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On the Representations of Solutions in the Plane of Theory of Thermodynamics with Microtemperatures

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In the present paper the mathematical model of the linear 2D dynamical theory of thermoelasticity with microtemperatures is considered. The representation of regular solution, the fundamental and singular solutions for a governing system of equations of this theory in the Laplace transform space are constructed. Finally, the single-layer, double-layer and volume potentials are presented.

Key words: Thermoelasticity with microtemperatures, Linear dynamical theory, Laplace transform method.

AMS Subject Classification: 74H45, 74H05.

1. Introduction

This paper concerns with the mathematical model of the linear 2D dynamical theory of thermoelasticity with microtemperatures. A thermodynamic theory for elastic materials with inner structure whose particles, in addition to microdeformations, possess microtemperatures that was proposed by Grot [1]. In [1] the thermodynamics of a continuum with microstructure is extended in that it is assumed that the temperatures of the particles of microcontinuum are different. Several mathematical models of continua with microtemperature have been formulated in which the deformation is described not only by the usual displacement vector field, but by other vector fields as well, see [2-3]. In [4] existence theorems were proved and the continuous dependence of solutions of the initial data and body loads was established. In [5] the mathematical model of the 3D linear dynamical theory of thermoelasticity with microtemperatures is considered and the basic initial-boundary value problems (BVPs) in the Laplace transform space are investigated. In [6-8] the fundamental solutions of the equations of theory of thermoelasticity with microtemperatures, the representations of Galerkin type and general solutions of equations of dynamic and steady vibrations were constructed. The basic theorems in the equilibrium theory of thermoelasticity with microtemperatures are proved. Effective solutions of the Dirichlet and the Neumann BVP of the linear theory of

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thermoelasticity with microtemperatures for a spherical ring are given in [9-10]. A wide class of external 3D BVPs of steady vibrations is investigated in [11].

The two-dimensional model of thermoelasticity with microtemperatures are devoted in [12-15]. In particular, fundamental and singular solutions of the system of equations of the equilibrium of the plane thermoelastisity theory with microtemperatures was constructed, uniqueness and existence theorems of some basic boundary value problems of the plane thermoelasticity with microtemperatures are proved. Explicit solutions of boundary value problems of 2D theory of thermoelasticity with microtemperatures for the half-plane are constructed in [15].

In the present paper the mathematical model of the linear 2D dynamical theory of thermoelasticity with microtemperatures is considered. The representation of regular solution, the fundamental and singular solutions for a governing system of equations of this theory in the Laplace transform space are constructed. Finally, the single-layer, double-layer and volume potentials are presented.

2. Basic equations

Let S be the smooth closed curve surrounding the finite domain D in Euclidean 2D space E_2. Let $\mathbf{x} = (x_1, x_2) \in E_2$, $\partial\mathbf{x} = \left(\frac{\partial}{\partial x_1}, \frac{\partial}{\partial x_2}\right)$, and let t denote the time variable $t > 0$. Let us assume that the domain D is filled with an isotropic thermoelastic material possessing microtemperatures.

In 2D space "rot" is defined as a scalar

$$rot\phi = \frac{\partial \phi_2}{\partial x_1} - \frac{\partial \phi_1}{\partial x_2}$$

for a vector $\phi = (\phi_1, \phi_2)$ and as a vector

$$rot\psi = \left(\frac{\partial \psi}{\partial x_2}, -\frac{\partial \psi}{\partial x_1}\right)$$

for a scalar ψ.

The basic homogeneous system of equations of motion in the linear theory thermoelasticity with microtemperatures for isotropic materials can be written as [4]

$$\begin{aligned} &\mu\Delta\mathbf{u} + (\lambda+\mu) graddiv\mathbf{u} - \beta grad\theta - \rho\frac{\partial^2\mathbf{u}}{\partial t^2} = 0, \\ &k_6\Delta\mathbf{w} + (k_4+k_5) graddiv\mathbf{w} - k_3\theta - k_2\mathbf{w} - b\frac{\partial \mathbf{w}}{\partial t} = 0, \\ &k\Delta\theta - cT_0\frac{\partial}{\partial t}\theta - \beta T_0\frac{\partial}{\partial t}div\mathbf{u} + k_1 div\mathbf{w} = 0, \end{aligned} \tag{1}$$

where $\mathbf{u} = (u_1, u_2)^T$ is the displacement vector, $\mathbf{w} = (w_1, w_2)^T$ is the microtemperature vector, θ is the temperature measured from the constant absolute temperature T_0 $(T_0 > 0)$ by the natural state, ρ is the reference mass density $(\rho > 0)$, Δ is the 2D Laplace operator. The superscript "T" denotes transposition,

