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ON ABSOLUTELY CLOSED MULTIVALUED MAPPINGS OF TOPOLOGICAL SPACES

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Abstract. We present results concerning absolute closeness of multivalued mappings for some well-known classes of pointwise closed mappings. The main results are the characterizations of absolute closeness for cofinally continuous and for residually continuous multivalued mappings. We found necessary and sufficient conditions so that the multivalued mapping $F : X \longrightarrow Y$ cannot be extended to a cofinally or a residually continuous mapping $F : X^* \longrightarrow Y$ from a space X^* in which X is a proper dense subset. We also proved some characterizations of cofinally and residually continuous mappings.

Our aim is to give, for multivalued mappings, some generalizations of the results obtained in [**1**], [**2**] and [**3**] related to the characterizations of absolute closeness for a singlevalued continuous mappings.

In [**4**, theorem 3], were proved some sufficient conditions for multivalued pointwise closed mapping $F : X \longrightarrow Y$ that cannot be extended to an upper semicontinuous (u.s.c.) or a lower semicontinuous (l.s.c.) mapping from a space X^* in which X is a dense proper subset, i.e., there were found sufficient conditions for the absolute u.s.c. or absolute l.s.c. closeness of multivalued mappings.

We prove a criterion for absolute closeness of mappings for the class of multivalued pointwise closed *cofinal continuous* mappings in which the class of u.s.c. mappings is contained, if the space Y is T_3-space. Also, we prove a criterion for absolute closeness of mappings for the class of multivalued pointwise closed *residual continuous* mappings which is identical to the class of l.s.c. mappings.

The terminology and the notation that we shall use throughout this article will be as follows.

(1) Let $F : X \longrightarrow Y$ be a multivalued mapping of a topological space X onto a topological space Y. The *inverse image* of $y \in Y$ by the mapping F is the set

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$F'y = \{x \mid y \in Fx\}$. The *inverse mapping* $F' : Y \longrightarrow X$ of the mapping F is defined pointwise by the inverse images $F'y$, for all $y \in Y$. The *image* of a set $A \subseteq X$ by the mapping F is the set $FA = \bigcup\{Fx \mid x \in A\} = \{y \mid F'y \cap A \neq \emptyset\}$ and the *small image* of the same set by the mapping F is the set $F^{\#}A = \{y \mid F'y \subseteq A\}$ $= Y \setminus F(X \setminus A)$. The *inverse image* of a set $B \subseteq Y$ by the mapping F is the set $F'B = \bigcup\{F'y \mid y \in B\} = \{x \mid Fx \cap B \neq \emptyset\}$ and the *small inverse image* of the same set by the mapping F is the set $F^{\flat}B = \bigcup\{x \mid Fx \subseteq B\} = X \setminus F'(Y \setminus B)$.

In addition, we shall give some well-known inclusions, implications and definitions concerning multivalued mappings that will be used frequently in the sequel. If $A \subseteq X$is a set, then $F^{\#}A \subseteq FA$ and $F^{\flat}F^{\#}A \subseteq F'F^{\#}A \subseteq A \subseteq F^{\flat}FA \subseteq F'FA$. If $B \subseteq Y$ is a set, then $F^{\flat}B \subseteq F'B$ and $F^{\#}F^{\flat}B \subseteq FF^{\flat}B \subseteq B \subseteq F^{\#}F'B \subseteq FF'B$. If $A, A' \subseteq Y$, then $A \cap A' = \emptyset \implies FA \cap F^{\#}A' = \emptyset$, and if $B, B' \subseteq Y$, then $B \cap B' = \emptyset \implies F'B \cap F^{\flat}B' = \emptyset$.

(2) Let us remark that multivalued cofinal and residual mappings are usually defined locally, by nets in a topological space, as follows.

Previously note, that a net $\varphi : (\mathfrak{D}, \leq) \longrightarrow X$ in a topological space X *converges* to $x \in X$ (written $\varphi(\mathfrak{D}, \leq) \longrightarrow x$) provided that it is *residual* in every nhood U of x, i.e., if for each nhood $U \in \mathfrak{U}(x)$ there is a $d_0 \in \mathfrak{D}$ such that $d_0 \leq d \implies \varphi(d) = x_d \in U$ for each $d \in \mathfrak{D}$. If $F : X \longrightarrow Y$ is a multivalued mapping of a space X onto a space Y and $\varphi : (\mathfrak{D}, \leq) \longrightarrow X$ is a net in X converging to an $x \in X$, then $\limsup_d Fx_d$ and $\liminf_d Fx_d$ denote the topological *limes superior*, respectively *limes inferior* of the net $\{Fx_d \mid d \in \mathfrak{D}\}$ in Y. Then:

(3) $y \in \limsup_d Fx_d$ iff for each nhood $V \in \mathfrak{V}(y)$ in Y the set

$$\Delta(V) = \{d \mid Fx_d \cap V \neq \emptyset\}$$

is *cofinal* in the directed set $(\mathfrak{D}, \leq)$, i.e., for each index $d \in \mathfrak{D}$ there is some $d' \in \Delta(V)$ so that $d \leq d'$;

$y \in \liminf_d Fx_d$ iff for each nhood $V \in \mathfrak{V}(y)$ in Y the set $\Delta(V)$ is *residual* in the directed set $(\mathfrak{D}, \leq)$, i.e., there is some $d \in \mathfrak{D}$ so that $d' \geq d$ implies $d' \in \Delta(V)$.

Note that $\liminf_d Fx_d \subseteq \limsup_d Fx_d$, since the set $\Delta(V)$ is cofinal in the directed set $(\mathfrak{D}, \leq)$, if it is residual in the directed set $(\mathfrak{D}, \leq)$.

(4) A multivalued mapping $F : X \longrightarrow Y$ is *cofinally continuous* (*residually continuous*) iff $\limsup_d Fx_d \subseteq Fx$ $(Fx \subseteq \liminf_d Fx_d)$ for each $x \in X$ and each net $\varphi : (\mathfrak{D}, \leq) \longrightarrow X$ converging to x

(5) The mapping F is Y*-compact* if the image Fx of each $x \in X$ is compact and F is X*-compact* if the inverse image $F'y$ of each $y \in Y$ is compact. The mapping F is *closed* (*regularly closed*) if the image of each closed (regularly closed) set is closed. Furthermore we shall assume that all mappings are pointwise closed, i.e., the image Fx of each point $x \in X$ is a closed set.

