COINCIDENCE AND COMMON FIXED POINT OF F-CONTRACTIONS VIA CLR_{ST} PROPERTY

Anita Tomar, Giniswamy, C. Jeyanthi, P. G. Maheshwari

Abstract. The aim of this paper is to establish the existence of coincidence and common fixed point of F-contractions via CLR_{ST} property. Our results generalize, extend and improve the results of Wardowski [D. Wardowski, Fixed points of a new type of contractive mappings in complete metric spaces, Fixed Point Theory and Applications (2012) 2012:94, 6 pages, doi: 10.1186/1687-1812-2012-94], Batra et al. [Coincidence Point Theorem for a New Type of Contraction on Metric Spaces, Int. Journal of Math. Analysis, Vol. 8(27) 2014, 1315-1320] and others existing in literature. Examples are also given in support of our results.

1 Introduction

The concept of F-contraction for a single valued map on a complete metric space was introduced by Wardowski [17] as a generalization of the famous type of map called Banach Contraction. Recently several authors have obtained various extensions and generalizations of F-contraction for single valued as well as multiple valued maps (for instance [5], [11], [12], [13], [15], [17], [18]). The aim of this paper is to establish the existence and uniqueness of coincidence and common fixed point of two pairs of weakly compatible maps using a new type of contraction, called F-contraction via CLR_{ST} property. Our results generalize, extend and improve the results of Wardowski [17], Batra et al. [13] and others existing in literature (for instance Minak et al. [11], Wardowski and Dung [18], Cosentino and Vetro [5], Ćirić [3], Hardy-Rogers [6], Kannan [9], Chatterjee [2], Reich [14] and references there in) without completeness or closedness of space/subspace, containment and continuity requirement of involved maps. In this paper we use Ćirić type F- contraction and Hardy-Roger type F-contraction for two pairs of self maps, which are more general than the contraction introduced by Wardowski [17] and others.

2 Preliminaries

We denote the set of all real numbers by R, the set of all positive real numbers by R^+ and the set of all natural numbers by N. Let (X, d) be a metric space.

Definition 1. *[3] A self map A on X is called Ćirić type contraction if*
$d(Ax, Ay) \leq c \, max\{d(x, y), d(x, Ax), d(y, Ay), d(x, Ay), d(y, Ax)\}$, where $0 \leq c < 1$, *for all $x, y \in X$.*

Definition 2. *[6] A self map A on X is called Hardy-Roger type contraction if*
$d(Ax, Ay)) \leq \alpha d(Ax, x) + \beta d(Ay, y) + \gamma d(x, y) + \delta d(Ax, y) + Ld(Ay, x))$, where
$\alpha + \beta + \gamma + \delta + L < 1$ and $\alpha, \beta, \gamma, \delta, L > 0$, for all $x, y \in X$.

Definition 3. *[17] A self map A on X is an F-contraction if there exist $\tau > 0$ such that*

$$d(Ax, Ay) > 0 \Rightarrow \tau + F(d(Ax, Ay)) \leq F(d(x, y)) \qquad (2.1)$$

for all $x, y \in X$ where $F : R^+ \to R$ is a function satisfying:

(F1) F is strictly increasing, i.e. for all $\alpha, \beta \in R$ such that $\alpha < \beta$, $F(\alpha) < F(\beta)$;

*(F2) For each sequence $\{\alpha_n\}$, $n \in N$ of positive numbers, the following holds:
$lim_{n \to \infty} \alpha_n = 0$ if and only if $lim_{n \to \infty} F(\alpha_n) = -\infty$;*

(F3) There exist $k \in (0, 1)$ such that $lim_{\alpha \to 0^+}(\alpha^k F(\alpha)) = 0$.

From (F1) and (2.1) it is easy to conclude that every F-contraction A is a contractive map and hence necessarily continuous. We denote by $\mathcal{F}$, the family of all $F : R^+ \to R$ satisfying the conditions (F1)-(F3). Taking different functions F, we obtain a variety of F-contractions, some of them being already known in the literature. Some examples of the functions belonging to $\mathcal{F}$ are:

(1) $F(\alpha) = \ln \alpha$;   (2) $F(\alpha) = \ln \alpha + \alpha, \alpha > 0$;

(3) $F(\alpha) = \dfrac{-1}{\sqrt{\alpha}}, \alpha > 0$;   (4) $F(\alpha) = \ln(\alpha^2 + \alpha), \alpha > 0$.

