *network at* x [2], if $x \in P$ for every $P \in \mathcal{P}$, and if $x \in U$ with U open in X, there exists $P \in \mathcal{P}$ such that $x \in P \subset U$.

$\mathcal{P}$ is *point-countable* [7], if for each $x \in X$, $\mathcal{P}_x$ is countable. $\mathcal{P}$ is a *cs-cover for* X [11], if for each convergent sequence S converging to x in X, there exists some $P \in \mathcal{P}$ such that S is eventually in P. $\mathcal{P}$ is a *cs-network for* X [8], if for each convergent sequence S converging to $x \in U$ with U open in X, there exists some $P \in \mathcal{P}$ such that S is eventually in $P \subset U$.

It is clear that if $\mathcal{P}$ is a *cs*-network for X, then $\mathcal{P}$ is a *cs*-cover for X.

A space X is an $\aleph_0$*-space* [13], if X has a countable *cs*-network. For each $n \in \mathbf{N}$, let $\mathcal{P}_n$ be a cover for X. $\{\mathcal{P}_n : n \in \mathbf{N}\}$ is a *refinement sequence* for X, if $\mathcal{P}_{n+1}$ is a refinement of $\mathcal{P}_n$ for each $n \in \mathbf{N}$. A refinement sequence for X is a *refinement* of X in the sense of [5].

Let $\{\mathcal{P}_n : n \in \mathbf{N}\}$ be a refinement sequence for X. $\{\mathcal{P}_n : n \in \mathbf{N}\}$ is a *point-star network* for X, if $\{st(x, \mathcal{P}_n) : n \in \mathbf{N}\}$ is a network at x for each $x \in X$. Note that this notion is used without the assumption of a refinement sequence in [12], and in [9], $\bigcup \{\mathcal{P}_n : n \in \mathbf{N}\}$ is a σ*-strong network* for X.

Let $\{\mathcal{P}_n : n \in \mathbf{N}\}$ be a point-star network for X. For every $n \in \mathbf{N}$, put $\mathcal{P}_n = \{P_\alpha : \alpha \in A_n\}$, and A_n is endowed with the discrete topology. Put

$$M = \Big\{a = (\alpha_n) \in \prod_{n \in \mathbf{N}} A_n : \{P_{\alpha_n} : n \in \mathbf{N}\} \text{ forms a network at some point } x_a \text{ in } X\Big\}.$$

Then M, which is a subspace of the product space $\prod_{n \in \mathbf{N}} A_n$, is a metric space with metric d described as follows. Let $a = (\alpha_n), b = (\beta_n) \in M$, if $a = b$, then $d(a, b) = 0$, and if $a \neq b$, then $d(a, b) = 1/(\min\{n \in \mathbf{N} : \alpha_n \neq \beta_n\})$.

Define $f \colon M \longrightarrow X$ by choosing $f(a) = x_a$, then f is a mapping, and $(f, M, X, \{\mathcal{P}_n\})$ is a *Ponomarev's system* [12], and if without the assumption of a refinement sequence in the notion of point-star networks, then $(f, M, X, \{\mathcal{P}_n\})$ is a *Ponomarev's system* in the sense of [16].

Let $f \colon X \longrightarrow Y$ be a mapping. f is a *sequence-covering mapping* [14], if for every convergent sequence S of Y, there is a convergent sequence L of X such that $f(L) = S$. f is a *pseudo-open mapping* [1], if $y \in \operatorname{int} f(U)$ whenever $f^{-1}(y) \subset U$ with U open in X.

f is a π*-mapping* [2], if for every $y \in Y$ and for every neighborhood U of y in Y, $d(f^{-1}(y), X - f^{-1}(U)) > 0$, where X is a metric space with a metric d. f is an *s-mapping* [2], if $f^{-1}(y)$ is separable for every $y \in Y$. f is a *π-s-mapping* [10], if f is both π-mapping and s-mapping.

Let X be a space. We recall that X is *sequential* [4], if a subset A of X is closed if and only if any convergent sequence in A has a limit point in A. Also, X is *Fréchet* if for each $x \in \overline{A}$, there exists a sequence in A converging to x.

For terms which are not defined here, please refer to [3, 15].