A criterion of *cofinal continuity* of multivalued mappings will be proved first.

THEOREM 1. *A multivalued mapping $F : X \longrightarrow Y$ of a space X onto a space Y is cofinally continuous iff for each $x \in X$*

$$Fx = \{\overline{FU} \mid U \in \mathfrak{U}(x)\}, \tag{1}$$

For the proof of the theorem we need the following

LEMMA 1. *Let* $F : X \longrightarrow Y$ *be a multivalued mapping. Then for each* $x \in X$ *and each* $y \in \bigcap\{\overline{FU} \mid U \in \mathfrak{U}(x)\}$ *there is a net* $\varphi : (\mathfrak{D}, \leq) \longrightarrow X$ *converging to* x, *so that* $y \in \liminf_d Fx_d$.

Proof. At first we define a relation $\leq$ in the Cartesian product

$$\mathfrak{D} = \mathfrak{U}(x) \times \mathfrak{V}(y)$$

of the open neighbourhood system $\mathfrak{U}(x)$ of x and the open neighbourhood system $\mathfrak{V}(y)$ of y, putting

$$(*) \qquad (U, V) \leq (U', V') :\Leftrightarrow (U' \subseteq U) \wedge (V' \subseteq V).$$

It is easy to see that $\leq$ is a partial order and a *direction* on $\mathfrak{D}$. Indeed, let $(U, V), (U', V') \in \mathfrak{U}(x) \times \mathfrak{V}(y) = \mathfrak{D}$. Then from $U, U' \in \mathfrak{U}(x)$ and $U'' = U \cap U' \in \mathfrak{U}(x)$, follows $U'' \subseteq U$ and $U'' \subseteq U'$ and, also, from $V, V' \in \mathfrak{V}(y)$, $V'' = V \cap V' \in \mathfrak{V}(y)$, follows $V'' \subseteq V$ and $V'' \subseteq V'$. By the definition (*) we have

$$(U, V) \leq (U'', V'') \text{ and } (U', V') \leq (U'', V'').$$

If $y \in \bigcap\{\overline{FU} \mid U \in \mathfrak{U}(x)\}$, then $y \in \overline{FU}$ for every nhood $U \in \mathfrak{U}(x)$, so $V \cap FU \neq \emptyset$ for every nhood $V \in \mathfrak{V}(x)$. Since $V \cap FU \neq \emptyset \iff F'V \cap U \neq \emptyset$, the net $\varphi : (\mathfrak{D}, \leq) \longrightarrow X$ can be defined putting $\varphi(U, V) = x_d \in F'V \cap U$, for each $d = (U, V) \in \mathfrak{D}$, where x_d is any point in $F'V \cap U$. The net $\varphi : (\mathfrak{D}, \leq) \longrightarrow X$ converges to x. Indeed, if $U \in \mathfrak{U}(x)$ is any nhood, then $d_0 = (U, Y)$ is such an "index" that for each "index" $d' = (U', V')$

$$d_0 \leq d' \implies \varphi(d') = \varphi(U', V') = x_{d'} \in F'V' \cap U' \subseteq U' \subseteq U,$$

since $d_0 \leq d'$ implies $U' \subseteq U$.

Now let $V, V' \in \mathfrak{V}(y)$ so that $V' \subseteq V$. Then for each "index" $d' = (U', V')$, we have $d_0 = (X, V) \leq (U', V') = d'$ and

$$d_0 \leq d' \implies \varphi(U', V') = \varphi(d') = x_{d'} \in F'V' \cap U' \subseteq F'V' \subseteq F'V.$$

Since $x_{d'} \in F'V \iff Fx_{d'} \cap V \neq \emptyset$, we have proved that the set

$$\Delta(V) = \{d \mid Fx_d \cap V \neq \emptyset\}$$

is residual in the directed set $(\mathfrak{D}, \leq)$. Since $V \in \mathfrak{V}(x)$ is an arbitrary set, it follows that $y \in \liminf_d Fx_d$.

Proof of Theorem 1. (a) Let $F : X \longrightarrow Y$ be a cofinally continuous mapping and $y \in \bigcap\{\overline{FU} \mid U \in \mathfrak{U}(x)\}$. By Lemma 1 there is a net $\varphi : (\mathfrak{D}, \leq) \longrightarrow X$ converging to x so that $y \in \liminf_d Fx_d \subseteq \limsup_d Fx_d$. Since F is cofinally continuous, $\limsup_d Fx_d \subseteq Fx$, so $y \in Fx$ and we have proved the implication

$$y \in \bigcap\{\overline{FU} \mid U \in \mathfrak{U}(x)\} \implies y \in Fx,$$

from which follows the inclusion $\bigcap\{\overline{FU} \mid U \in \mathfrak{U}(x)\} \subseteq Fx$. Since the converse inclusion is obvious, the equality (1) is proved.

(b) Now let the equality (1) hold and let $\varphi : (\mathfrak{D}, \leq) \longrightarrow X$ be a net converging to x. We shall prove the inclusion

$$\limsup_d Fx_d \subseteq Fx. \tag{2}$$

Let $y \in \limsup_d Fx_d$. Then for every $V \in \mathfrak{V}(y)$ the set $\Delta(V) = \{d \mid Fx_d \cap V \neq \emptyset\}$ is cofinal in $(\mathfrak{D}, \leq)$. Since $\varphi(\mathfrak{D}, \leq) \longrightarrow x$, for each $U \in \mathfrak{U}(x)$, there is an index $d_U \in \mathfrak{D}$ so that $d_U \leq d \implies \varphi(d) = x_d \in U$, for every index $d \in \mathfrak{D}$,

As the set $\Delta(V)$ is cofinal in $(\mathfrak{D}, \leq)$ for d_U there is $d' \in \Delta(V)$ so that $d_U \leq d'$. Then $x_{d'} \in U$ and $Fx_{d'} \cap V \neq \emptyset$, so $\emptyset \neq Fx_{d'} \cap V \subseteq FU \cap V$ and, because $V \in \mathfrak{V}(y)$ is any set, $y \in \overline{FU}$. But since $U \in \mathfrak{U}(x)$ is an arbitrary set too,

$$y \in \bigcap\{\overline{FU} \mid U \in \mathfrak{U}(x)\} = Fx.$$

So the implication $y \in \limsup_d Fx_d \implies y \in Fx$, as well as the inclusion (2) are proved. By definition of cofinal continuity of multivalued mappings, F is cofinally continuous in x and Theorem 1 is proved. □

