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ON A NEW CLASS OF INTEGRALS INVOLVING GENERALIZED MITTAG-LEFFLER FUNCTION

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Abstract. In this paper, we aim at establishing two generalized integral formulae involving generalized Mittag-Leffler function which are expressed in terms of the generalized hypergeometric function and generalized (Wright) hypergeometric function. Some interesting special cases of our main results are also considered. The results are derived with the help of an interesting integral due to Lavoie and Trottier.

1 Introduction and Preliminaries

In recent years, many integral formulae involving a variety of special functions have been developed by various authors [2, 6, 8] for a very recent work, see also [1]. Integrals involving generalized Mittag-Leffler functions are of great importance since they are used in applied physics and in many branches of engineering.
First we begin by recalling some known Mittag-Leffler functions and earlier results including other special functions.

In 1903, the Swedish mathematician Gosta Mittag-Leffler [5] introduced the function $E_\alpha(z)$ known as one-parameter Mittag-Leffler function defined by

$$E_\alpha(z) = \sum_{k=0}^{\infty} \frac{z^k}{\Gamma(\alpha k + 1)}, \tag{1.1}$$

where z is a complex variable and $\Gamma(.)$ is a Gamma function. The Mittag-Leffler function is a direct generalization of the exponential function to which it reduces for $\alpha = 1$. For $0 < \alpha < 1$ it interpolates between the pure exponential and a hypergeometric function $\frac{1}{1-z}$. Its importance is realized during the last two decades due to its involvement in the problems of physics, chemistry, biology, engineering and applied sciences. Mittag-Leffler function naturally occurs as the solution of fractional order differential equation or fractional order integral equations.

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The two-parameter generalization of $E_\alpha(z)$ was studied by Wiman [11] as:

$$E_{\alpha,\beta}(z)=\sum_{k=0}^{\infty}\frac{z^k}{\Gamma\ (\alpha k+\beta)}\qquad (\alpha,\beta\in\mathbb{C};\Re(\alpha)>0,\Re(\beta)>0), \tag{1.2}$$

which is known as Wiman's function or generalized Mittag-Leffler function as

$$\mathrm{E}_{\alpha,1}(\mathrm{z})=\mathrm{E}_\alpha(\mathrm{z}).$$

The three-parameter generalization of (1.2) was introduced in terms of series representation by Prabhakar [7]

$$E^{\gamma}_{\alpha,\ \beta}(z)=\sum_{k=0}^{\infty}\left(\frac{(\gamma)_k z^k}{\Gamma\ (\alpha k+\beta)\mathrm{k}\ !}\right) \tag{1.3}$$

called usually Prabhakar function. Where $\alpha,\ \beta,\ \gamma\in\mathbb{C}$; $\mathfrak{R}(\alpha)>0$, $\mathfrak{R}(\beta)>0)$, $\mathfrak{R}$ $(\gamma)>0$, and $(\gamma)_k$ is the well-known Pochhammer symbol.

Further generalization of (1.3) is generalized Mittag-Leffler function $E^{\gamma,q}_{k,\ \beta}(z)$ due to Shukla and Prajapati [9], which is defined for $k,\ \beta,\ \gamma\in\mathbb{C}$; $\mathfrak{R}(k)>0$, $\mathfrak{R}(\beta)$ >0 , $\mathfrak{R}(\gamma)>0$, $\beta>k>0$ and q $\in$ (0, 1)U N is

$$E^{\gamma,q}_{k,\ \beta}(z)=\sum_{n=0}^{\infty}\left(\frac{(\gamma)_{qn}z^n}{\mathrm{n}\ !\Gamma\ (kn+\beta)}\right), \tag{1.4}$$

where $(\gamma)_{\mathrm{qn}}=\frac{\Gamma\ (\gamma+\mathrm{qn})}{\Gamma\ (\gamma)}$ denotes the generalized Pochhammer symbol which in particular reduces to

$$\mathrm{q}^{\mathrm{qn}}\prod_{\mathrm{r}=1}^{\mathrm{q}}\left(\frac{\gamma+\mathrm{r}-1}{\mathrm{q}}\right)_{\mathrm{n}}. \tag{1.5}$$

