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SOME ABSOLUTELY CONTINUOUS REPRESENTATIONS OF FUNCTION ALGEBRAS

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Abstract. In this paper we study some absolutely continuous representations of function algebras, which are weak ρ-spectral in the sense of [5] and [6], for a scalar $\rho > 0$. Precisely we investigate certain conditions for the existence of a spectral ρ-dilation of such representation. Among others we obtain different results which generalize the corresponding theorems of D. Gaşpar [3].

1 Preliminaries

Let X be a compact Hausdorff space,$C(X)$ (respectively $C_{\mathbb{R}}(X)$) be the Banach algebra of all complex (real) valued continuous functions on X.

Let A be a function algebra on X (that is a closed subalgebra of $C(X)$ containing the constants and separating the points of X) and $\overline{A}$ be the set of the complex conjugates of the functions from A. Denote by $M(A)$ the set of all nonzero complex homomorphisms of A and for $\gamma \in M(A)$ we put $A_\gamma = \ker\gamma$. Clearly, any $\gamma \in M(A)$ can be extended to a bounded linear functional on $A + \overline{A}$, also denoted by γ, which satisfies for $f, g \in A$:

$$\gamma(f + \overline{g}) = \gamma(f) + \overline{\gamma(g)}, \qquad |\gamma(f + \overline{g})| \leq 2\|f + \overline{g}\|.$$

Two homomorphisms $\gamma_0, \gamma_1 \in M(A)$ is called Gleason equivalent if

$$\|\gamma_0 - \gamma_1\| < 2.$$

The Gleason equivalence is a relation of equivalence in $M(A)$, and the corresponding equivalence classes are called the Gleason parts of A ([1], [9]).

If $\gamma \in M(A)$ we denote by M_γ the set of all representing measures for γ, that is a positive Borel measures on X satisfying

$$\gamma(f) = \int f dm \qquad (f \in A).$$

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Denote by $Bor(X)$ the family of all Borel sets of X, and M a set of positive Borel measures on X. A Borel measure ν on X is called M-absolutely continuous (M a.c.) if $\nu(\sigma) = 0$ for any M-null set $\sigma \in Bor(X)$ (that is with $\mu(\sigma) = 0$ for every $\mu \in M$). Also, on says that ν is M-singular (M s.) if ν is supported on a M-null set. It is known [1] that each Borel measure ν on X has a unique decomposition of the form

$$\nu = \nu_a + \nu_s$$

where ν_a is M a.c. and ν_s is M s. This decomposition is called the M - decomposition of ν. A measure ν is completely singular if it is M s. where $M = \underset{\gamma\in\mathcal{M}(A)}{\cup} M_\gamma$. We recall ([1]) that if γ_0 and γ_1 are Gleason equivalent, than the $M_{\gamma_j}-$ decompositions of ν coincide for $j = 0, 1$. When γ_0 and γ_1 are in different Gleason parts, than the M_{γ_j} a.c. component of ν is $M_{\gamma_{j-1}}$s., for $j = 0, 1$.

Let H be a complex Hilbert space, and $B(H)$ be the Banach algebra of all bounded linear operators on H.

A representation of a function algebra A on H is a multiplicative linear map Φ of A into $B(H)$ with $\Phi(1) = I$, the identity operator, and

$$\|\Phi(f)\| \leq c\,\|f\| \qquad (f \in A),$$

for some constant $c > 0$. When $c = 1$, Φ is a contractive representation.

If Φ is a representations of A on H, then by Hahn - Banach and Riesz - Kakutani theorems it follows that, for each $x, y \in H$ there exists a measure $\mu_{x,y}$ on X such that $\left\|\mu_{x,y}\right\| \leq c\,\|x\|\,\|y\|$ and

$$(1) \qquad <\Phi(f)\,x, y> = \int f d\mu_{x,y} \qquad (f \in A).$$

Such measures $\mu_{x,y}$ $(x, y \in \mathcal{H})$ are called elementary measures for Φ. Also, if M $\subset M_\gamma$ for some $\gamma \in M(A)$, one says that Φ is M - absolutely continuous (M a.c.), respectively Φ is M - singular (M s.), if there exist M a.c., respectively M s. elementary measures $\mu_{x,y}$ of Φ for any $x, y \in H$. When Φ is M_γ s. for every $\gamma \in M(A)$, Φ is called completely singular.

