

ABSTRACT. This is the first of two interconnected parts: Part I contains the geometric theory of *generalized modular forms* and their connections with the cooperation algebra for elliptic cohomology, Ell_*Ell , while Part II is devoted to the more algebraic theory associated with Hecke algebras and stable operations in elliptic cohomology.

We investigate the structure of the stable operation algebra Ell^*Ell by first determining the dual cooperation algebra Ell_*Ell . A major ingredient is our identification of the cooperation algebra Ell_*Ell with a ring of generalized modular forms whose exact determination involves understanding certain integrality conditions; this is closely related to a calculation by N. Katz of the ring of all ‘divided congruences’ amongst modular forms. We relate our present work to previous constructions of Hecke operators in elliptic cohomology. We also show that a well known operator on modular forms used by Ramanujan, Swinnerton-Dyer, Serre and Katz cannot extend to a stable operation.