Every F-contraction A is a contractive map, i.e. $d(Ax, Ay) < d(x, y)$ for all $x, y \in X$, $Ax \neq Ay$. The Banach contraction [1] is a particular case of F-contraction. Meanwhile there exist F-contractions, which are not Banach contractions [17].

Following Wardowski, Shukla and Radenovic [15] introduced the concept of Ćirić type F-contraction for a pair of map in 0-complete partial metric space without giving any name to it. Later on Minak et al. [11] and Wardowski and Dung [18] independently introduced Ćirić type F-contraction for a single map in metric space. Minak et al. called it Ćirić type generalized F-contraction and Wardowski and Dung called it F-weak contraction. Later Cosentino and Vetro [5] introduced Hardy-Rogers

type F-contraction.

If there exists $F \in \mathcal{F}$ and $\tau > 0$ such that, for all $x, y \in X$,

(a) $p(Ax, By) > 0 \Rightarrow \tau + F(p(Ax, By)) \leq F(max\{p(x, y), p(x, Ax), p(y, By), \frac{p(x, By) + p(y, Ax)}{2}\})$, where p is partial metric, Shukla and Radenovic [15];

(b) $d(Ax, Ay) > 0 \Rightarrow \tau + F(d(Ax, Ay)) \leq F(m(x, y))$, where $m(x, y) = max\{d(x, y), d(x, Ax), d(y, Ay), \frac{d(x, Ay) + d(y, Ax)}{2}\}$, is the Ćirič type F-contraction [11] and if $m(x, y) = \alpha d(x, y) + \beta d(x, Ax) + \gamma d(y, Ay) + \delta d(x, Ay) + Ld(y, Ax)$, where $\alpha + \beta + \gamma + 2\delta = 1$, $\gamma \neq 1$ and $L \geq 0$, is the Hardy-Rogers type F-contraction [6].

On the other hand, Piri and Kumam [12] introduced Suzuki type F-contraction. If there exists $F \in \mathcal{F}$ and $\tau > 0$ such that for all $x, y \in X$,

(c) $d(Ax, Ay) > 0, \frac{1}{2}d(x, Ax) < d(x, y) \Rightarrow \tau + F(d(Ax, Ay)) \leq F(d(x, y))$

Furthermore Malhotra et al. [10] introduced a F-g-contraction in partially ordered metric-like spaces and called it ordered F-g weak contraction and Batra et al. [13] introduced F-g contraction in metric space. If there exists $F \in \mathcal{F}$ and $\tau > 0$ such that for all $x, y \in X$,

(d) $\sigma(Ax, Ay) > 0 \Rightarrow \tau + F(\sigma(Ax, Ay)) \leq F(max\{\sigma(gx, gy), \sigma(gx, Ax), \sigma(gy, Ay)\})$, where σ is metric like, is ordered F-g weak contraction (Malhotra et al. [10]) and

(e) $\tau + F(d(Tx, Ty)) \leq F(d(gx, gy))$ with $gx \neq gy$ and $Tx \neq Ty$, is F-g contraction (Batra et al. [13]).

Note that every F-contraction is a Ćirič type F-contraction (Ex 2.3 [18]), Hardy-Rogers type F-contraction [6], Suzuki type F-contraction but the reverse implication does not hold. On substituting g = I(the identity map of X) in F-g contraction we get F-contraction.

Definition 4. *A pair of self-maps (A, S) on a metric space (X, d) is*

(a) **compatible** *[7] if $lim_{n \to \infty} d(ASx_n, SAx_n) = 0$ whenever $\{x_n\}$ is a sequence in X such that $lim_{n \to \infty} Ax_n = lim_{n \to \infty} Sx_n = t$ for some $t \in X$;*

(b) **non-compatible** *if there exists a sequence $\{x_n\}$ in X such that $lim_{n \to \infty} Ax_n = lim_{n \to \infty} Sx_n = t$ for some $t \in X$ but either $lim_{n \to \infty} d(ASx_n, SAx_n) \neq 0$ or this limit does not exist;*

(c) **weakly compatible** [8] if the pair commute on the set of their coincidence points i.e. for $x \in X$, $Ax = Sx$ implies $ASx = SAx$;

(d) satisfy the **common limit range property** [16] with respect to S denoted by CLR_S if there exists a sequence $\{x_n\}$ in X such that $lim_{n \to \infty} Ax_n = lim_{n \to \infty} Sx_n = t$ where $t \in SX$.