For $\rho > 0$, and $\gamma \in M(A)$ a contractive representation $\widetilde{\Phi}$ of $C(X)$ on a Hilbert space $K \supset H$ is called a $\gamma-$ spectral $\rho-$ dilation of a representation Φ of A on H if

$$(2) \qquad \Phi(f) = \rho P_{\mathcal{H}}\,\widetilde{\Phi}(f)\,|\mathcal{H} \qquad (f \in A_\gamma),$$

where $P_{\mathcal{H}}$ is the orthogonal projection of K on H. When $\rho = 1$, such a representation $\widetilde{\Phi}$ is called a spectral dilation of Φ (that is $\widetilde{\Phi}$ is a $\varphi-$ spectral 1 - dilation of Φ, for any $\varphi \in M(A)$).

According to [2], one says that a representation Φ of A on H is of class $C_\rho(A_\gamma, \mathcal{H})$ if Φ has a $\gamma-$ spectral ρ - dilation. Clearly, if Φ is of class $C_\rho(A_\gamma, \mathcal{H})$ then

$$(3) \qquad \|\Phi(f)\| \leq \|\rho f + (1-\rho)\,\gamma(f)\| \qquad (f \in A),$$

but the converse assertion is not true, in general (even if $\rho = 1$). D. Gaşpar

([2] and[3]) obtains certain conditions under which (3) assures the existence of a $\gamma-$ spectral ρ - dilation for Φ. This happens for instance, if A is a Dirichlet algebra on X (that is $A+\overline{A}$ is dense in $C(X)$), or more general, when γ has a unique representing measure m and Φ is m a.c. Also, T. Nakazi [7], [8] gives other equivalent conditions with the existence of a $\gamma-$ spectral ρ - dilation, if A is a hypo - Dirichlet algebra (that is $A+\overline{A}$ has finite codimension in $C(X)$).

In this paper we generalize some results of D. Gaşpar [3] by investigating a weakly condition than (3), namely the condition

$$w\left(\Phi\left(f\right)\right) \leq \left\|\rho f+\left(1-\rho\right)\gamma\left(f\right)\right\| \qquad \left(f\in A\right), \tag{4}$$

here $w(T)$ is the numerical radius for $T \in B(H)$.

A representation Φ of A on H satisfying (4) is called weak $\rho-$ spectral with respect to γ. When Φ satisfies (3) it simply called spectral with respect to γ.

In [5] and [6] were given different characterizations for that a representation Φ to be weak $\rho-$ spectral with respect to γ. This happens if and only if for any $x \in H$ there exists a positive measure μ_x on X with $\mu_x\left(X\right)=\|x\|^2$ such that

$$<\Phi\left(f\right)x,x>=\int\left(\rho f+\left(1-\rho\right)\gamma\left(f\right)\right)d\mu_x \qquad \left(f\in A\right). \tag{5}$$

Such a measure μ_x is called a weak $\rho-$ spectral measure attached to x by Φ and γ.

The aim of this paper is to further investigate the weak $\rho-$ spectral representations for the weak$^*-$ Dirichlet function algebras. Recall [10] that m is a probability measure on X and $A\subset L^\infty(m)$ is a subalgebra, then A is called a weak$^*-$ Dirichlet algebra in $L^\infty(m)$ if m is multiplicative on A and $A+\overline{A}$ is weak* dense in $L^\infty(m)$.

2 Representations with Spectral ρ-Dilations

In this section we refer to some weak $\rho-$ spectral representations which have spectral $\rho-$ dilations. In fact we generalize certain results concerning the $\rho-$ spectral representations in the case of unique representing measure ([3]).

We begin with the following

Theorem 1. *Let A be a function algebra on X which is weak$^*-$ Dirichlet in $L^\infty(m)$ for some representing measure m for $\gamma\in M\left(A\right)$. If Φ is a representing of A on H such that for any $x\in H$ there exists a m a.c. weak $\rho-$ spectral measure attached to x by Φ and γ, then Φ has a $\gamma-$ spectral $\rho-$ dilation. Moreover, in this case there exists a unique $B(H)$ - valued and m a.c. semispectral measure F on X satisfying*

$$<\Phi\left(f\right)x,y>=\int\left(\rho f\left(\xi\right)+\left(1-\rho\right)\gamma\left(f\right)\right)d\left(F\left(\xi\right)x,y\right) \tag{6}$$

for $f\in A$ and $x,y\in H$.