COROLLARY 1.1. *A multivalued mapping $F : X \longrightarrow Y$ of a space X onto a space Y is cofinally continuous iff the inverse mapping $F' : Y \longrightarrow X$ is cofinally continuous.*

The proof of the corollary follows from Theorem 1 and the next

LEMMA 2. *Let $F : X \longrightarrow Y$ be a multivalued mapping of a space X onto a space Y. Then*

$$(\forall x \in X)(Fx = \bigcap\{\overline{FU} \mid U \in \mathfrak{U}(x)\}) \tag{a}$$

iff

$$(\forall y \in Y)(F'y = \bigcap\{\overline{F'V} \mid V \in \mathfrak{V}(y)\}). \tag{b}$$

Proof. Let the equality (a) hold and $y \in Y$. Then

$$\begin{aligned} x \in \bigcap\{\overline{F'V} \mid V \in \mathfrak{V}(y)\} &\iff (\forall V \in \mathfrak{V}(y))(x \in \overline{F'V}) \\ &\iff (\forall V \in \mathfrak{V}(y))(\forall U \in \mathfrak{U}(x))(U \cap F'V \neq \emptyset) \\ &\iff (\forall U \in \mathfrak{U}(x))(\forall V \in \mathfrak{V}(y))(FU \cap V \neq \emptyset) \\ &\iff (\forall U \in \mathfrak{U}(x))(y \in \overline{FU}) \\ &\iff (y \in \bigcap\{\overline{FU} \mid U \in \mathfrak{U}(x)\} = Fx), \end{aligned}$$

by (a). Since $y \in Fx \iff x \in F'y$, it is proved the implication

$$x \in \bigcap\{\overline{F'V} \mid V \in \mathfrak{V}(y)\} \implies x \in F'y,$$

which implies the inclusion $\bigcap\{\overline{F'V} \mid V \in \mathfrak{V}(y)\} \subseteq F'y$. As the converse inclusion is obvious, the implication $(a) \implies (b)$ is proved. The proof of the implication $(b) \implies (a)$ is similar. $\square$

Proof of Corollary 1. By Theorem 1 and Lemma 2 the mapping F is cofinally continuous iff $(\forall y \in Y)(F'y = \bigcap\{\overline{F'V} \mid V \in \mathfrak{V}(y)\})$. But, by Theorem 1 the mapping F' is cofinally continuous iff $(\forall y \in Y)(F'y = \bigcap\{\overline{F'V} \mid V \in \mathfrak{V}(y)\})$, so the mapping F is cofinally continuous iff the mapping F' is cofinally continuous.

COROLLARY 1.2. *A multivalued u.s.c. mapping $F : X \longrightarrow Y$ of a space X onto a space Y is cofinally continuous, if: (i) Y is a T_3-space or (ii) Y is a T_2-space and the mapping F is Y-compact.*

Proof. The corollary will be proved if we show

$$Fx = \bigcap\{\overline{FU} \mid U \in \mathfrak{U}(x)\} \tag{1}$$

for each $x \in X$. Let $y \in Y \setminus Fx$. From the assumptions (i) or (ii) there exists an open set W so that $Fx \subseteq W \subseteq \overline{W} \subseteq Y \setminus \{y\}$. Then it follows that $x \in F^\flat Fx \subseteq F^\flat W = U_0 \in \mathfrak{U}(x)$, since U_0 is open because F is u.s.c. mapping. Hence, $\bigcap\{\overline{FU} \mid U \in \mathfrak{U}(x)\} \subseteq \overline{FU_0} \subseteq \overline{FF^\flat W} \subseteq \overline{W} \subseteq Y \setminus \{y\}$ and the implication $y \notin Fx \implies y \notin \bigcap\{\overline{FU} \mid U \in \mathfrak{U}(x)\}$ and the inclusion $\bigcap\{\overline{FU} \mid U \in \mathfrak{U}(x)\} \subseteq Fx$ are proved. Since the converse inclusion is obvious, the corollary is proved. $\square$

THEOREM 2. *Let $F : X \longrightarrow Y$ be a multivalued mapping of a space X onto a space Y and let X^* be a space in which X is a dense subspace so that*

$$(\forall x^* \in X^*)\Big(\bigcap\{\overline{F(U^* \cap X)} \mid U^* \in \mathfrak{U}^*(x^*)\} \neq \emptyset\Big),$$

where $\mathfrak{U}^*(x^*)$ denotes the open neighbourhood system of $x^* \in X^*$. Then the mapping $\overline{F} : X^* \longrightarrow Y$ defined by $\overline{F}x^* = \bigcap\{\overline{F(U^* \cap X)} \mid U^* \in \mathfrak{U}^*(x^*)\}$, is cofinally continuous, for each $x^* \in X^*$.

If F is cofinally continuous, then $\overline{F}$ is a cofinally continuous extension of F.

Proof. Let $\varphi : (\mathfrak{D}, \leq) \longrightarrow X^*$ be a net in X^* converging to an $x^* \in X^*$. If $\limsup_d \overline{F}x_d^* \not\subseteq \overline{F}x^*$, then there is a $y \in \limsup_d \overline{F}x_d^* \setminus \overline{F}x^*$. But then $y \notin \overline{F}x^* = \bigcap\{\overline{F(U^* \cap X)} \mid U^* \in \mathfrak{U}^*(x^*)\}$, and there is a $U_0^* \in \mathfrak{U}^*(x^*)$ so that $y \notin \overline{F(U_0^* \cap X)}$. Then $y \in Y \setminus \overline{F(U_0^* \cap X)} = V_0 \in \mathfrak{V}(y)$, since V_0 is open in Y. Because $y \in \limsup_d \overline{F}x_d^*$, the set $\Delta(V_0) = \{d \mid \overline{F}x_d^* \cap V_0 \neq \emptyset\}$ is cofinal in the directed set $(\mathfrak{D}, \leq)$.