Definition 5. [4] Two pairs of self maps (A, S) and (B, T) of a metric space (X, d) satisfy the common limit range property with respect to S and T denoted by CLR_{ST} if there exist two sequences $\{x_n\}$ and $\{y_n\}$ in X such that $lim_{n \to \infty} Ax_n = lim_{n \to \infty} Sx_n = lim_{n \to \infty} By_n = lim_{n \to \infty} Ty_n = t$ where $t \in SX \cap TX$.

3   Main Results

Definition 6. Two pairs of self maps (A, S) and (B, T) of a metric space (X, d) are said to satisfy Ćirić type F-contraction condition if there exist $F \in \mathcal{F}$ and $\tau > 0$ such that for all $x, y \in X$

$$d(Ax, By) > 0 \Rightarrow \tau + F(d(Ax, By)) \leq F(max\{d(Ax, Sx), d(By, Ty), d(Ty, Sx),$$
$$d(Ty, Ax), d(By, Sx)\}).$$

(3.1)

It is interesting to point out here that every Ćirić type contraction condition [3] is also Ćirić type F-contraction condition. However converse need not be true.

Theorem 7. Let (A, S) and (B, T) be two pairs of self maps of a metric space (X, d) satisfying CLR_{ST} property and Ćirić type F- contraction condition 3.1. Then the pairs (A, S) and (B, T) have a coincidence point in X if F is continuous. Moreover A, B, S and T have a unique common fixed point in X provided that both the pairs (A, S) and (B, T) are weakly compatible.

Proof. Let (A, S) and (B, T) be pairs of self maps satisfying CLR_{ST} property then there exist two sequences $\{x_n\}$ and $\{y_n\}$ in X such that $lim_{n \to \infty} Ax_n = lim_{n \to \infty} Sx_n = lim_{n \to \infty} By_n = lim_{n \to \infty} Ty_n = t$ where $t \in SX \cap TX$. Since $t \in SX$, there exist $v \in X$ such that $Sv = t$.

If $d(Av, By_n) = 0$, then as $n \to \infty$, $d(Av, t) = 0$ which implies $Av = t$ and if $d(Av, By_n) > 0$, then using (3.1),

$$F(d(Av, By_n)) \leq F(max\{d(Av, Sv), d(By_n, Ty_n), d(Sv, Ty_n),$$
$$d(Av, Ty_n), d(By_n, Sv)\}) - \tau.$$

As $n \to \infty$ and since F is continuous,

$$F(d(Av, t)) \leq F(max\{d(Av, t), d(t, t), d(t, t), d(Av, t), d(t, t)\}) - \tau$$
$$= F(d(Av, t)) - \tau < F(d(Av, t)),$$

which is a contradiction. Hence $Av = Sv = t$, i.e. the pair (A, S) have a coincidence point in X.

Since $t \in TX$, there exist $u \in X$ such that $Tu = t$. Let $d(Bu, t) > 0$, then using (3.1),

$$F(d(t, Bu)) = F(d(Av, Bu))$$
$$\leq F(max\{d(Av, Sv), d(Bu, Tu), d(Sv, Tu), d(Av, Tu), d(Bu, Sv)\}) - \tau$$
$$= F(d(Bu, t)) - \tau < F(d(Bu, t)),$$

which is a contradiction. Hence $Bu = Tu = t$, the pair (B, T) have a coincidence point in X.

Since (A, S) and (B, T) are weakly compatible, $At = ASv = SAv = St$, $Bt = BTu = TBu = Tt$.

Let $At \neq t$, then using (3.1)

$$F(d(At, t)) = F(d(At, Bu))$$
$$\leq F(max\{d(At, St), d(Bu, Tu), d(St, Tu), d(At, Tu), d(Bu, St)\}) - \tau$$
$$= F(d(At, t)) - \tau < F(d(At, t)),$$

which is a contradiction. Hence At = St = t. Similarly we can show that Bt = Tt = t. Thus At = St = Bt = Tt = t, i.e. t is the common fixed point of A, B, S and T.

Let t and w be two fixed points and let $d(t, w) > 0$. Consider,

$$F(d(t, w)) = F(d(At, Bw))$$
$$\leq F(max\{d(At, St), d(Bw, Tw), d(St, Tw), d(At, Tw), d(Bw, St)\}) - \tau$$
$$= F(max\{d(t, t), d(w, w), d(t, w), d(t, w), d(w, t)\}) - \tau$$
$$= F(d(t, w)) - \tau < F(d(t, w)),$$

which is a contradiction. Hence $t = w$. Therefore the common fixed point t is unique. $\qquad \square$

We now give an example to illustrate theorem 7.