2. Results

Lemma 2.1. *Let $f\colon X \longrightarrow Y$ be a mapping, and $\mathcal{P}$ be a collection of subsets of X. If f is a sequence-covering mapping and $\mathcal{P}$ is a cs-cover for X, then $f(\mathcal{P})$ is a cs-cover for Y.*

Proof. Let S be a convergent sequence in Y. Then $S = f(L)$ for some convergent sequence L in X. Since $\mathcal{P}$ is a *cs*-cover for X, L is eventually in some $P \in \mathcal{P}$. It implies that S is eventually in $f(P) \in f(\mathcal{P})$. Then $f(\mathcal{P})$ is a *cs*-cover for Y. ■

Let $\{X_\lambda : \lambda \in \Lambda\}$ be a cover for a space X such that each X_λ has a refinement sequence $\{\mathcal{P}_{\lambda,n} : n \in \mathbf{N}\}$. $\{X_\lambda : \lambda \in \Lambda\}$ is a *double cs-cover* for X, if $\{X_\lambda : \lambda \in \Lambda\}$ is a *cs*-cover for X, and each $\mathcal{P}_{\lambda,n}$ is a countable *cs*-cover for X_λ.

$\{X_\lambda : \lambda \in \Lambda\}$ has *π-property*, if $\{\mathcal{P}_n\}_{n\in\mathbf{N}}$ is a point-star network of X, where $\mathcal{P}_n = \bigcup_{\lambda\in\Lambda} \mathcal{P}_{\lambda,n}$ for each $n \in \mathbf{N}$.

Theorem 2.2. *The following are equivalent for a space X.*

(1) *X is a sequence-covering π-s-image of a locally separable metric space,*

(2) *X has a point-countable double cs-cover $\{X_\lambda : \lambda \in \Lambda\}$ having π-property of $\aleph_0$-spaces (i.e., each X_λ is an $\aleph_0$-space).*

Proof. (1) $\Rightarrow$ (2). Let $f\colon M \longrightarrow X$ be a sequence-covering π-s-mapping from a locally separable metric space M with metric d onto X. Since M is a locally separable metric space, $M = \bigoplus_{\lambda\in\Lambda} M_\lambda$ where each M_λ is a separable metric space by [3, 4.4.F]. For each $\lambda \in \Lambda$, let D_λ be a countable dense subset of M_λ, and put

$$f_\lambda = f|_{M_\lambda}, X_\lambda = f_\lambda(M_\lambda).$$

For each $a \in M_\lambda$ and $n \in \mathbf{N}$, put

$$B_\lambda(a, 1/n) = \{b \in M_\lambda : d(a,b) < 1/n\},$$
$$\mathcal{B}_{\lambda,n} = \{B_\lambda(a,1/n) : a \in D_\lambda\}, \quad \mathcal{Q}_{\lambda,n} = f_\lambda(\mathcal{B}_{\lambda,n}).$$

Then $\{\mathcal{Q}_{\lambda,n} : n \in \mathbf{N}\}$ is a cover sequence of countable covers for X_λ, and for each $\lambda \in \Lambda$ and $n \in \mathbf{N}$, $\mathcal{Q}_{\lambda,n+1}$ is a refinement of $\mathcal{Q}_{\lambda,n}$.

For each $\lambda \in \Lambda$, put $\Lambda_\lambda = \{\alpha \in \Lambda : X_\alpha \cap f(D_\lambda) \neq \emptyset\}$, for each $\lambda \in \Lambda$ and $n \in \mathbf{N}$, put

$$\mathcal{P}_{\lambda,n} = \{Q \cap X_\lambda : Q \in \mathcal{Q}_{\alpha,n}, \alpha \in \Lambda_\lambda\},$$

and for each $n \in \mathbf{N}$, put $\mathcal{P}_n = \bigcup\{\mathcal{P}_{\lambda,n} : \lambda \in \Lambda\}$.

It is clear that $\{X_\lambda : \lambda \in \Lambda\}$ is a cover for X such that each X_λ has a refinement sequence $\{\mathcal{P}_{\lambda,n} : n \in \mathbf{N}\}$.

(a) $\{X_\lambda : \lambda \in \Lambda\}$ *is point-countable.*

Since f is an s-mapping, $\{X_\lambda : \lambda \in \Lambda\}$ is point-countable.

(b)$\{X_\lambda : \lambda \in \Lambda\}$ *is a cs-cover for X.*

Note that $\{M_\lambda : \lambda \in \Lambda\}$ is a cs-cover for M, then $\{X_\lambda : \lambda \in \Lambda\}$ is a cs-cover for X by Lemma 2.1.

(c) *For every* $\lambda \in \Lambda$ *and* $n \in \mathbf{N}$, $\mathcal{P}_{\lambda,n}$ *is a countable cs-cover for* X_λ.