Since $\varphi(\mathfrak{D}, \leq) \longrightarrow x^*$, for the set $U_0^* \in \mathfrak{U}^*(x^*)$, there is an index $d_0 \in \mathfrak{D}$ so that for each $d \in \mathfrak{D}$, $d_0 \leq d \implies \varphi(d) = x_d^* \in U_0^*$. Then as the set U_0^* is open, $U_0^* \in \mathfrak{U}^*(x_d^*)$. So if $d_0 \leq d$, then

$$\overline{F}x_d^* = \bigcap\{\overline{F(U^* \cap X)} \mid U^* \in \mathfrak{U}^*(x_d^*)\} \subseteq \overline{F(U_0^* \cap X)} \subseteq Y \setminus V_0. \tag{3}$$

By cofinality of $\Delta(V_0)$ it follows that, for the index d_0, there is an index $d' \in \Delta(V_0)$ so that $d_0 \leq d'$ and $\overline{F}x_{d'}^* \cap V_0 \neq \emptyset$. Hence, by (3) $\emptyset \neq \overline{F}x_{d'}^* \cap V_0 \subseteq (Y \setminus V_0) \cap V_0$ and we have contradiction. Therefore for every $x^* \in X^*$ and every net $\varphi : (\mathfrak{D}, \leq) \longrightarrow X^*$ converging to x^* we have $\limsup_d \overline{F}x_d^* \subseteq \overline{F}x^*$, and the mapping $\overline{F}$ is cofinally continuous.

Now we show that $\overline{F}$ is an extension of F, i.e., $\overline{F}x = Fx$, for each $x \in X$. Since $x \in U^* \cap X$ for each $U^* \in \mathfrak{U}^*(x)$, we have

$$Fx \subseteq \bigcap\{\overline{F(U^* \cap X)} \mid U^* \in \mathfrak{U}^*(x^*)\} = \overline{F}x.$$

If $y \in \overline{F}x = \bigcap\{\overline{F(U^* \cap X)} \mid U^* \in \mathfrak{U}^*(x^*)\}$, then by Lemma 1 there is a net $\varphi : (\mathfrak{D}, \leq) \longrightarrow X$ converging to x, so that

$$y \in \limsup_d Fx_d. \tag{4}$$

Since F is cofinally continuous, we have $\limsup_d Fx_d \subseteq Fx$, and $y \in Fx$ by (4). So the implication $y \in \overline{F}x \implies y \in Fx$, as well as the inclusion $\overline{F}x \subseteq Fx$ are proved and the proof of the theorem is completed. □

Observe that by [**6**] a topological space X is *absolutely closed* iff there does not exist any space X^* with X as a dense subspace. Also by [**6**], a topological space X is *absolutely closed* iff every free open ultrafilter $\mathfrak{U}$ (i.e., maximal centred system of open sets) has a nonempty adherence $\operatorname{ad}\mathfrak{U} = \bigcap\{\overline{U} \mid U \in \mathfrak{U}\} \neq \emptyset$. Let now $F : X \longrightarrow Y$ be a multivalued mapping of a space X onto a space Y. We call the mapping F *absolutely cofinally (residually, u.s.c., l.s.c.) closed* iff there is *no cofinally continuous (residually continuous, u.s.c., l.s.c.)* extension $\overline{F} : X^* \longrightarrow Y$ of the mapping F to a space X^* in which X is dense subset.

The next two theorems present a basic characterizations of absolute cofinal and residual closeness of the multivalued mappings and are generalizations of corresponding criteria for continuous single valued mappings.

THEOREM 3. *Let $F : X \longrightarrow Y$ be a multivalued cofinally continuous mapping of a T_3-space X onto a space Y. The mapping F is absolutely cofinally closed iff (a) F is X-compact and (b) F is closed mapping.*

Proof. (1) (a) Let F be an absolutely cofinally closed mapping. If F were not X-compact, then it would exist a point $y \in Y$ so that the set $F'y$ would not be compact. Therefore, there is an open cover $\mathfrak{U} = \{U_\alpha \mid \alpha \in A\}$ of $F'y$ so that for all its finite subfamilies

$$\mathfrak{U}^* = \{U_{\alpha(i)} \mid i = 1, \ldots, n\} \neq \emptyset,$$
$$F'y \setminus |\mathfrak{U}^*| = F'y \setminus \bigcup \{U_{\alpha(i)} \mid i = 1, \ldots, n\} \neq \emptyset.$$

Since X is a T_3-space, it may be assumed that also

$$F'y \setminus \bigcup \{\overline{U}_{\alpha(i)} \mid i = 1, \ldots, n\} \neq \emptyset \tag{1}$$

for every finite subfamily $\mathfrak{U}^* = \{U_{\alpha(i)} \mid i = 1, \ldots, n\} \neq \emptyset \subseteq \mathfrak{U}$.

Now let $\mathfrak{B}_1$ be the family of all sets $B_\alpha = X \setminus \overline{U}_\alpha$. Then, by (1), $F'y \setminus \overline{U}_\alpha \neq \emptyset$ and hence $F'y \cap B_\alpha \neq \emptyset$. But then from $F'y \cap B_\alpha \neq \emptyset \iff y \in FB_\alpha \subseteq \overline{FB_\alpha}$ it follows that

$$y \in \bigcup \{\overline{FB_\alpha} \mid \alpha \in A\}. \tag{2}$$

Since $B_\alpha = X \setminus \overline{U}_\alpha \iff \overline{B}_\alpha = X \setminus U_\alpha$ and $F'y \subseteq |\mathfrak{U}| = \cup\{U_\alpha \mid \alpha \in A\}$, we have

$$\bigcap \{\overline{B}_\alpha \mid \alpha \in A\} = \bigcap \{X \setminus U_\alpha \mid \alpha \in A\} = X \setminus \bigcup \{U_\alpha \mid \alpha \in A\} \subseteq X \setminus F'y.$$

Since the mapping F is cofinally continuous, then $F'y = \{\overline{F'V} \mid V \in \mathfrak{V}(y)\}$ by Lemma 2. If $x \notin \overline{F'V}$, then there is an open set $B_x \subset X$ so that $\overline{F'V} \subseteq B_x \subseteq \overline{B}_x \subseteq X \setminus \{x\}$ and $\overline{F'V} \subseteq \bigcap \{\overline{B}_x \mid x \in X \setminus \overline{F'V}\}$.