Example 8. *Let $X = [3, 15)$ and d be the usual metric on X.*
Define $A, B, S, T : X \to X$ as follows:

$$Ax = \begin{cases} 3 & if \ x = 3, 8 < x < 15 \\ 5 & if \ 3 < x \leq 8 \end{cases} \qquad Bx = \begin{cases} 3 & if \ x = 3, 8 < x < 15 \\ 6 & if \ 3 < x \leq 8 \end{cases}$$

$$Sx = \begin{cases} 3 & if \ x = 3 \\ 10 & if \ 3 < x \leq 8 \\ \frac{x+1}{3} & if \ 8 < x < 15 \end{cases} \qquad Tx = \begin{cases} 3 & if \ x = 3 \\ 12 & if \ 3 < x \leq 8 \\ x - 5 & if \ 8 < x < 15 \end{cases}$$

Let $\{x_n\}$ and $\{y_n\}$ be two sequences in X such that $x_n = 8 + \frac{1}{n}$ and $y_n = 3$ then $lim_{n\to\infty}Ax_n = lim_{n\to\infty}Sx_n = lim_{n\to\infty}By_n = lim_{n\to\infty}Ty_n = 3 \in SX \cap TX$. Hence A, B, S and T satisfy the CLR_{ST} property. Also $A3 = S3 = 3$, $B3 = T3 = 3$, i.e. pairs (A, S) and (B, T) have coincidence points in X. Also $AS3 = SA3 = 3$, $BT3 = TB3 = 3$, so the pairs (A, S) and (B, T) are weakly compatible. Also A, B, S and T satisfy Ćirić type F-contraction condition (3.1) for $\tau = log2$ and $F(\alpha) = log\alpha$. Hence all the conditions of theorem 7 are satisfied and $x = 3$ is a unique common fixed point of A, B, S and T. One may verify that self maps A, B, S and T does not satisfy Ćirić type contraction condition, for example, for $x = 3$, $3 < y \le 8$ and $c = \frac{1}{8}$.

Definition 9. *Two pairs of self maps (A, S) and (B, T) of a metric space (X, d) are said to satisfy Hardy-Rogers type F-contraction condition if there exist $F \in \mathcal{F}$ and $\tau > 0$ such that for all $x, y \in X$,*

$$d(Ax, By) > 0 \Rightarrow \tau + F(d(Ax, By)) \le F(\alpha d(Ax, Sx) + \beta d(By, Ty) + \gamma d(Sx, Ty) + \delta d(Ax, Ty) + Ld(By, Sx))$$

$$(3.2)$$

where $\alpha + \beta + \gamma + \delta + L < 1$, $\alpha, \beta, \gamma, \delta, L > 0$.

It is interesting to point out here that every Hardy-Rogers type contraction condition [6] is also Hardy-Rogers type F-contraction condition. However converse need not be true.

Theorem 10. *Let (A, S) and (B, T) be two pairs of self maps of a metric space (X, d) satisfying CLR_{ST} property and Hardy-Roger type F-contraction condition 3.2. Then the pairs (A, S) and (B, T) have a coincidence point in X if F is continuous. Moreover A, B, S and T have a unique common fixed point in X provided that both the pairs (A, S) and (B, T) are weakly compatible.*

Proof. Let (A, S) and (B, T) be pairs of self maps satisfying CLR_{ST} property then there exist two sequences $\{x_n\}$ and $\{y_n\}$ in X such that $lim_{n\to\infty}Ax_n = lim_{n\to\infty}Sx_n = lim_{n\to\infty}By_n = lim_{n\to\infty}Ty_n = t$, where $t \in SX \cap TX$. Since $t \in SX$ there exist $v \in X$ such that $Sv = t$.
If $d(Av, By_n) = 0$, then as $n \to \infty$, $d(Av, t) = 0$ which implies $Av = t$ and if $d(Av, By_n) > 0$, then using 3.2,

$$F(d(Av, By_n)) \le F(\alpha d(Av, Sv) + \beta d(By_n, Ty_n) + \gamma d(Sv, Ty_n) + \delta d(Av, Ty_n) + Ld(By_n, Sv)) - \tau.$$

As $n \to \infty$ and since F is continuous,

$$F(d(Av, t)) \le F(\alpha d(Av, t) + \beta d(t, t) + \gamma d(t, t) + \delta d(Av, t) + Ld(t, Sv)) - \tau$$
$$= F((\alpha + \delta)(d(Av, t)) - \tau$$
$$< F(d(Av, t)),$$

which is a contradiction. Hence $Av = Sv = t$, i.e. the pair (A, S) have a coincidence point in X.