An interesting generalizations of the generalized hypergeometric series ${}_p\mathrm{F}_q$ are due to Fox [3] and Wright [12, 13, 14], who studied the asymptotic expansion of the generalized (Wright) hypergeometric function defined by [10]:

$${}_p\Psi_q\left[\begin{array}{c}(\alpha_1,A_1),\ldots,(\alpha_p,A_p)\ ;z\\(\beta_1,B_1),\ldots,(\beta_q,B_q);\end{array}\right]=\sum_{k=0}^{\infty}\frac{\prod_{j=1}^{p}\ \Gamma\ (\alpha_j+A_jk)}{\prod_{j=1}^{q}\Gamma\ (\beta_j+B_jk)}\frac{z^k}{\mathrm{k}!}, \tag{1.6}$$

where the coefficients $A_1,\ldots,A_p$ and $B_1,\ldots,B_q$ are real positive numbers such that

$$1+\sum_{j=1}^{q}B_j-\sum_{j=1}^{p}A_j\geqq 0 \tag{1.7}$$

A special case of (1.6) when $A_i=B_j=1\quad(i=1,\ldots,p\ and\ j=1,\ldots,q)$

$$ {}_p\Psi_q \left[\begin{array}{c} (\alpha_1,1),\ldots,(\alpha_p,1)\ ;z \\ (\beta_1,1),\ldots,(\beta_q,1)\,; \end{array} \right] = \frac{\prod_{j=1}^{p}\Gamma\ (\alpha_j)}{\prod_{j=1}^{q}\Gamma\ (\beta_j)}\, {}_p\mathrm{F}_q \left[\begin{array}{c} \alpha_1,\ldots,\alpha_p\ ;z \\ \beta_1,\ldots,\beta_q\ ; \end{array} \right], \tag{1.8} $$

where${}_p\mathrm{F}_q$ is the generalized hypergeometric series defined by (see [10]):

$$ {}_p\mathrm{F}_q \left[\begin{array}{c} \alpha_1,\ldots,\alpha_p\ ;z \\ \beta_1,\ldots,\beta_q\ ; \end{array} \right] = \sum_{n=0}^{\infty} \frac{(\alpha_1)_n\cdots(\alpha_p)_n z^n}{(\beta_1)_n\cdots(\beta_q\ )_n \mathrm{n}!} \tag{1.9} $$

$$ = {}_p\mathrm{F}_q\left(\alpha_1,\ldots,\alpha_p;\beta_1,\ldots,\beta_q;z\right). $$

We also recall Lavoie-Trottier integral formula [4] for our present study

$$ \int_0^1 x^{\alpha-1}(1-x)^{2\beta-1}\left(1-\frac{x}{3}\right)^{2\alpha-1}\left(1-\frac{x}{4}\right)^{\beta-1}dx $$

$$ = \left(\frac{2}{3}\right)^{2\alpha}\frac{\Gamma\ (\alpha)\Gamma\ (\beta)}{\Gamma\ (\alpha+\beta)}\ ,\ \left(\Re\left(\alpha\right)>0\ \mathit{and}\ \Re\left(\beta\right)>0\right). \tag{1.10} $$

In this present paper, we establish two general integral formulae involving generalized Mittag-Leffler function (1.4) by using Lavoie-Trottier integral (1.10) in terms of the generalized hypergeometric and generalized (Wright) hypergeometric functions. Certain corollaries and special cases are also obtained.

2 Main Results

In this section, we established two generalized integral formulae which are expressed in terms of the generalized hypergeometric function (1.9), by inserting generalized Mittag-Leffler function (1.4) with the suitable arguments into integrand of (1.10).

Theorem 1. *For* $k,\ \beta,\ \gamma,\ \rho,\ j\in\mathbb{C};\Re(k)>0,\Re(\beta)>0,\Re(\gamma)>0,\ \beta>k>0,\ \Re(\rho)>0,\Re(\rho+j)>0,\Re(\rho+n)>0,\ x>0$ *and* $q\in(0,1)\cup\mathbb{N}$*, the following integral formula holds true:*

$$ \int_0^1 x^{\rho+j-1}(1-x)^{2\rho-1}\left(1-\frac{x}{3}\right)^{2(\rho+j)-1}\left(1-\frac{x}{4}\right)^{\rho-1} E_{k,\ \beta}^{\gamma,q}\left(y\left(1-\frac{x}{4}\right)(1-x)^2\right)dx $$