Let now $\mathfrak{B}_2$ be the collection of all open sets B_x, if $x \in X \setminus F'y$. Then we have

$$\operatorname{ad} \mathfrak{B}_2 = \bigcap \{\overline{B}_x \mid x \in X \setminus F'y\} = \bigcap \{\overline{F'V} \mid V \in \mathfrak{V}(y)\} = F'y.$$

Note that, if $B_\alpha, B_{\alpha'} \in \mathfrak{B}_1$, then

$$\begin{aligned} B_\alpha \cap B_{\alpha'} &= (X \setminus U_\alpha) \cap (X \setminus U_{\alpha'}) = \\ &= X \setminus (U_\alpha \cup U_{\alpha'}) \supseteq F'y \setminus (U_\alpha \cup U_{\alpha'}) \neq \emptyset; \end{aligned}$$

if $B_x, B_{x'} \in \mathfrak{B}_2$, then there are $V, V' \in \mathfrak{V}(y)$ so that

$$B_x \cap B_{x'} \supseteq \overline{F'V} \cap \overline{F'V'} \supseteq F'y \neq \emptyset,$$

and if $B_\alpha \in \mathfrak{B}_1$ and $B_x \in \mathfrak{B}_2$, then

$$B_\alpha \cap B_x \supseteq B_\alpha \cap \overline{F'V} \supseteq B_\alpha \cap F'y \neq \emptyset.$$

Because of that, the family $\mathfrak{B} = \mathfrak{B}_1 \cup \mathfrak{B}_2$ may by taken as a base of an open ultrafilter $\mathfrak{W}$ in X. Observe that the ultrafilter $\mathfrak{W}$ is free, since

$$\begin{aligned}\operatorname{ad}\mathfrak{W} \subseteq \operatorname{ad}\mathfrak{B} &= \Big[\bigcap\{\overline{B}_\alpha \mid \alpha \in A\}\Big] \cap \Big[\bigcap\{\overline{B}_x \mid x \in X \setminus F'y\}\Big] \\ &\subseteq (X \setminus F'y) \cap F'y = \emptyset.\end{aligned}$$

Now if $B_x \in \mathfrak{B}_2$, then there is a set $V \in \mathfrak{V}(y)$ so that $F'y \subseteq F'V \subseteq \overline{F'V} \subseteq B_x$ and $y \in FF'y \subseteq FB_x \subseteq \overline{FB}_x$. Therefore $y \in \bigcap\{\overline{FB}_x \mid B_x \in \mathfrak{B}_2\}$ and by (2) we have

$$y \in \Big[\bigcap\{\overline{FB}_\alpha \mid \alpha \in A\}\Big] \cap \Big[\bigcap\{\overline{FB}_x \mid B_x \in \mathfrak{B}_2\}\Big] = \bigcap\{\overline{FW} \mid W \in \mathfrak{W}\} \neq \emptyset.$$

Now we put $\{\mathfrak{W}\} = x^*$ and $X^* = X \cup \{x^*\}$. On the set X^* we define a topology $\mathfrak{T}^*$ keeping on X the existing topology and taking at the point x^* as the neighbourhood system $\mathfrak{U}^*(x^*)$ the family $\{W \cup \{x^*\} \mid W \in \mathfrak{W}\}$. Then X is a dense subset in the space $(X^*, \mathfrak{T}^*)$ and we may define the mapping $\overline{F} : X^* \longrightarrow Y$ setting $\overline{F}x^* = \bigcap\{\overline{FW} \mid W \in \mathfrak{W}\}$ and $\overline{F}x = Fx$ if $x \in X$. Then $y \in \overline{F}x^* \neq \emptyset$ and by Theorem 1, the mapping $\overline{F}$ is cofinally continuous at x^*. So $\overline{F}$ is a cofinally continuous extension of F, contrary to the assumption that F is absolutely closed. Hence, the mapping F is X-compact.

(b) Further, we shall show that F is closed if it is absolutely closed. Let $C \subseteq X$ be any closed set. We need only to prove the inclusion $\overline{FC} \subseteq FC$.

If $y \in \overline{FC} \setminus FC$, then $(\forall V \in \mathfrak{V}(y))(V \cap FC \neq \emptyset)$ and $y \notin FC$. But $y \notin FC \iff F'y \cap C = \emptyset$ and, since F is X-compact, there is an open set $B_0 \subseteq X$ so that $C \subseteq B_0 \subseteq \overline{B}_0 \subseteq X \setminus F'y$. Let now $\mathfrak{B} = \mathfrak{B}_2 \cup \{B_0\}$, where $\mathfrak{B}_2$ is the same family as in (a). Then $B_x \cap B_0 \supseteq \overline{F'V} \cap C \neq \emptyset$, since for each $B_x \in \mathfrak{B}_2$, $B_x \supseteq \overline{F'V}$ for some $V \in \mathfrak{V}(y)$ and

$$\begin{aligned}(\forall V \in \mathfrak{V}(y))(V \cap FC \neq \emptyset) &\iff (\forall V \in \mathfrak{V}(y))(F'V \cap C \neq \emptyset) \\ &\implies (\forall V \in \mathfrak{V}(y))(\overline{F'V} \cap C \neq \emptyset).\end{aligned}$$

So the family $\mathfrak{B}$ may be accepted as a base of an open ultrafilter $\mathfrak{W}$ in the space X. The ultrafilter $\mathfrak{W}$ is free, since

$$\begin{aligned}\operatorname{ad}\mathfrak{W} &= \bigcap\{\overline{W} \mid W \in \mathfrak{W}\} \subseteq \operatorname{ad}\mathfrak{B} = \Big[\bigcap\{\overline{B}_x \mid x \in X \setminus F'y\}\Big] \cap \overline{B}_0 \\ &= \Big[\bigcap\{\overline{F'V} \mid V \in \mathfrak{V}(y)\}\Big] \cap \overline{B}_0 = F'y \cap \overline{B}_0 \subseteq F'y \cap (X \setminus F'y) = \emptyset.\end{aligned}$$