Since $t \in TX$ there exist $u \in X$ such that $Tu = t$. If possible let $d(t, Bu) > 0$. Using (3.2),

$$
\begin{aligned}
F(d(t, Bu)) &= F(d(Av, Bu)) \\
&\leq F(\alpha d(Av, Sv) + \beta d(Bu, Tu) + \gamma d(Sv, Tu) + \delta d(Av, Tu) + L d(Bu, Sv)) - \tau \\
&= F((\beta + L)d(Bu, t)) - \tau \\
&< F(d(Bu, t)),
\end{aligned}
$$

which is a contradiction. Hence $Bu = Tu = t$, i.e. the pair (B, T) have a coincidence point in X. Thus $Av = Sv = Bu = Tu = t$. Since (A, S) and (B, T) are weakly compatible, $At = ASv = SAv = St$, $Bt = BTu = TBu = Tt$.

Let $d(At, t) > 0$, then using (3.2)

$$
\begin{aligned}
F(d(At, t)) &= F(d(At, Bu)) \\
&\leq F(\alpha d(At, St) + \beta d(Bu, Tu) + \gamma d(St, Tu) + \delta d(At, Tu) + L d(Bu, St)) - \tau \\
&= F((\gamma + \delta + L)d(At, t)) - \tau < F(d(At, t)).
\end{aligned}
$$

which is a contradiction. Hence At = St = t. Similarly we can show that Bt = t. Thus At = St = Bt = Tt = t, i.e. t is the common fixed point of A, B, S and T. Uniqueness of the common fixed point is easy consequence of (3.2).

$\square$

The following example illustrates theorem 10.

Example 11. *Let $X = [2, 16)$ and d be the usual metric on X.*
Define $A, B, S, T : X \to X$ as follows:

$$
Ax = \begin{cases} 2 & if \ \ x = 2, x \geq 5 \\ 5 & if \ \ 2 < x < 5 \end{cases}
\qquad
Bx = \begin{cases} 2 & if \ \ x = 2, x \geq 5 \\ 6 & if \ \ 2 < x < 5 \end{cases}
$$

$$
Sx = \begin{cases} 2 & if \ \ x = 2 \\ 10 & if \ \ 2 < x < 5 \\ \frac{x-1}{2} & if \ \ x \geq 5 \end{cases}
\qquad
Tx = \begin{cases} 2 & if \ \ x = 2 \\ 14 & if \ \ 2 < x < 5 \\ x - 4 & if \ \ x \geq 5 \end{cases}
$$

Let $\{x_n\}$ and $\{y_n\}$ be two sequences in X such that $x_n = 5 + \frac{1}{n}$ and $y_n = 2$ then $lim_{n \to \infty} Ax_n = lim_{n \to \infty} Sx_n = lim_{n \to \infty} By_n = lim_{n \to \infty} Ty_n = 2 \in SX \cap TX$. Hence A, B, S and T satisfy the CLR_{ST} property. Also $A2 = S2 = 2$, $B2 = T2 = 2$, i.e. pairs (A, S) and (B, T) have coincidence points in X. Also $AS2 = SA2 = 2$, $BT2 = TB2 = 2$, so the pairs (A, S) and (B, T) are weakly compatible. Also A, B, S and

T satisfy Hardy-Rogers type F-contraction condition (3.2) for $\tau = \frac{1}{35}$, $F(x) = -\frac{1}{\sqrt{x}}$, $\alpha = \frac{1}{5}, \beta = \frac{1}{6}, \gamma = \frac{1}{10}, \delta = \frac{1}{10}$ and $L = \frac{1}{5}$. Hence all the conditions of theorem 10 are satisfied and $x = 2$ is a unique common fixed point of A, B, S and T. One may verify that self maps A, B, S and T does not satisfy Hardy-Rogers type contraction condition, for example, for $x = 2$, $2 < y \leq 5$ and $\alpha = \beta = \gamma = \delta = L = \frac{1}{10}$.