Proof. Let Φ a representation of A on H and we suppose that for $x\in H$ there exists a m a.c. measure $\mu_x\geq 0$ with $\mu_x\left(X\right)=\|x\|^2$ and

$$\Phi(f)x,x>=\int(\rho f+(1-\rho)\gamma(f))\,d\mu_x \qquad (f\in A).$$

For $f\in A$ and $x,y\in H$ we have

$$\begin{aligned}
&\int(\rho f+(1-\rho)\gamma(f))\,d\left(\mu_{x+y}+\mu_{x-y}\right)\\
=\ & <\Phi(f)(x+y),x+y>+<\Phi(f)(x-y),x-y>\\
=\ & 2\left(<\Phi(f)x,x>+<\Phi(f)y,y>\right)\\
=\ & 2\int(\rho f+(1-\rho)\gamma(f))\,d\left(\mu_x+\mu_y\right),
\end{aligned}$$

or equivalently

$$\begin{aligned}
&\rho\int fd\left(\mu_{x+y}+\mu_{x-y}\right)+(1-\rho)\gamma(f)\left(\|x+y\|^2+\|x-y\|^2\right)\\
=\ & 2\rho\int fd\left(\mu_x+\mu_x\right)+(1-\rho)\gamma(f)\left[2\left(\|x\|^2+\|y\|^2\right)\right].
\end{aligned}$$

This yields for each $x,y\in H$,

$$\int fd\left(\mu_{x+y}+\mu_{x-y}\right)=\int fd\left(2\mu_x+2\mu_y\right) \qquad (f\in A),$$

and since the measures $\mu_{x+y}+\mu_{x-y}$ and $\mu_x+\mu_y$ are m a.c., by Gleason - Whitney theorem [10] it follows that

$$\mu_{x+y}+\mu_{x-y}=2\left(\mu_x+\mu_y\right) \qquad (x,y\in\mathcal{H}).$$

Now, if we define the measure

$$\mu_{x,y}=\frac{1}{4}\left[\mu_{x+y}-\mu_{x-y}+i\left(\mu_{x+iy}-\mu_{x-iy}\right)\right],$$

then it is known ([9]) that the $B(H)$ valued measure F on X defined by

$$<F(\sigma)x,y>=\mu_{x,y}(\sigma)$$

for $\sigma\in Bor(X)$ and $x,y\in H$ is a semispectral measure which clearly satisfies

$$< \Phi(f)\,x,y > = \int (\rho f(\xi) + (1-\rho)\,\gamma(f))\,d\,(F(\xi)\,x,y) \qquad (f \in A)\,.$$

Next by Naimark dilation theorem (see [9]) there exists a contractive representation $\widetilde{\Phi}$ of $C(X)$ on a Hilbert space $K \supset H$ such that

$$< \widetilde{\Phi}(g)\,x,y > = \int g(\xi) d\,(F(\xi)\,x,y) \qquad (g \in C(X), x,y \in \mathcal{H})\,.$$

Thus for $f \in A_\gamma$ and $x,y \in H$ one infers

$$< \Phi(f)\,x,y > = \rho \int f(\xi) d\,(F(\xi)\,x,x) = \rho < \widetilde{\Phi}(f)\,x,y >,$$

whence we get

$$\Phi(f) = \rho P_{\mathcal{H}} \widetilde{\Phi}(f)\,|\mathcal{H} \qquad (f \in A_\gamma)\,.$$

Hence $\widetilde{\Phi}$ is a $\gamma-$ spectral $\rho-$ dilation of Φ.

Obviously, the above semispectral measure F is m a.c. and the uniqueness property of F as a m a.c. semispectral measure satisfying (6) also follows from Gleason - Whitney theorem. This ends the proof. □

As an application the following result can be obtained, which completes the [3, Theorem 2] of D. Gaşpar (the equivalence $(ii) \Leftrightarrow (i)$ below).