λ, μ, β, b, c, k, k_j, $j = 1, ..., 6$, are constitutive coefficients and they satisfy the following conditions

$$\begin{array}{c} \lambda + \mu > 0 \quad \mu > 0 \quad k_5 \pm k_6 > 0 \quad (k_1 + T_0 k_3)^2 < 4T_0 k k_2 \\ b > 0 \quad c > 0 \quad k > 0 \quad 2k_4 + k_5 + k_6 > 0. \end{array}$$

Let $\tau = \sigma + i\omega$ be a complex variable, $\Pi_{\sigma_0} = \{\tau : \sigma > \sigma_0 > 0, -\infty < \omega < +\infty\}$ is the half-plane. By applying formally the Laplace transform, as in the classical theory of thermoelasticity, from Eq.(1) we obtain (for details see [5],[16])

$$\begin{aligned} &\mu \Delta \tilde{\mathbf{u}} + (\lambda + \mu) graddiv \tilde{\mathbf{u}} - \beta grad \tilde{\theta} - \rho \tau^2 \tilde{\mathbf{u}} = 0, \\ &k_6 \Delta \tilde{\mathbf{w}} + (k_4 + k_5) graddiv \tilde{\mathbf{w}} - k_3 grad \tilde{\theta} - k_8 \tilde{\mathbf{w}} = 0, \\ &(k\Delta - a_0)\tilde{\theta} - \beta_0 div \tilde{\mathbf{u}} + k_1 div \tilde{\mathbf{w}} = 0, \end{aligned} \tag{2}$$

where $k_8 = k_2 - b\tau$, $b > 0$, $a_0 = -cT_0\tau$, $\beta_0 = -\tau\beta T_0$, $\tilde{\mathbf{u}} = (\tilde{u}_1, \tilde{u}_2)$, $\tilde{\mathbf{w}} = (\tilde{w}_1, \tilde{w}_2)$.

We assume that the inversion formulas

$$\begin{aligned} \mathbf{u}(\mathbf{x}, t) &= \frac{1}{2\pi i} \int_{\sigma - i\infty}^{\sigma + i\infty} \tilde{\mathbf{u}}(x_1, x_2, \tau) \exp(\tau t) d\tau, \\ \mathbf{w}(\mathbf{x}, t) &= \frac{1}{2\pi i} \int_{\sigma - i\infty}^{\sigma + i\infty} \tilde{\mathbf{w}}(x_1, x_2, \tau) \exp(\tau t) d\tau, \\ \theta(\mathbf{x}, t) &= \frac{1}{2\pi i} \int_{\sigma - i\infty}^{\sigma + i\infty} \tilde{\theta}(x_1, x_2, \tau) \exp(\tau t) d\tau \end{aligned}$$

are valid at any point t.

3. A representation of regular solutions

Let further $\mathbf{U} = (\tilde{\mathbf{u}}, \tilde{\mathbf{w}}, \tilde{\theta})$, is a regular solutions of the homogeneous system (2).