Observe here, that $y \in \big[\bigcap\{\overline{FB}_x \mid B_x \in \mathfrak{B}_2\}\big] \cap \overline{FB}_0 \neq \emptyset$, since by $C \subseteq B_0$ it follows that $y \in \overline{FC} \implies y \in \overline{FB}_0$ and, as is proved in (a), $y \in \cap\{\overline{FB}_x \mid B_x \in \mathfrak{B}_2\}$. As in the part (a) of the proof we may define the mapping $\overline{F} : X^* \longrightarrow Y$ setting $\overline{F}x^* = \bigcap\{\overline{FW} | W \in \mathfrak{W}\}$ and $\overline{F}x = Fx$ if $x \in X$, where $\{\mathfrak{W}\} = x^*$ and $X^* = X \cup \{x^*\}$. Then $\overline{F}x^* \neq \emptyset$, since $y \in \bigcap\{\overline{FB_x} \mid B_x \in \mathfrak{B}_2\} = \bigcap\{\overline{FW} |\ W \in \mathfrak{W}\}$. The mapping $\overline{F}$ is a cofinally continuous extension of F, contrary to the assumption that F is absolutely closed. Hence, F is closed.

(2) Now let both conditions (a) and (b) hold and let $\mathfrak{U} = \{U_\alpha \mid \alpha \in A\}$ be any open ultrafilter with $\operatorname{ad}\mathfrak{U} = \bigcap\{\overline{U}_\alpha |\ \alpha \in A\} = \emptyset$. If $y \in Y$ is a point, then for each $\alpha \in A$, it may not be $F'y \cap \overline{U}_\alpha \neq \emptyset$, since the family $\mathfrak{F} = \{F'y \cap \overline{U}_\alpha \mid U_\alpha \in \mathfrak{U}\}$ is a closed filter in the compact subspace $F'y$ by (a). Then it must be

$$\emptyset \neq \bigcap\{F'y \cap \overline{U}_\alpha \mid \alpha \in A\} = F'y \cap \Big[\bigcap\{\overline{U}_\alpha \mid \alpha \in A\}\Big] = F'y \cap \operatorname{ad}\mathfrak{U}$$

what contradicts the assumption $\operatorname{ad}\mathfrak{U} = \emptyset$. Hence, there is a set $U_{\alpha'} \in \mathfrak{U}$ so that $F'y \cap \overline{U}_{\alpha'} = \emptyset$. Now let $W_0 = X \setminus \overline{U}_{\alpha'}$. Then $F'y \subseteq W_0$ and $W_0 \cap U_{\alpha'} = \emptyset$. The set $F^\# W_0 = V_0$ is open in Y, since W_0 is open and F closed. But $y \in F^\# F'y \subseteq F^\# W_0 = V_0$, so V_0 is an open nhood of y in Y. Since

$$W_0 \cap U_{\alpha'} = \emptyset \implies F^\# W_0 \cap FU_{\alpha'} = \emptyset,$$

i.e., $V_0 \cap FU_{\alpha'} = \emptyset$, it follows that $y \notin \overline{FU_{\alpha'}} \supseteq \bigcap\{\overline{FU_\alpha} |\ \alpha \in A\}$. Because the point $y \in Y$ is arbitrary, there must be $\bigcap\{\overline{FU_\alpha} \mid \alpha \in A\} = \emptyset$, and F cannot be extended to a cofinal continuous mapping of any overspace X^* in which X is a dense subspace. So the mapping F is cofinally absolutely closed. □

COROLLARY 3.1. *Let $F : X \longrightarrow Y$ be an u.s.c. mapping of a T_3-space X onto a compact T_2-space Y. Then F is absolutely u.s.c. closed iff (a) F is X-compact and (b) F is closed.*

The corollary follows from the well known fact that, if Y is a compact T_2-space, then the multivalued cofinally continuous mapping $F : X \longrightarrow Y$ is an u.s.c. mapping.

COROLLARY 3.2. *Let $F : X \longrightarrow Y$ be a mapping of a T_3-space X onto a T_3-space Y. The mapping F is on both sides cofinally absolutely closed iff it is perfect ($\equiv F$ and F' are closed and F is X-compact and Y-compact).*

Note that Corollary 3.2 also presents a characterization of multivalued perfect mappings.

Let us remark that the next criterion of absolute residual closeness of the multivalued mappings, which we shall prove further, is a criterion of absolute lower semicontinuous closeness too, since the class of residual continuous multivalued mappings is the same as the class of lower semicontinuous mappings. Although that fact has for a long time been known, for sake of completeness we shall, however, give a short proof of it.

Let $F : X \longrightarrow Y$ be a multivalued residual continuous mapping. Since F is l.s.c. iff the inverse F' is open or, equivalently, iff the small inverse $F^\flat$ of F is closed, it is enough to prove that $F^\flat$ is closed. Therefore we only needs to prove the inclusion $\overline{F^\flat B} \subseteq F^\flat B$, where $B \subseteq Y$ is any closed set.

Let us note that $(\mathfrak{U}(x), \leq)$ is a directed set, if in $\mathfrak{U}(x)$ we define partial order $\leq$ by $U \leq U' \iff U' \subseteq U$, for $U, U' \in \mathfrak{U}(x)$. Since

$$x \in \overline{F^\flat B} \iff (\forall U \in \mathfrak{U}(x))(U \cap F^\flat B \neq \emptyset),$$

we may define a net $\varphi : (\mathfrak{U}(x), \leq) \longrightarrow X$, converging to $x \in \overline{F^\flat B}$, by putting $\varphi(U) = x_U \in U \cap F^\flat B$, for each $U \in \mathfrak{U}(x)$. Further, we shall prove the inclusion $\liminf_U Fx_U \subseteq B$. Let $y \in \liminf_U Fx_U$ by any point. Since

$$\varphi(U) = x_U \in U \cap F^\flat B \subseteq F^\flat B \iff FX_u \subseteq B,$$

if $U \in \Delta(V) = \{U \mid Fx_U \cap V \neq \emptyset\}$, it follows that $\emptyset \neq Fx_u \cap V \subseteq B \cap V$, for each $V \in \mathfrak{V}(y)$. So $y \in \overline{B} = B$ and the implication

$$y \in \liminf_U Fx_U \implies y \in B$$

as the inclusion $\liminf_U Fx_U \subseteq B$ are proved. Since the mapping F is residually continuous at x iff $Fx \subseteq \liminf_U Fx_U$ we have the inclusion $Fx \subseteq B$, i.e., $x \in F^\flat B$. So we proved the implication $x \in \overline{F^\flat B} \implies x \in F^\flat B$ and the inclusion $\overline{F^\flat B} \subseteq F^\flat B$.