Theorem 2. *Let A be a function algebra on X and $\gamma \in M(A)$ such that γ has a unique representing measure m. Then for a m a.c. representation Φ of A on H the following statements are equivalent:*

(i) Φ has a $\gamma-$ spectral $\rho-$ dilation;

(ii) Φ is a $\rho-$ spectral with respect to γ;

(iii) Φ is weak $\rho-$ spectral with respect to γ.

Proof. Since the implications $(i) \Rightarrow (ii) \Rightarrow (iii)$ are trivial, it remains to prove the implication $(iii) \Rightarrow (i)$.

Suppose that the statement (iii) holds and let μ_x be a weak $\rho-$ spectral attached to $x \in H$ by Φ and γ. As Φ is a m a.c. representation there exists a system $\{\nu_{x,y}\}_{x,y\in\mathcal{H}}$ of m a.c. elementary measures for Φ. If $\nu_x = \nu_{x,x}$ $(x \in \mathcal{H})$ then it follows that

$$\int fd\left(\nu_x - \rho\mu_x\right) = 0 \qquad\qquad (f \in A_\gamma)\,,$$

that is $\nu_x - \rho\mu_x$ is orthogonal to A_γ. Now if $\mu_x = \mu_x^a + \mu_x^s$ is m decomposition of μ_x then by M. and F. Riesz theorem ([1] and [4]) one has that $\rho\ \mu_x^s$ is orthogonal to A, since $\mu_x^s \geq 0$ it results $\mu_x^s = 0$. Thus $\mu_x = \mu_x^a$ that is μ_x is m a.c. for any $x \in H$, and then by Theorem 1 the representation Φ has a $\gamma-$ spectral $\rho-$ dilation. This ends the proof. □

From Theorem 1 we infer also the following

Corollary 3. *Let A be a function algebra on X and $\xi \in X$ a peak point for A such that A is weak*$-$ Dirichlet in $L^\infty(m)$ for some $m \in M_\xi$. Suppose that the Gleason part of A containing ξ is reduced to $\{\xi\}$. Then any m a.c. representation Φ of A on H which is weak $\rho-$ spectral with respect to ξ is a contractive spectral representation. Moreover, we have*

$$\Phi\left(f\right) = f\left(\xi\right) I \qquad\qquad (f \in A)\,.$$

Proof. Let Φ as above and $\{\nu_{x,y}\}_{x,y\in\mathcal{H}}$ be a system of m a.c. measures for Φ, where we denote $\nu_x = \nu_{x,x}$. Let also $\{\mu_x\}$ be a system of weak $\rho-$ spectral measures attached to the points $x \in H$ by Φ and ξ. If $\mu_x = \mu_x^a + \mu_x^s$ is the M_ξ decomposition of μ_x, then the M_ξ decomposition of $\nu_x - \rho\mu_x$ is

$$\nu_x - \rho\mu_x = (\nu_x - \rho\mu_x^a) - \rho\mu_x^s$$

because ν_x being m a.c. it is also M_ξ a.c. Since $\nu_x - \rho\mu_x$ is orthogonal to

$$A_\xi = \{f \in A : f\left(\xi\right) = 0\}$$

by M. and F. Riesz ([1] and [4]) we have that $\rho\mu_x^s$ is orthogonal to A, hence $\mu_x^s = 0$ because $\mu_x^s \geq 0$. Therefore $\mu_x = \mu_x^a$ is M_ξ a.c. and also the measure

$$\nu_x - \rho\mu_x - (1-\rho)\left\|x\right\|^2 m$$

is M_ξ a.c. Since this measure is orthogonal to A and by hypothesis ξ is a peak point and $\{\xi\}$ is a Gleason part for A, from a result in [4] it follows that

$$\nu_x - \rho\mu_x - (1-\rho)\left\|x\right\|^2 m = 0.$$

But this implies that μ_x is m a.c., for any $x \in H$ and by Theorem 1 there exists a m a.c. semispectral measure F on X satisfying

$$< \Phi\left(f\right)x, y > = \int \left(\rho f\left(\eta\right) + (1-\rho) f\left(\xi\right)\right) d\left(F\left(\eta\right)x, y\right)$$

for $f \in A$ and $x, y \in H$. Since ξ is a peak point and F is m a.c. one infers that

$$\begin{aligned} < \quad & \Phi(f)\, x, y > = (\rho f(\xi) + (1-\rho) f(\xi)) < F(\{\xi\})\, x, y > \\ = \quad & < f(\xi)\, F(\{\xi\})\, x, y > \end{aligned}$$

and so $\Phi(f) = f(\xi) F(\{\xi\}), f \in A$. In particular it follows that $F(\{\xi\}) = I$ and consequently $\Phi(f) = f(\xi) I$, for $f \in A$. The proof is finished. □