Theorem 3.1: *The regular solution $\boldsymbol{U} = (\tilde{\boldsymbol{u}}, \tilde{\boldsymbol{w}}, \tilde{\theta})$ of the system (2) admits in the domain of regularity a representation*

$$\boldsymbol{U} = (\boldsymbol{u}^{(1)} + \boldsymbol{u}^{(2)}, \boldsymbol{w}^{(1)} + \boldsymbol{w}^{(2)}, \tilde{\theta}),$$

where $\boldsymbol{u}^{(1)}$, $\boldsymbol{u}^{(2)}$, $\boldsymbol{w}^{(1)}$ *and* $\boldsymbol{w}^{(2)}$ *are the regular vectors, satisfying the equations*

$$(\Delta-\lambda_4^2)\boldsymbol{u}^{(2)}=0, \quad (\Delta-\lambda_5^2)\boldsymbol{w}^{(2)}=0, \quad rot\boldsymbol{u}^{(1)}=0,$$
$$rot\boldsymbol{w}^{(1)}=0, \quad div\boldsymbol{u}^{(2)}=0, \quad div\boldsymbol{w}^{(2)}=0,$$
$$(\Delta+\lambda_1^2)(\Delta+\lambda_2^2)(\Delta+\lambda_3^2)\boldsymbol{u}^{(1)}=0,$$
$$(\Delta+\lambda_1^2)(\Delta+\lambda_2^2)(\Delta+\lambda_3^2)\boldsymbol{w}^{(1)}=0,$$
$$(\Delta+\lambda_1^2)(\Delta+\lambda_2^2)(\Delta+\lambda_3^2)\tilde{\theta}=0.$$

λ_j^2, $j=1,2,3,$ *are roots of equation* $detD(-\xi)=0$, *where*

$$detD(\Delta)=(\mu_0\Delta-\rho\tau^2)k_1k_3\Delta+(k_7\Delta-k_8)[(\mu_0\Delta-\rho\tau^2)(k\Delta-a_0)-\beta\beta_0\Delta],$$

$$\lambda_1^2+\lambda_2^2+\lambda_3^2=-\frac{1}{\mu_0kk_7}\left[\mu_0(a_0k_7+kk_8-k_1k_3)+\rho\tau^2kk_7+\beta\beta_0k_7\right],$$

$$\lambda_1^2\lambda_2^2+\lambda_1^2\lambda_3^2+\lambda_2^2\lambda_3^2=\frac{1}{\mu_0kk_7}\left[k_8(\mu_0a_0+\beta\beta_0)+\rho\tau^2(a_0k_7+kk_8-k_1k_3)\right],$$

$$\lambda_1^2\lambda_2^2\lambda_3^2=-\frac{a_0k_8\rho\tau^2}{\mu_0kk_7}, \quad \mu_0=\lambda+2\mu, \quad k_7=k_4+k_5+k_6>0,$$

and the constants λ_4^2 *and* λ_5^2 *are determined by the formulas*

$$\lambda_4^2:=\frac{\tau^2\rho}{\mu}, \qquad \lambda_5^2:=\frac{k_8}{k_6}.$$

The quantities λ_j^2, $j=1,..,5$ are complex numbers and are chosen so as to ensure positivity of their imaginary part, i.e. it is assumed that $Im\lambda_j^2>0$.

Proof: Let $\mathbf{U}=(\tilde{\mathbf{u}},\tilde{\mathbf{w}},\tilde{\theta})$ be a regular solution of the system (2). Taking into account the identity

$$\Delta\mathbf{v}=graddiv\mathbf{v}-rotrot\mathbf{v}, \tag{3}$$

where

$$rotrot\mathbf{v}:=\left(\frac{\partial}{\partial x_2}\left(\frac{\partial v_2}{\partial x_1}-\frac{\partial v_1}{\partial x_2}\right),-\frac{\partial}{\partial x_1}\left(\frac{\partial v_2}{\partial x_1}-\frac{\partial v_1}{\partial x_2}\right)\right),$$

from the equation (2) we obtain

$$\tilde{\mathbf{u}}=\frac{\mu_0}{\rho\tau^2}graddiv\tilde{\mathbf{u}}-\frac{\mu}{\rho\tau^2}rotrot\tilde{\mathbf{u}}-\frac{\beta}{\rho\tau^2}grad\tilde{\theta},$$

$$\tilde{\mathbf{w}} = \frac{k_7}{k_8} graddiv\tilde{\mathbf{w}} - \frac{k_6}{k_8} rotrot\tilde{\mathbf{w}} - \frac{k_3}{k_8} grad\tilde{\theta}.$$