$$ = \left(\frac{2}{3}\right)^{2(\rho+j)}\frac{\Gamma\ \ (\rho+j)\,\Gamma\ \ (\rho)}{\Gamma\ \ (\beta)\,\Gamma\ \ (2\rho+j)}\,{}_{q+1}\mathrm{F}_{k+1}\left[\begin{array}{c} \Delta\left(q;\gamma\right),\left(\rho\right) \\ \Delta\left(k;\beta\right),\left(2\rho+j\right) \end{array} ;\frac{yq^q}{k^k}\right], \tag{2.1} $$

where

$$ \Delta(q;\gamma)\ \mathit{is}\ q-\mathit{tuple}\left(\frac{\gamma}{q}\right),\left(\frac{\gamma+1}{q}\right),\ldots,\left(\frac{\gamma+q-1}{q}\right) $$

and

$$\Delta\left(k;\beta\right)\ is\ k-tuple \left(\frac{\beta}{k}\right),\left(\frac{\beta+1}{k}\right),\ldots,\left(\frac{\beta+k-1}{k}\right).$$

Proof. By applying (1.4) in the integrand of (2.1) and interchanging the order of integral and summation which is valid under uniform convergence of the involved series with the given condition, we get

$$\int_0^1 x^{\rho+j-1}(1-x)^{2\rho-1}\left(1-\frac{x}{3}\right)^{2(\rho+j)-1}\left(1-\frac{x}{4}\right)^{\rho-1}E^{\gamma,q}_{k,\ \beta}\left(y\left(1-\frac{x}{4}\right)(1-x)^2\right)dx$$

$$=\sum_{n=0}^{\infty}\frac{(\gamma)_{qn}}{\Gamma\ (kn+\beta)}\frac{y^n}{n!}\int_0^1 x^{\rho+j-1}(1-x)^{2(\rho+n)-1}\left(1-\frac{x}{3}\right)^{2(\rho+j)-1}\left(1-\frac{x}{4}\right)^{(\rho+n)-1}dx.$$

Applying integral formula (1.10) and then using (1.5) we obtain

$$\int_0^1 x^{\rho+j-1}(1-x)^{2\rho-1}\left(1-\frac{x}{3}\right)^{2(\rho+j)-1}\left(1-\frac{x}{4}\right)^{\rho-1}E^{\gamma,q}_{k,\ \beta}\left(y\left(1-\frac{x}{4}\right)(1-x)^2\right)dx$$

$$=\left(\frac{2}{3}\right)^{2(\rho+j)}\frac{\Gamma\ (\rho+j)\Gamma\ (\rho)}{\Gamma\ (\beta)\,\Gamma\ (2\rho+j)}\sum_{n=0}^{\infty}\frac{q^{qn}\ \prod_{i=1}^{q}\left(\frac{\gamma+i-1}{q}\right)_n(\rho)_n y^n}{k^{kn}\ n!\prod_{l=1}^{k}\left(\frac{\beta+l-1}{k}\right)_n(2\rho+j)_n}$$

$$=\left(\frac{2}{3}\right)^{2(\rho+j)}\frac{\Gamma\ (\rho+j)\Gamma\ (\rho)}{\Gamma\ (\beta)\,\Gamma\ (2\rho+j)}{}_{q+1}\mathrm{F}_{k+1}\left[\begin{array}{c}\left(\frac{\gamma}{q},\frac{\gamma+1}{q},\ldots,\frac{\gamma+q-1}{q}\right),(\rho)\,;\\ \left(\frac{\beta}{k},\frac{\beta+1}{k},\ldots,\frac{\beta+k-1}{k}\right),(2\rho+j)\,;\end{array}\ \frac{yq^q}{k^k}\right].$$

This completes the proof of Theorem 1. □

Theorem 2. *The following integral formula holds true:*

$$\int_0^1 x^{\rho-1}(1-x)^{2(\rho+j)-1}\left(1-\frac{x}{3}\right)^{2\rho-1}\left(1-\frac{x}{4}\right)^{(\rho+j)-1}E^{\gamma,q}_{k,\ \beta}\left(y\ x\left(1-\frac{x}{3}\right)^2\right)dx$$

$$=\left(\frac{2}{3}\right)^{2\rho}\frac{\Gamma\ (\rho+j)\,\Gamma\ (\rho)}{\Gamma\ (\beta)\,\Gamma\ (2\rho+j)}\ {}_{q+1}\mathrm{F}_{k+1}\left[\begin{array}{c}\Delta\left(q;\gamma\right),(\rho)\\ \Delta\left(k;\beta\right),(2\rho+j)\end{array};\frac{4yq^q}{9k^k}\right],\qquad(2.2)$$

where $k,\ \beta,\ \gamma,\ \rho,\ j\in\mathbb{C};\Re(k)>0,\Re(\beta)>0,\Re(\gamma)>0,\ \beta>k>0,\Re(\rho)>0,\Re(\rho+j)>0,\Re(\rho+n)>0,\ x>0$ *and* $q\in(0,1)\cup\mathbb{N}$.