Note that this corollary is a generalized version of the [3, Corollary 1] because our algebra A is supposed to be weak*- Dirichlet in $L^\infty(m)$ and so that m is not necessary the unique representing for the peak point ξ for A. Also, we only assume that the representation Φ is weak $\rho-$ spectral with respect to ξ, a weaker condition than in [3], where Φ is ρ-spectral with respect to ξ. An example for which the above corollary can be applied is the following.

Example. Let $A_1(\mathbb{T})$ be the algebra of all continuous functions on the unit circle T which have analytic extensions $\widetilde{f}$ to the open unit disc such that $\widetilde{f}(0) = f(1)$. Then $A_1(\mathbb{T})$ is a function algebra on T which is weak*$-$ Dirichlet in $L^\infty(m_1)$ where m_1 is the Haar measure on T. Clearly, the measure m_1, the Dyrac measure δ_1 which is supported in $\{1\}$ and also $\mu = \frac{1}{2}(m_1 + \delta_1)$ are representing measures for the homomorphism of evaluation at 1. But any point $\lambda \in T$ is a peak point for $A_1(\mathbb{T})$ and the evaluation e_λ at $\lambda \neq 1$ has a unique representing measure m_λ relative to $A_1(\mathbb{T})$. Also, $\{e_\lambda\}$ forms a Gleason part of $A_1(\mathbb{T})$ for every $\lambda \in T$. Thus by Corollary 3 it follows that the only m_λ a.c. representation of $A_1(\mathbb{T})$ on H which is weak $\rho-$ spectral with respect to e_λ is Φ_λ given by $\Phi_\lambda(f) = f(\lambda) I$, $f \in A_1(\mathbb{T})$, for any $\lambda \in T$.

Now we obtain in our context the following version of [3, Theorem 3].

Theorem 4. *Let A be a function algebra on X which is weak*$-$ Dirichlet in $L^\infty(m)$ for some $m \in M_\gamma$ and $\gamma \in M(A)$. Suppose $\gamma' \in M(A)$ such that γ' is not in the same Gleason part with γ. Then any m a.c. representation Φ of A on H which is weak $\rho-$ spectral with respect to γ' is a contractive and dilatable representation.*

Proof. We use the idea from the proof of [3, Theorem 3]. Let Φ be a representation of A on H for which there exist a system $\{\nu_{x,y}\}_{x,y\in\mathcal{H}}$ of m a.c. elementary measures and a weak $\rho-$ spectral measure μ_x attached to every $x \in H$ by Φ and γ'. Putting $\nu_x = \nu_{x,x}$, $x \in H$ one has that $\nu_x - \rho\mu_x$ is orthogonal to $A_{\gamma'}$. If $\mu_x = \mu_x^a + \mu_x^s$ is the $M_{\gamma'}$ decomposition of μ_x, then by M. and F. Riesz theorem ([1] and [4]) it follows that $\nu_x - \rho\mu_x^s$ is orthogonal to A, since ν_x being m a.c. it is also M_γ a.c. and ν_x is $M_{\gamma'}$ s. because γ and γ' belong to different Gleason parts of A (by [1, Theorem vi.2.2]).

Let now $\mu_x^s = \mu_x^{sa} + \mu_x^{ss}$ be the M_γ decomposition of μ_x^s. Then applying also the M. and F. Riesz theorem we infer that the measures $\nu_x - \rho\mu_x^{sa}$ and $\rho\mu_x^{ss}$ are orthogonal to A, hence $\mu_x^{ss} = 0$ because $\mu_x^{ss} \geq 0$. Next, as ν_x is a Hahn - Banach extension to $C(X)$ of the functional $f \to < \Phi(f)x, x >$ on A, and since $\mu_x^{sa} \geq 0$ and $\int f d\nu_x = \rho \int f d\mu_x^{sa}$ we get

$$\begin{aligned} \|\nu_x\| &= \sup_{\substack{f\in A \\ \|f\|=1}} \left| \int f d\nu_x \right| = \rho \sup_{\substack{f\in A \\ \|f\|=1}} \left| \int f d\mu_x^{sa} \right| \leq \rho \left\| \mu_x^{sa} \right\| \\ &= \rho\mu_x^{sa}(1) = \nu_x(1) \leq \|\nu_x\|, \end{aligned}$$

whence

$$\|\nu_x\| = \nu_x(1) = \rho \left\| \mu_x^{sa} \right\|.$$

This means that the measures ν_x are positive, for any $x \in H$.