We introduce the notations

$$\boldsymbol{u}^{(1)} = \frac{\mu_0}{\rho\tau^2} graddiv\tilde{\mathbf{u}} \ - \frac{\beta}{\rho\tau^2} grad\tilde{\theta}, \tag{4}$$

$$\boldsymbol{u}^{(2)} = -\frac{\mu}{\rho\tau^2} rotrot\tilde{\mathbf{u}}, \tag{5}$$

$$\boldsymbol{w}^{(1)} = \frac{k_7}{k_8} graddiv\tilde{\mathbf{w}} - \frac{k_3}{k_8} grad\tilde{\theta}, \tag{6}$$

$$\boldsymbol{w}^{(2)} = -\frac{k_6}{k_8} rotrot\tilde{\mathbf{w}}. \tag{7}$$

Clearly

$$\tilde{\mathbf{u}} = \boldsymbol{u}^{(1)} + \boldsymbol{u}^{(2)}, \quad \tilde{\mathbf{w}} = \boldsymbol{w}^{(1)} + \boldsymbol{w}^{(2)}. \tag{8}$$

Acting with the operator *rot* on (4) and (6) and considering the identity $rotgrad \equiv 0$ and with the operator *div* on (5), (7) we have

$$rot\boldsymbol{u}^{(1)} = 0, \quad rot\boldsymbol{w}^{(1)} = 0, \quad div\boldsymbol{u}^{(2)} = 0, \quad and \quad div\boldsymbol{w}^{(2)} = 0 \tag{9}$$

respectively. Taking into account the last equalities, we have from equations (5)-(7)

$$(\Delta - \lambda_4^2)\boldsymbol{u}^{(2)} = 0, \quad (\Delta - \lambda_5^2)\boldsymbol{w}^{(2)} = 0. \tag{10}$$

Applying the operator *div* to system (2), and taking into account the identity $divgrad \equiv \Delta$, we obtain

$$\begin{aligned}
&(\mu_0\Delta - \rho\tau^2) div\tilde{\mathbf{u}} - \beta\Delta\tilde{\theta} = 0,\\
&(k_7\Delta - k_8) div\tilde{\mathbf{w}} - k_3\Delta\tilde{\theta} = 0,\\
&k\Delta\tilde{\theta} + k_1 div\tilde{\mathbf{w}} - \beta_0 div\tilde{\mathbf{u}} - a_0\tilde{\theta} = 0.
\end{aligned} \tag{11}$$

The system (11) may be written as

$$D(\Delta)\Psi := \begin{pmatrix} \mu_0\Delta - \rho\tau^2 & 0 & -\beta\Delta \\ 0 & k_7\Delta - k_8 & -k_3\Delta \\ -\beta_0 & k_1 & k\Delta - a_0 \end{pmatrix} \Psi = 0, \tag{12}$$

where $\Psi = (div\tilde{\mathbf{u}}, div\tilde{\mathbf{w}}, \tilde{\theta})^T$. It is easily seen that

$$\begin{aligned}
&(\Delta+\lambda_1^2)(\Delta+\lambda_2^2)(\Delta+\lambda_3^2)div\tilde{\mathbf{u}}=0,\\
&(\Delta+\lambda_1^2)(\Delta+\lambda_2^2)(\Delta+\lambda_3^2)div\tilde{\mathbf{w}}=0,\\
&(\Delta+\lambda_1^2)(\Delta+\lambda_2^2)(\Delta+\lambda_3^2)\tilde{\theta}=0,
\end{aligned} \tag{13}$$

where λ_j^2 are the roots of the equation $D(-\xi)=0$ (with respect to ξ.) Applying the operator $(\Delta+\lambda_1^2)(\Delta+\lambda_2^2)(\Delta+\lambda_3^2)$ to Eqs. (4),(6) and taking into account the the latter relations we obtain

$$\begin{aligned}
&(\Delta+\lambda_1^2)(\Delta+\lambda_2^2)(\Delta+\lambda_3^2)\mathbf{u}^{(\mathbf{1})}=\mathbf{0},\\
&(\Delta+\lambda_1^2)(\Delta+\lambda_2^2)(\Delta+\lambda_3^2)\mathbf{w}^{(\mathbf{1})}=\mathbf{0},\\
&(\Delta+\lambda_1^2)(\Delta+\lambda_2^2)(\Delta+\lambda_3^2)\tilde{\theta}=0.
\end{aligned} \tag{14}$$