Proof. It is easy to see that a similar argument as in the proof of Theorem 1 will establish the integral formula (2.2). Therefore, we omit the details of the proof of this theorem. □

Corollary 3. *Let the condition of Theorem 1 be satisfied, then the following integral formula holds true:*

$$\int_0^1 x^{\rho+j-1}(1-x)^{2\rho-1}\left(1-\frac{x}{3}\right)^{2(\rho+j)-1}\left(1-\frac{x}{4}\right)^{\rho-1} E^{\gamma,q}_{k,\ \beta}\left(y\left(1-\frac{x}{4}\right)(1-x)^2\right)dx$$

$$= \left(\frac{2}{3}\right)^{2(\rho+j)}\frac{\Gamma\ (\rho+j)}{\Gamma\ (\gamma)}\ {}_2\Psi_2\left[\begin{array}{c}(\gamma,q),(\rho,1)\\(\beta,k),(2\rho+j,1)\end{array};y\right]. \tag{2.3}$$

Corollary 4. *Let the condition of Theorem 2 be satisfied, then the following integral formula holds true:*

$$\int_0^1 x^{\rho-1}(1-x)^{2(\rho+j)-1}\left(1-\frac{x}{3}\right)^{2\rho-1}\left(1-\frac{x}{4}\right)^{(\rho+j)-1} E^{\gamma,q}_{k,\ \beta}\left(y\ x\left(1-\frac{x}{3}\right)^2\right)dx$$

$$= \left(\frac{2}{3}\right)^{2\rho}\frac{\Gamma\ (\rho+j)}{\Gamma\ (\gamma)}\ {}_2\Psi_2\left[\begin{array}{c}(\gamma,q),(\rho,1)\\(\beta,k),(2\rho+j,1)\end{array};\frac{4y}{9}\right]. \tag{2.4}$$

Following the proofs of Theorem 1 and 2, we can easily establish the integral formulas of Corollary 3 and Corollary 4 by using the definition (1.6). Therefore, we omit the details of the proof of corollaries.

3 Special cases

In this section, we derive some new integral formulae by using some known generalized Mittag-Leffler functions, which are given in corollaries 5 to 12.
If we employ the same method as in getting Theorems 1 and 2 and Corollaries 3 and 4, we obtain the following four corollaries with the help of (1.3) which is well known generalized Mittag-Leffler function due to Prabhakar [7].

Corollary 5. *Let the condition of Theorem 1 be satisfied and for $q=1$, Theorem 1 reduces in following form*

$$\int_0^1 x^{\rho+j-1}(1-x)^{2\rho-1}\left(1-\frac{x}{3}\right)^{2(\rho+j)-1}\left(1-\frac{x}{4}\right)^{\rho-1} E^{\gamma}_{k,\ \beta}\left(y\left(1-\frac{x}{4}\right)(1-x)^2\right)dx$$

$$= \left(\frac{2}{3}\right)^{2(\rho+j)}\frac{\Gamma\ (\rho+j)\,\Gamma\ (\rho)}{\Gamma\ (\beta)\,\Gamma\ \ (2\rho+j)}\ {}_2\mathrm{F}_{k+1}\left[\begin{array}{c}(\gamma),(\rho)\\\Delta(k;\beta),2\rho+j\end{array};\frac{y}{k^k}\right]. \tag{3.1}$$

Corollary 6. *Let the condition of Theorem 2 be satisfied and for $q=1$, Theorem 1 reduces in following form*

$$\int_0^1 x^{\rho-1}(1-x)^{2(\rho+j)-1}\left(1-\frac{x}{3}\right)^{2\rho-1}\left(1-\frac{x}{4}\right)^{(\rho+j)-1} E^{\gamma}_{k,\ \beta}\left(y\ x\left(1-\frac{x}{3}\right)^2\right)dx$$

$$=\left(\frac{2}{3}\right)^{2\rho}\frac{\Gamma\ (\rho+j)\,\Gamma\ (\rho)}{\Gamma\ (\beta)\,\Gamma\ (2\rho+j)}\ {}_2\mathrm{F}_{k+1}\left[\begin{array}{c}(\gamma)\,,(\rho)\\ \Delta\,(k;\beta)\,,2\rho+j\end{array};\frac{4y}{9k^k}\right]. \tag{3.2}$$

