Surveys in Mathematics and its Applications

ISSN 1842-6298 (electronic), 1843-7265 (print)
Volume 6 (2011), 203 – 219

LOG-CONCAVITY PROPERTY FOR SOME WELL-KNOWN DISTRIBUTIONS

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Abstract. Interesting properties and propositions, in many branches of science such as economics have been obtained according to the property of cumulative distribution function of a random variable as a concave function. Caplin and Nalebuff (1988 [10],1989 [11]), Bagnoli and Khanna (1989 [7]) and Bagnoli and Bergstrom (1989 [4], 1989 [5], 2005 [6]) have discussed the log-concavity property of probability distributions and their applications, especially in economics.

Log-concavity concerns twice differentiable real-valued function g whose domain is an interval on extended real line. g as a function is said to be log-concave on the interval (a, b) if the function $ln(g)$ is a concave function on (a, b). Log-concavity of g on (a, b) is equivalent to $g^{'}/g$ being monotone decreasing on (a, b) or $(ln(g))^{''} < 0$. Bagnoli and Bergstrom (2005 [6]) have obtained log-concavity for distributions such as normal, logistic, extreme-value, exponential, Laplace, Weibull, power function, uniform, gamma, beta, Pareto, log-normal, Student's t, Cauchy and F distributions. We have discussed and introduced the continuous versions of the Pearson family, also found the log-concavity for this family in general cases, and then obtained the log-concavity property for each distribution that is a member of Pearson family. For the Burr family these cases have been calculated, even for each distribution that belongs to Burr family. Also, log-concavity results for distributions such as generalized gamma distributions, Feller-Pareto distributions, generalized Inverse Gaussian distributions and generalized Log-normal distributions have been obtained.

Full text

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2010 Mathematics Subject Classification: 91B02; 62H10; 62P20.
Keywords: Log-concavity; Log-convexity; Continuous distributions; Pearson family; Burr family; Generalized gamma distributions; Generalized inverse Gaussian.

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