Using the fact that $\nu_{x,y}$ are elementary measures for Φ we obtain for $x, x', y \in H$ and $\alpha, \beta \in C$ that

$$\int f d\nu_{\alpha x + \beta x', y} = \int f d\left(\alpha\nu_{x,y} + \beta\nu_{x',y}\right) \qquad (f \in A).$$

But this implies by Gleason - Whitney theorem ([10]) that

$$\nu_{\alpha x + \beta x', y} = \alpha\nu_{x,y} + \beta\nu_{x',y}$$

because the measures $\nu_{z,z'}$ are m a.c. for any $z, z' \in H$. Similary, one infers that

$$\nu_{x', \alpha x + \beta y} = \overline{\alpha}\nu_{x',x} + \overline{\beta}\nu_{x',y}.$$

Thus, for $\sigma \in Bor(X)$ the functional $(x, y) \to \nu_{x,y}(\sigma)$ is linear in $x \in H$, antilinear in $y \in H$ and also we have

$$|\nu_{x,y}(\sigma)| \leq \|\nu_{x,y}\| \leq \|\Phi\| \, \|x\| \, \|y\|$$

because $\nu_{x,y}$ is an elementary measure for Φ. Hence we can define the map $F : Bor(X) \to B(H)$ by

$$< F(\sigma)x, y >= \nu_{x,y}(\sigma) \qquad (\sigma \in Bor(X), x, y \in \mathcal{H})$$

and it is immediate that F is a semispectral measure for Φ. Finally, by the Naimark theorem ([9]), it follows that Φ has a spectral dilation, necessarily a contractive one. Consequently, Φ is a contractive representation, and the proof is finished. □

As an application we have the following result which generalized [3, Theorem 3] because our hypothesis on Φ is weaker than the assumption from [3].

Corollary 5. *Let A be a function algebra on X and $\gamma, \gamma' \in M(A)$ belonging to different Gleason parts of A, such that γ has a unique representing measure. Then any M_γ a.c. representation of A on H which is weak $\rho-$ spectral with respect to γ' is a contractive representation and it has a spectral dilation.*

Proof. If $M_\gamma = \{m\}$ then A is weak$^*-$ Dirichlet in $L^\infty(m)$. So we can apply Theorem 4 to any m a.c. representation which is weak $\rho-$ spectral with respect to γ' and the conclusion follows. □

Finally, we prove the following

Theorem 6. *Let A be a function algebra on X with the property that the only measure orthogonal to A which is singular to all representing measures for the homomorphisms in $M(A)$ is the zero measure. Then every completely singular representation of A on H which is weak $\rho-$ spectral with respect to some $\gamma \in M(A)$ is a spectral one.*

Proof. Let Φ and γ as above, and for $x, y \in H$ let $\nu_{x,y}$ be a completely singular elementary measure for Φ. If μ_x is a weak $\rho-$ spectral measure attached to $x \in H$ by Φ and γ then $\nu_x - \rho\mu_x$ is orthogonal to A_γ. So, if $\mu_x = \mu_x^a + \mu_x^s$ is the M_γ decomposition of μ_x, by M. and F. Riesz theorem one has that $\nu_x - \rho\mu_x^s$ is orthogonal to A because ν_x is also M_γ s. (being completely singular). Next, as in the proof of Theorem 4 we deduce that ν_x is a positive measure. Also, using the hypothesis on A we infer that the map $(x, y) \to \nu_{x,y}$ is linear in $x \in H$ and it is antilinear in $y \in H$. This leads (as in the proof of Theorem 1) to the fact Φ is a contractive representation which has a spectral dilation, and [3, Theorem 4] implies that Φ is even a spectral representation. This ends the proof. □

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