Corollary 7. *Let the condition of Theorem 1 be satisfied and for $q=1$, Corollary 3 reduces in following form*

$$\int_0^1 x^{\rho+j-1}(1-x)^{2\rho-1}\left(1-\frac{x}{3}\right)^{2(\rho+j)-1}\left(1-\frac{x}{4}\right)^{\rho-1} E^{\gamma}_{k,\ \beta}\left(y\left(1-\frac{x}{4}\right)(1-x)^2\right)dx$$

$$=\left(\frac{2}{3}\right)^{2(\rho+j)}\frac{\Gamma\ (\rho+j)}{\Gamma\ (\gamma)}\ {}_2\Psi_2\left[\begin{array}{c}(\gamma,1)\,,(\rho,1)\\ (\beta,k)\,,(2\rho+j,1)\end{array};y\right]. \tag{3.3}$$

Corollary 8. *Let the condition of Theorem 2 be satisfied and for $q=1$, Corollary 4 reduces in following form*

$$\int_0^1 x^{\rho-1}(1-x)^{2(\rho+j)-1}\left(1-\frac{x}{3}\right)^{2\rho-1}\left(1-\frac{x}{4}\right)^{(\rho+j)-1} E^{\gamma}_{k,\ \beta}\left(y\ x\left(1-\frac{x}{3}\right)^2\right)dx$$

$$=\left(\frac{2}{3}\right)^{2\rho}\frac{\Gamma\ (\rho+j)}{\Gamma\ (\gamma)}\ {}_2\Psi_2\left[\begin{array}{c}(\gamma,1)\,,(\rho,1)\\ (\beta,k)\,,(2\rho+j,1)\end{array};\frac{4y}{9}\right]. \tag{3.4}$$

Again, if we set $q=\gamma=1$ and use definition (1.2), we obtain the following results involving generalized Mittag-Leffler function due to Wiman [11]:

Corollary 9. *Let the condition of Theorem 1 be satisfied and for $q=\gamma=1$, then Theorem 1 reduces in following form:*

$$\int_0^1 x^{\rho+j-1}(1-x)^{2\rho-1}\left(1-\frac{x}{3}\right)^{2(\rho+j)-1}\left(1-\frac{x}{4}\right)^{\rho-1}\frac{1}{n!}E_{\ k,\ \beta}\left(y\left(1-\frac{x}{4}\right)(1-x)^2\right)dx$$

$$=\left(\frac{2}{3}\right)^{2(\rho+j)}\frac{\Gamma\ (\rho+j)\,\Gamma\,(\rho)}{\Gamma\ (\beta)\,\Gamma\ (2\rho+j)}\ {}_1\mathrm{F}_{k+1}\left[\begin{array}{c}(\rho)\\ \Delta\,(k;\beta)\,,(2\rho+j)\end{array};\frac{y}{k^k}\right]. \tag{3.5}$$

Corollary 10. *Let the condition of Theorem 2 be satisfied and for $q = \gamma = 1$, Theorem 2 reduces in following form*

$$\int_0^1 x^{\rho-1}(1-x)^{2(\rho+j)-1}\left(1-\frac{x}{3}\right)^{2\rho-1}\left(1-\frac{x}{4}\right)^{(\rho+j)-1}\frac{1}{n!}E_{\,k,\,\beta}\left(y\,x\left(1-\frac{x}{3}\right)^2\right)dx$$

$$=\left(\frac{2}{3}\right)^{2\rho}\frac{\Gamma\ (\rho+j)\,\Gamma\ (\rho)}{\Gamma\ (\beta)\,\Gamma\ (2\rho+j)}\,{}_1\mathrm{F}_{k+1}\left[\begin{matrix}(\rho)\\ \Delta\,(k;\beta)\,,(2\rho+j)\end{matrix}\,;\frac{4y}{9k^k}\right]. \tag{3.6}$$

