

## APPLICATION OF HOMOTOPY ANALYSIS METHOD FOR SOLVING NONLINEAR CAUCHY PROBLEM

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**Abstract.** In this paper, by means of the homotopy analysis method (HAM), the solutions of some nonlinear Cauchy problem of parabolic-hyperbolic type are exactly obtained in the form of convergent Taylor series. The HAM contains the auxiliary parameter  $\hbar$  that provides a convenient way of controlling the convergent region of series solutions. This analytical method is employed to solve linear examples to obtain the exact solutions. The results reveal that the proposed method is very effective and simple.

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