

REPRESENTATION THEOREM FOR STOCHASTIC  
DIFFERENTIAL EQUATIONS IN HILBERT SPACES AND  
ITS APPLICATIONS

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**Abstract.** In this survey we recall the results obtained in [16] where we gave a representation theorem for the solutions of stochastic differential equations in Hilbert spaces. Using this representation theorem we obtained deterministic characterizations of exponential stability and uniform observability in [16], [17] and we will prove a result of Datko type concerning the exponential dichotomy of stochastic equations.

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2000 Mathematics Subject Classification: 93E15, 34D09, 93B07.

Keywords: Lyapunov equations, stochastic differential equations, uniform exponential stability, uniform observability, uniform exponential dichotomy.

*This research was supported by grant CEEEX-code PR-D11-PT00-48/2005 from the Romanian Ministry of Education and Research*

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