Since D_λ is countable and $\{X_\alpha : \alpha \in \Lambda\}$ is point-countable, Λ_λ is countable. Then $\mathcal{P}_{\lambda,n}$ is countable. Let $\{x_i : i \in \omega\}$ be a convergent sequence converging to x_0 in X_λ. Since $M_\lambda = \overline{D_\lambda}$, there exists a sequence $\{a_i : i \in \mathbf{N}\} \subset D_\lambda$ such that $a_i \to a_0$. Then $\{f(a_i) : i \in \mathbf{N}\} \subset f(D_\lambda)$ and $f(a_i) \to x_0$. For every $i \in \mathbf{N}$, put

$$z_{2i} = x_i, z_{2i+1} = f(a_i).$$

Then $S = \{z_i : i \in \mathbf{N}\} \cup \{x_0\}$ is a convergent sequence converging to x_0 in X_λ. Since f is sequence-covering, $S = f(L)$ for some convergent sequence in M. Thus, there exists some $\alpha \in \Lambda$, and some $a \in M_\alpha$ such that L is eventually in $B_\alpha(a, 1/n)$. It implies that S is eventually in $f(B_\alpha(a, 1/n)) \in \mathcal{Q}_{\alpha,n}$, and then, S is eventually in $f(B_\alpha(a, 1/n)) \cap X_\lambda$. From this fact we get that $\alpha \in \Lambda_\lambda$, and $\{x_i : i \in \omega\}$ is eventually in $f(B_\alpha(a, 1/n)) \cap X_\lambda \in \mathcal{P}_{\lambda,n}$.

Hence, $\mathcal{P}_{\lambda,n}$ is a countable cs-cover for X_λ.

(d) $\{X_\lambda : \lambda \in \Lambda\}$ *has* π*-property.*

Since $\{\mathcal{P}_{\lambda,n} : n \in \mathbf{N}\}$ is a refinement sequence for X_λ for each $\lambda \in \Lambda$, $\{\mathcal{P}_n : n \in \mathbf{N}\}$ is a refinement sequence for X. For each $x \in U$ with U open in X. Since f is a π-mapping, $d(f^{-1}(x), M - f^{-1}(U)) > 2/n$ for some $n \in \mathbf{N}$. Then, for each $\lambda \in \Lambda$ with $x \in X_\lambda$, we get $d(f_\lambda^{-1}(x), M_\lambda - f_\lambda^{-1}(U_\lambda)) > 2/n$ where $U_\lambda = U \cap X_\lambda$. Let $a \in D_\lambda$ and $x \in f_\lambda(B_\lambda(a, 1/n)) \in \mathcal{Q}_{\lambda,n}$. We shall prove that $B_\lambda(a, 1/n) \subset f_\lambda^{-1}(U_\lambda)$. In fact, if $B_\lambda(a, 1/n) \not\subset f_\lambda^{-1}(U_\lambda)$, then pick $b \in B_\lambda(a, 1/n) - f_\lambda^{-1}(U_\lambda)$. Note that $f_\lambda^{-1}(x) \cap B_\lambda(a, 1/n) \neq \emptyset$, pick $c \in f_\lambda^{-1}(x) \cap B_\lambda(a, 1/n)$, then $d(f_\lambda^{-1}(x), M_\lambda - f_\lambda^{-1}(U_\lambda)) \leq d(c, b) \leq d(c, a) + d(a, b) < 2/n$. It is a contradiction. So $B_\lambda(a, 1/n) \subset f_\lambda^{-1}(U_\lambda)$, then $f_\lambda(B_\lambda(a, 1/n)) \subset U_\lambda$. It implies that $st(x, \mathcal{Q}_{\lambda,n}) \subset U_\lambda$, and hence $st(x, \mathcal{Q}_n) = \bigcup\{st(x, \mathcal{Q}_{\lambda,n}) : \lambda \in \Lambda \text{ with } x \in X_\lambda\} \subset U$. For every $P \in \mathcal{P}_{\lambda,n}$ with $x \in P$, we have $P = Q \cap X_\lambda$ for some $Q \in \mathcal{Q}_{\alpha,n}$ with $\alpha \in \Lambda_\lambda$. It implies that $P \subset Q$ and $x \in Q$. Then $st(x, \mathcal{P}_{\lambda,n}) \subset st(x, \mathcal{Q}_n)$. Therefore $st(x, \mathcal{P}_n) = \bigcup\{st(x, \mathcal{P}_{\lambda,n}) : \lambda \in \Lambda \text{ with } x \in X_\lambda\} \subset st(x, \mathcal{Q}_n) = \bigcup\{st(x, \mathcal{Q}_{\lambda,n}) : \lambda \in \Lambda \text{ with } x \in X_\lambda\} \subset U$.