Corollary 11. *Let the condition of Theorem 1 be satisfied and for $q = \gamma = 1$, then Corollary 3 reduces in following form:*

$$\int_0^1 x^{\rho+j-1}(1-x)^{2\rho-1}\left(1-\frac{x}{3}\right)^{2(\rho+j)-1}\left(1-\frac{x}{4}\right)^{\rho-1}\frac{1}{n!}E_{\,k,\,\beta}\left(y\left(1-\frac{x}{4}\right)(1-x)^2\right)dx$$

$$=\left(\frac{2}{3}\right)^{2(\rho+j)}\frac{\Gamma\ (\rho+j)}{\Gamma\ (\gamma)}\,{}_1\Psi_2\left[\begin{matrix}(\rho,1)\\ (\beta,k)\,,(2\rho+j,1)\end{matrix}\,;y\right]. \tag{3.7}$$

Corollary 12. *Let the condition of Theorem 2 be satisfied and for $q = \gamma = 1$, Corollary 4 reduces in following form*

$$\int_0^1 x^{\rho-1}(1-x)^{2(\rho+j)-1}\left(1-\frac{x}{3}\right)^{2\rho-1}\left(1-\frac{x}{4}\right)^{(\rho+j)-1}\frac{1}{n!}E_{\,k,\,\beta}\left(y\,x\left(1-\frac{x}{3}\right)^2\right)dx$$

$$=\left(\frac{2}{3}\right)^{2\rho}\frac{\Gamma\ (\rho+j)}{\Gamma\ (\gamma)}\,{}_1\Psi_2\left[\begin{matrix}(\rho,1)\\ (\beta,k)\,,(2\rho+j,1)\end{matrix}\,;\frac{4y}{9}\right]. \tag{3.8}$$

References

[1] P. Agarwal, S. Jain, S. Agarwaland M. Nagpal, *On a new class of integrals involving Bessel functions of the first kind*, ISPACS, **2014** (2014), 1-7.

[2] J. Choi, A. Hasanov, H. M. Srivastava and M. Turaev, *Integral representations for Srivastava's triple hypergeometric functions*, *Taiwanese J. Math*, **15**(2011), 2751-2762. MR2896142. Zbl 1250.33009.

[3] C. Fox, *The asymptotic expansion of generalized hypergeometric functions*, Proc. London Math. Soc, **27**(1928), 389-400. MR1575398. JFM 54.0392.03.

[4] J. L. Lavoie and G. Trottier, *On the sum of certain Appell's series*, Ganita, **20** (1969), 43-46. MR0267147. Zbl 0208.08201.

[5] G. M. Mittag-Leffler, *Sur la nouvelle fonction* $E_\alpha(x)$, C. R. Acad. Sci. Paris **137** (1903), 554-558.

[6] F. W. L. Olver, D. W. Lozier, R. F. Boisvert and C. W. Clark, *NIST Handbook of Mathematical Functions*, Cambridge University Press, 2010. MR2723248(2012a:33001). Zbl 1198.00002.

[7] T.R. Prabhakar, *A singular integral equation with a generalized Mittag-Leffler function in the kernel*, Yokohama Math. J. **19** (1971), 7-15. MR0293349. Zbl 0221.45003.

[8] M. A. Rakha, A. K. Rathie, M. P. Chaudhary and S. Ali, *On A New Class of integrals involving hypergeometric function*, J. Inequal. Spec. Funct., **3**(2012), 10-27. MR2914525. Zbl 1312.33013.

[9] A. K. Shukla and J. C. Prajapati, *On a generalization of Mittag-Leffler function and its properties*, J. Math. Anal. Appl., **336**(2007), 797-811. MR2352981(2008m:33055). Zbl 1122.33017.

[10] H. M. Srivastava and P. W. Karlsson, *Multiple Gaussian Hypergeometric Series*, Halsted Press (Ellis Horwood Limited, Chichester), John Wiley and Sons, New York, Chichester, Brisbane, and Toronto, 1985. MR0834385(87f:33015). Zbl 0552.33001.

[11] A. Wiman, *Über den fundamental Satz in der Theorie der Funktionen* $E_\alpha(x)$, Acta Math. **29**(1905), 191-201. MR1555014. JFM 36.0471.01.

[12] E. M. Wright, *The asymptotic expansion of the generalized hypergeometric functions*, J. London Math. Soc, **10**(1935), 286-293.

[13] E. M. Wright, *The asymptotic expansion of integral functions defined by Taylor series*, Philos. Trans. Roy. Soc. London, A. **238**(1940), 423-451. MR0001296. Zbl 0023.14002.

[14] E. M. Wright, *The asymptotic expansion of the generalized hypergeometric function II*, Proc. London Math. Soc., **46**(1940), 389-408. MR0003876.

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