Let now F be a l.s.c. mapping and $\varphi : (\mathfrak{D}, \leq) \longrightarrow X$ a net converging to $x \in X$. But the mapping F is l.s.c. iff the set $F'V \subseteq X$ is open as the set $V \subseteq Y$ is open. So, if $V \in \mathfrak{V}(y)$ is an open nhood of a $y \in Fx$, then $x \in F'y \subseteq F'V \in \mathfrak{U}(x)$. Since the net converges to x, for the nhood $F'V = U \in \mathfrak{U}(x)$, there is an index $d_V \in \mathfrak{D}$ so that, for each $d \in \mathfrak{D}$,

$$d_V \leq d \implies \varphi(d) = x_d \in F'V, \quad i.e., \quad d_V \leq D \implies Fx_d \cap V \neq \emptyset.$$

Thus we have proved that the set $\Delta(V) = \{d \mid Fx_U \cap V \neq \emptyset\}$ is residual in the directed set $(\mathfrak{D}, \leq)$ and, as the set $V \in \mathfrak{V}(y)$ is arbitrary, it follows that $y \in \liminf_U Fx_U$. So we have proved the implication $y \in Fx \implies y \in \liminf_U Fx_U$ from which follows the inclusion $Fx \subseteq \liminf_U Fx_U$ and that F is residually continuous in $x \in X$.

The next criterion of absolute residual closeness of the multivalued mappings is also a criterion of absolute lower semicontinuous closeness, since, as it has been showed, the class of residual continuous multivalued mappings coincides with the class of lower semicontinuous (l.s.c.) mappings.

THEOREM 4. *Let $F : X \longrightarrow Y$ be a multivalued residually continuous mapping of a T_2-space X onto a T_2-space Y. The mapping F is absolutely residually closed iff (a) for each free open ultrafilter $\mathfrak{U}$ in X with $\operatorname{ad}\mathfrak{U} = \emptyset$ and each $y \in Y$ there is a $U \in \mathfrak{U}$ so that $F'y \cap \overline{U} = \emptyset$; (b) F is a regular closed mapping.*

Proof. (1) (a) Let the mapping F be absolutely residual closed and let $\mathfrak{U}$ be any free open ultrafilter with $\operatorname{ad}\mathfrak{U} = \emptyset$. Let us note that, for each $y \in Y$,

$$(*) \qquad F'y = \bigcap\{F'V \mid V \in \mathfrak{V}(y)\},$$

since the mapping F is pointwise closed. Indeed, since the set Fx is closed and $x \notin F'y \iff y \notin Fx$, the set $V_0 = Y \setminus Fx$ is an open nhood of y. Then $F'y \subseteq F'V_0$, as $y \in V_0$ and, because of $V_0 \cap Fx = \emptyset \iff x \notin F'V_0$, we have proved the implication $x \notin F'y \implies x \notin F'V_0$ from which follows the inclusion

$$\bigcap\{F'V \mid V \in \mathfrak{V}(y)\} \subseteq F'V_0 \subseteq F'y$$

and the equality $(*)$.

Let now $y \in Y$ be any point. If we suppose that $F'y \cap \overline{U} \neq \emptyset$, for each $U \in \mathfrak{U}$, then by $(*)$ we have

$$\emptyset \neq F'y \cap \overline{U} = \Big[\bigcap\{F'V \mid V \in \mathfrak{V}(y)\}\Big] \cap \overline{U} = \bigcap\{F'V \cap \overline{U} \mid V \in \mathfrak{V}(y)\}.$$

So $F'V \cap \overline{U} \neq \emptyset$, for each $U \in \mathfrak{U}$ and $V \in \mathfrak{V}(y)$. Since the mapping F is residually continuous iff it is l.s.c., the set $F'V$ is open, so $F'V \cap U \neq \emptyset$, for each $U \in \mathfrak{U}$ and each $V \in \mathfrak{V}(y)$. From the supposition that $\mathfrak{U}$ is an ultrafilter it follows that $F'V \in \mathfrak{U}$, for each set $V \in \mathfrak{V}(y)$.

Put now $\{\mathfrak{U}\} = x^*$, $X^* = X \cup \{x^*\}$ and assume for the nhoods of $x^* \in X^*$ all the sets $U^* = U \cup \{x^*\}$, $U \in \mathfrak{U}$. If on X we keep the existing topology, then on X^* is defined a topology in which X is a dense subset.

Further on X^* we define the mapping $\overline{F} : X^* \longrightarrow Y$, by putting $\overline{F}x^* = \{y\}$ and $\overline{F}x = Fx$, for each $x \in X$. We shall prove that $\overline{F}$ is a residually continuous extension of F. Indeed, it is obvious that $\overline{F}$ is residually continuous at each $x \in X$ and we need only to prove that $\overline{F}$ is residually continuous at x^*.

Let $\varphi : (\mathfrak{D}, \leq) \longrightarrow X^*$ and be a net converging to x^*. If $V \in \mathfrak{V}(y)$, then $x^* \in \overline{F}'y \subseteq \overline{F}'V = \{x^*\} \cup F'V = U^*$ and so U^* is an open nhood of x^*. Then there is an index $d_0 \in \mathfrak{D}$ so that for each $d \in \mathfrak{D}$

$$d \geq d_0 \implies \varphi(d) = x_d \in F'V, \quad \text{i.e.,} \quad d \geq d_0 \implies Fx_d \cap V \neq \emptyset.$$

Since for each $x \in X$, $Fx = \overline{F}x$, it follows that $d \geq d_0 \implies \overline{F}x_d \cap V \neq \emptyset$. Thus we have proved that the set $\Delta(V) = \{d \mid \overline{F}x_d \cap V \neq \emptyset\}$ is residual in the directed set $(\mathfrak{D}, \leq)$, for any set $V \in \mathfrak{V}(y)$. Then $y \in \liminf_d \overline{F}x_d$, thus we proved the implication $y \in \{y\} = \overline{F}x^* \implies y \in \liminf_d \overline{F}x_d$ as well as the inclusion $\{y\} = \overline{F}x^* \subseteq \liminf_d \overline{F}x_d$, which shows that the mapping $\overline{F}$ is residually continuous at x^*, since $\varphi : (\mathfrak{D}, \leq) \longrightarrow X^*$ is an arbitrary net converging to x^*. But then the mapping F is not absolutely residually closed, contrary to the supposition. So F satisfies the condition (a).