Hence, $\{\mathcal{P}_n\}_{n \in \mathbf{N}}$ is a point-star network for X, i.e., $\{X_\lambda : \lambda \in \Lambda\}$ has π-property.

(e) *For every* $\lambda \in \Lambda$, X_λ *is an* $\aleph_0$*-space.*

We shall prove that $\mathcal{P}_\lambda = \bigcup\{\mathcal{P}_{\lambda,n} : n \in \mathbf{N}\}$ is a countable cs-network for X_λ. Since each $\mathcal{P}_{\lambda,n}$ is countable, $\mathcal{P}_\lambda$ is countable. Let $\{x_i : i \in \omega\}$ be a convergent sequence converging to $x_0 \in U_\lambda$ with U_λ open in X_λ, and let $x_0 = f(a_0)$ for some $a_0 \in M_\lambda$. Since $M_\lambda = \overline{D_\lambda}$, there exists a sequence $\{a_i : i \in \mathbf{N}\} \subset D_\lambda$ such that $a_i \to a_0$. Then $\{f(a_i) : i \in \mathbf{N}\} \subset f(D_\lambda)$ and $f(a_i) \to x_0$. For every $i \in \mathbf{N}$, put

$$z_{2i} = x_n, z_{2i+1} = f(a_i).$$

Then $S = \{z_i : i \in \mathbf{N}\} \cup \{x_0\}$ is a convergent sequence converging to x_0 in X_λ. Since f is sequence-covering, $S = f(L)$ for some convergent sequence in M. Thus, there exists some $\alpha \in \Lambda$, some $a \in M_\alpha$, and some $n \in \mathbf{N}$ such that L is eventually in $B_\alpha(a, 1/n) \subset f^{-1}(U)$, where U is open in X and $U \cap X_\lambda = U_\lambda$. It implies that S is eventually in $f(B_\alpha(a, 1/n)) \subset U$, and then, S is eventually in $f(B_\alpha(a, 1/n)) \cap X_\lambda \subset U \cap X_\lambda = U_\lambda$. From this fact we get $\alpha \in \Lambda_\lambda$, and $\{x_i : i \in \omega\}$ is eventually in $f(B_\alpha(a, 1/n)) \cap X_\lambda \subset U_\lambda$. Then $\mathcal{P}_\lambda$ is a countable cs-network for X_λ.

(2) $\Rightarrow$ (1). For each $\lambda \in \Lambda$, since each X_λ is an $\aleph_0$-space, X_λ has a countable cs-network $\mathcal{Q}_\lambda$. For each $\lambda \in \Lambda$ and $n \in \mathbf{N}$, put

$$\mathcal{R}_{\lambda,n} = \mathcal{P}_{\lambda,n} \cap \mathcal{Q}_\lambda = \{P \cap Q : P \in \mathcal{P}_{\lambda,n}, Q \in \mathcal{Q}_\lambda\}.$$

Then each $\mathcal{R}_{\lambda,n}$ is countable and, for each $\lambda \in \Lambda$, $\{\mathcal{R}_{\lambda,n} : n \in \mathbf{N}\}$ is a refinement sequence for X_λ. Let $x \in U_\lambda$ with U_λ open in X_λ. We get $U_\lambda = U \cap X_\lambda$ with some U open in X. Since $st(x, \mathcal{P}_n) \subset U$ for some $n \in \mathbf{N}$, $st(x, \mathcal{P}_{\lambda,n}) \subset U_\lambda$. Note that $st(x, \mathcal{R}_{\lambda,n}) \subset st(x, \mathcal{P}_{\lambda,n})$, then $st(x, \mathcal{R}_{\lambda,n}) \subset U_\lambda$. It implies that $\{\mathcal{R}_{\lambda,n} : n \in \mathbf{N}\}$ is a point-star network for X_λ. Then the Ponomarev's system $(f_\lambda, M_\lambda, X_\lambda, \{\mathcal{R}_{\lambda,n}\})$ exists. Since each $\mathcal{R}_{\lambda,n}$ is countable, M_λ is a separable metric space with metric d_λ described as follows. For $a = (\alpha_n), b = (\beta_n) \in M_\lambda$, if $a = b$, then $d_\lambda(a, b) = 0$, and if $a \neq b$, then $d_\lambda(a, b) = 1/(\min\{n \in \mathbf{N} : \alpha_n \neq \beta_n\})$.