Definition 12. *A pair of self maps (A, S) of a metric space (X, d) is said to satisfy Ciric type F-contraction condition if there exist $F \in \mathcal{F}$ and $\tau > 0$ such that for all $x, y \in X$,*

$$d(Ax, Ay) > 0 \Rightarrow \tau + F(d(Ax, Ay)) \leq F(max\{d(Ax, Sx), d(Ay, Sy), d(Sy, Sx),$$
$$d(Sy, Ax), d(Ay, Sx)\}).$$
(3.3)

Theorem 13. *Let (A, S) be a pair of self maps of a metric space (X, d) satisfying CLR_S property and Ciric type contraction condition 3.3. Then the pair (A, S) has a coincidence point in X if F is continuous. Moreover A and S have a unique common fixed point in X provided that the pair (A, S) is weakly compatible.*

Proof. Proof is an easy consequence of Theorem 7 taking A=B and S=T. □

Definition 14. *A pair of self maps (A, S) of a metric space (X, d) is said to satisfy Hardy-Rogers type F-contraction condition if $F \in \mathcal{F}$ and $\tau > 0$ such that for all $x, y \in X$,*

$$d(Ax, Ay) > 0 \Rightarrow \tau + F(d(Ax, Ay)) \leq F(\alpha d(Ax, Sx) + \beta d(Ay, Sy) + \gamma d(Sx, Sy)$$
$$+ \delta d(Ax, Sy) + Ld(Ay, Sx)),$$
(3.4)

where $\alpha + \beta + \gamma + \delta + L < 1$ and $\alpha, \beta, \gamma, \delta, L > 0$.

Theorem 15. *Let (A, S) be a pair of self maps of a metric space (X, d) satisfying CLR_S property and Hardy-Roger type F-contraction condition(3.4). Then the pair (A, S) has a coincidence point in X if F is continuous. Moreover A and S have a unique common fixed point in X provided that the pair (A, S) is weakly compatible.*

Proof. Proof is an easy consequence of Theorem 10 taking A=B and S=T. □

Remark 16. *Taking B=A, S=T=I(the identity map of X) in Theorem 10 and*

(i) *$\gamma < 1, \alpha = \beta = \delta = L = 0$, we obtain Theorem 2.1 of Wardowski [17].*

(ii) *$\gamma = \delta = L = 0, \alpha + \beta < 1, \beta \neq 0$,we obtain the following version of Kannan's [9] contractive condition: $\tau + F(d(Ax, Ay)) \leq F(\alpha d(Ax, x) + \beta d(Ay, y))$, for all x, y in X and $Ax \neq Ay$ where $\alpha + \beta < 1$.*

(iii) $\alpha = \beta = \gamma = 0, \delta = 1/2$, we obtain the following version of Chatterjee's [2] contractive condition: $\tau + F(d(Ax, Ay)) \leq F(1/2(d(Ax, y)) + Ld(Ay, x))$, for all x, y in X and $Ax \neq Ay$, $L < 1/2$.

(iv) $\delta = L = 0$, we obtain the following version of Reich's [14] contractive condition: $\tau + F(d(Ax, Ay)) \leq F(\alpha d(Ax, x) + \beta d(Ay, y) + \gamma d(x, y))$ for all x, y in X and $Ax \neq Ay$ where $\alpha + \beta + \gamma < 1$.

Remark 17. *Batra et al. [13] proved unique coincidence point for a pair of self map satisfying F-g contraction by taking containment of range space of involved maps, completeness of space along with continuity and commutativity of both the maps. We have established existence and uniqueness of coincidence and common fixed point for two pairs of discontinuous self maps without containment of range space of involved maps and completeness of underlying space. Moreover both the pairs are weakly compatible, which is weaker than commutativity of a pair of maps.*

Remark 18. *Since F-contraction is proper generalization of ordinary contraction, our results generalize, extend and improve the results of Wardowski [17] and others existing in literature (for instance Minak et al. [11], Wardowski and Dung [18], Cosentino and Vetro [5], Ċiriċ [3], Hardy-Rogers [6], Kannan [9], Chatterjee [2], Reich [14]) without using completeness of space/subspace, containment requirement of range space and continuity of involved maps.*

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Anita Tomar,
Government P.G. College, Dakpathar(Dehradun), India.
e-mail: anitatmr@yahoo.com

Giniswamy,
P. E. S College of Science, Arts and Commerce, Mandya, India.
e-mail: gswamypes@gmail.com

C. Jeyanthi,
Teresian College, Mysore.
e-mail: jaiprab@yahoo.co.in

P. G. Maheshwari,
Government First Grade College, Vijayanagara, Bangalore.
e-mail: maheshwari616@yahoo.com

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