(b) Let $C = \overline{\operatorname{Int} C} \subseteq X$ be any regularly closed set. If we prove the inclusion $\overline{FC} \subseteq FC$, then the condition (b) will be proved. Let $y \in \overline{FC}$ be any point. Then $V \cap FC \neq \emptyset$ for each nhood $V \in \mathfrak{V}(y)$ and also $F'V \cap \operatorname{Int} C \neq \emptyset$, since

$$V \cap FC \neq \emptyset \iff \emptyset \neq F'V \cap C = F'V \cap \overline{\operatorname{Int} C} \iff F'V \cap \operatorname{Int} C \neq \emptyset.$$

If $V, V' \in \mathfrak{V}(y)$, then $V'' = V \cap V' \in \mathfrak{V}(y)$ and

$$\emptyset \neq F'V'' \cap \operatorname{Int} C \subseteq [F'V \cap \operatorname{Int} C] \cap [F'V' \cap \operatorname{Int} C].$$

Then we may assume that the family $\mathfrak{B} = \{F'V \cap \operatorname{Int} C \mid V \in \mathfrak{V}(y)\}$ as a base of an open ultrafilter $\mathfrak{W}$ of all open sets $W \subseteq X$ containing any set $F'V \cap \operatorname{Int} C \in \mathfrak{B}$. Then, obviously, $\operatorname{Int} C \in \mathfrak{U}$ and $\{F'V \mid V \in \mathfrak{V}(y)\} \subseteq \mathfrak{U}$.

If we suppose that $y \notin FC$, then $\emptyset = F'y \cap C \supseteq F'y \cap \operatorname{Int} C$ and

$$\begin{aligned} \bigcap \mathfrak{W} &= \bigcap \{W \mid W \in \mathfrak{W}\} \subseteq \bigcap \{F'V \cap \operatorname{Int} C \mid V \in \mathfrak{V}(y)\} \\ &= \Big[\bigcap \{F'V \mid V \in \mathfrak{V}(y)\}\Big] \cap \operatorname{Int} C = F'y \cap \operatorname{Int} C = \emptyset, \end{aligned}$$

and it follows that the ultrafilter $\mathfrak{W}$ is free. Then, as in the part (a), putting $\{\mathfrak{W}\} = x^*$, $X^* = X \cup \{x^*\}$ and defining a topology on X^* and the mapping $\overline{F} : X^* \longrightarrow Y$ as in (a), we obtain that $\overline{F}$ is a residual continuous extension of F contrary to the supposition that F is absolutely residually closed. So F satisfies the condition (b).

(2) Now let F satisfy both (a) and (b), and suppose F is not absolutely residually closed. Then, there is a residual continuous extension $\overline{F} : X^* \longrightarrow Y$ of F, where X^* is any space in which X is a dense subset.

If $x^* \in X^* \setminus X$, then the family $\mathfrak{U}^*(x^*)$ of open nhoods of x^* in X^* is an open ultrafilter and the family $\mathfrak{U} = \{U \mid U = U^* \cap X, U^* \in \mathfrak{U}^*(x^*)\}$ is a free open ultrafilter in the subspace X, since $\bigcap\{U^* \mid U^* \in \mathfrak{U}^*(x^*)\} = \{x^*\}$ in each T_2-space.

If $y \in Fx^*$ then by (a) there is a set $U_0 \in \mathfrak{U}$ so that $F'y \cap \overline{U}_0 = \emptyset$. But then $y \notin F\overline{U}_0$ and, since F is by (b) regularly closed, the set $F\overline{U}_0$ is closed in Y. So $V_0 = Y \setminus F\overline{U}_0$ is an open nhood of y, since $y \in Y \setminus F\overline{U}_0 = V_0$. Because $\overline{F}$ is a residually continuous mapping iff is l.s.c., the set $\overline{F}'V_0$ is in X^* an open set, as the set V_0 is open. The set $\overline{F}'V_0$ is, also, an nhood of the point x in the space X^*, since from $y \in \overline{F}x^*$ and $y \in V_0$ it follows that $\overline{F}x^* \cap V_0 \neq \emptyset$ and $x^* \in \overline{F}V_0$.

So $\overline{F}V_0 \in \mathfrak{U}^*(x^*)$ and $\overline{F}V_0 \cap X = FV_0 \in \mathfrak{U}$ and, as $\mathfrak{U}$ is an ultrafilter, $FV_0 \cap U \neq \emptyset$, for each $U \in \mathfrak{U}$. But since $FV_0 \cap U \neq \emptyset \iff V_0 \cap FU \neq \emptyset$ and $U_0 \in \mathfrak{U}$, it must be also $\emptyset \neq V_0 \cap FU_0 \subseteq V_0 \cap F\overline{U}_0 = (Y \setminus F\overline{U}_0) \cap F\overline{U}_0$, what is impossible. Hence, the mapping F is absolutely residually closed. $\square$

COROLLARY 4.1. *A multivalued continuous ($\equiv$ u.s.c. & l.s.c.) mapping $F : X \longrightarrow Y$ of a T_2-space X onto a T_2-space Y is absolutely closed iff (a) for each free open ultrafilter $\mathfrak{U}$ in X with* $\operatorname{ad}\mathfrak{U} = \emptyset$ *and each $y \in Y$, there is a $U \in \mathfrak{U}$ so that $F'y \cap \overline{U} = \emptyset$; (b) F is a regular closed mapping.*

COROLLARY 4.2. *A singlevalued continuous mapping $f : X \longrightarrow Y$ of a T_2-space X onto a T_2-space Y is absolutely closed iff (a) for each free open ultrafilter $\mathfrak{U}$ in X with $\operatorname{ad}\mathfrak{U} = \emptyset$ and each $y \in Y$ there is a $U \in \mathfrak{U}$ so that $f^{-1}y \cap \overline{U} = \emptyset$; (b) F is a regular closed mapping.*

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