Put $M = \oplus_{\lambda \in \Lambda} M_\lambda$ and define $f \colon M \longrightarrow X$ by choosing $f(a) = f_\lambda(a)$ for every $a \in M_\lambda$ with some $\lambda \in \Lambda$. Then f is a mapping and M is a locally separable metric space with metric d as follows. For $a, b \in M$, if $a, b \in M_\lambda$ for some $\lambda \in \Lambda$, then $d(a, b) = d_\lambda(a, b)$, and otherwise, $d(a, b) = 1$.

We shall prove that f is a sequence-covering π-s-mapping.

(a) *f is a π-mapping.*

Let $x \in U$ with U open in X, then $st(x, \mathcal{P}_n) \subset U$ for some $n \in \mathbf{N}$. So, for each $\lambda \in \Lambda$ with $x \in X_\lambda$, we get $st(x, \mathcal{R}_{\lambda,n}) \subset st(x, \mathcal{P}_{\lambda,n}) \subset U_\lambda$ where $U_\lambda = U \cap X_\lambda$. It is implies that $d_\lambda(f_\lambda^{-1}(x), M_\lambda - f_\lambda^{-1}(U_\lambda)) \geq 1/n$. In fact, if $a = (\alpha_k) \in M_\lambda$ such that $d_\lambda(f_\lambda^{-1}(x), a) < 1/n$, then there is $b = (\beta_k) \in f_\lambda^{-1}(x)$ such that $d_\lambda(a, b) < 1/n$. So $\alpha_k = \beta_k$ if $k \leq n$. Note that $x \in R_{\beta_n} \subset st(x, \mathcal{R}_{\lambda,n}) \subset U_\lambda$. Then $f_\lambda(a) \in R_{\alpha_n} = R_{\beta_n} \subset st(x, \mathcal{R}_{\lambda,n}) \subset U_\lambda$. Hence $a \in f_\lambda^{-1}(U_\lambda)$. It implies that $d_\lambda(f_\lambda^{-1}(x), a) \geq 1/n$ if $a \in M_\lambda - f_\lambda^{-1}(U_\lambda)$, i.e., $d_\lambda(f_\lambda^{-1}(x), M_\lambda - f_\lambda^{-1}(U_\lambda)) \geq 1/n$. Therefore

$$\begin{aligned} d(f^{-1}(x), M - f^{-1}(U)) &= \inf\{d(a, b) : a \in f^{-1}(x), b \in M - f^{-1}(U)\} \\ &= \min\big\{1, \inf\{d_\lambda(a, b) : a \in f_\lambda^{-1}(x), b \in M_\lambda - f_\lambda^{-1}(U_\lambda), \lambda \in \Lambda\}\big\} \geq 1/n > 0. \end{aligned}$$

It implies that f is a π-mapping.

(b) *f is an s-mapping.*

For each $x \in X$, since $\{X_\lambda : \lambda \in \Lambda\}$ is point-countable, $\Lambda_x = \{\lambda \in \Lambda : x \in X_\lambda\}$ is countable. Then, for each $\lambda \in \Lambda_x$, $f_\lambda^{-1}(x)$ is separable by the fact that M_λ is separable metric. Therefore $f^{-1}(x) = \bigcup\{f_\lambda^{-1}(x) : \lambda \in \Lambda_x\}$ is separable. It implies that f is an s-mapping.

(c) *f is sequence-covering.*

For each $\lambda \in \Lambda$, let S be a convergent sequence in X_λ. For each $n \in \mathbf{N}$, since $\mathcal{P}_{\lambda,n}$ and $\mathcal{Q}_\lambda$ are cs-covers for X_λ, S is eventually in $P \cap Q$ for some $P \in \mathcal{P}_{\lambda,n}$ and some $Q \in \mathcal{Q}_\lambda$. Then $\mathcal{R}_{\lambda,n}$ is a cs-cover for X_λ. It follows from [16, Lemma 2.2] that f_λ is sequence-covering.

Let L be a convergent sequence in X. Since $\{X_\lambda : \lambda \in \Lambda\}$ is a cs-cover for X, L is eventually in some X_λ. Since f_λ is sequence-covering, $L \cap X_\lambda = f_\lambda(L_\lambda)$ for some convergent sequence L_λ in M_λ. On the other hand, $L - X_\lambda = f(F)$ for some finite F in M. Put $K = F \cup L_\lambda$, then K is a convergent sequence in M satisfying $f(K) = L$. It implies that f is sequence-covering. ■

Corollary 2.3. *The following are equivalent for a space X.*

(1) *X is a sequence-covering quotient (resp. pseudo-open) π-s-image of a locally separable metric space,*

(2) *X is a sequential (resp. Fréchet) space with a point-countable double cs-cover $\{X_\lambda : \lambda \in \Lambda\}$ having π-property of $\aleph_0$-spaces.*

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