The latter formulas prove the theorem. □

Theorem 3.2: *The regular solution $\boldsymbol{U}=(\tilde{\boldsymbol{u}},\tilde{\boldsymbol{w}},\tilde{\theta})\in C^2(D)$ of system (2) is represented as the sum*

$$\tilde{\boldsymbol{u}}=\sum_{j=1}^{4}\boldsymbol{u}^{(j)}(\boldsymbol{x}),\quad \tilde{\boldsymbol{w}}=\sum_{j=1,2,3,5}\boldsymbol{w}^{(j)}(\boldsymbol{x}),\quad \tilde{\theta}=\sum_{j=1}^{3}\varphi_j, \tag{15}$$

where $\boldsymbol{u}^{(j)}, \boldsymbol{w}^{(j)}$and φ_j are regular functions satisfying the following equations

$$\begin{aligned}
&(\Delta+\lambda_j^2)\boldsymbol{u}^{(j)}=0,\quad (\Delta+\lambda_l^2)\boldsymbol{w}^{(l)}=0,\quad (\Delta+\lambda_m^2)\varphi_m=0,\quad (\Delta-\lambda_4^2)\boldsymbol{u}^{(4)}=0,\\
&(\Delta-\lambda_5^2)\boldsymbol{w}^{(5)}=0,\quad j,l,m=1,2,3,\quad div\boldsymbol{u}^4=0,\quad div\boldsymbol{w}^5=0.
\end{aligned} \tag{16}$$

Proof: Applying to both sides of the equation (2) the operator $(\Delta+\lambda_1^2)(\Delta+\lambda_2^2)(\Delta+\lambda_3^2)$ and taking into account the relations (11),(12) and (13), we obtain

$$\begin{aligned}
&(\Delta+\lambda_1^2)(\Delta+\lambda_2^2)(\Delta+\lambda_3^2)(\Delta-\lambda_4^2)\tilde{\mathbf{u}}=0,\\
&(\Delta+\lambda_1^2)(\Delta+\lambda_2^2)(\Delta+\lambda_3^2)(\Delta-\lambda_5^2)\tilde{\boldsymbol{w}}=0,\\
&(\Delta+\lambda_1^2)(\Delta+\lambda_2^2)(\Delta+\lambda_3^2)\tilde{\theta}=0.
\end{aligned} \tag{17}$$

We introduce the notations

$$\mathbf{u}^{(1)}=-\frac{(\Delta+\lambda_2^2)(\Delta+\lambda_3^2)(\Delta-\lambda_4^2)}{(\lambda_1^2-\lambda_2^2)(\lambda_1^2-\lambda_3^2)(\lambda_1^2+\lambda_4^2)}\tilde{\mathbf{u}},\ \ \mathbf{u}^{(2)}=-\frac{(\Delta+\lambda_1^2)(\Delta+\lambda_3^2)(\Delta-\lambda_4^2)}{(\lambda_2^2-\lambda_1^2)(\lambda_2^2-\lambda_3^2)(\lambda_2^2+\lambda_4^2)}\tilde{\mathbf{u}},$$

$$\mathbf{u}^{(3)}=-\frac{(\Delta+\lambda_1^2)(\Delta+\lambda_2^2)(\Delta-\lambda_4^2)}{(\lambda_3^2-\lambda_1^2)(\lambda_3^2-\lambda_2^2)(\lambda_3^2+\lambda_4^2)}\tilde{\mathbf{u}},\ \ \mathbf{u}^{(4)}=\frac{(\Delta+\lambda_1^2)(\Delta+\lambda_2^2)(\Delta+\lambda_3^2)}{(\lambda_4^2+\lambda_1^2)(\lambda_4^2+\lambda_2^2)(\lambda_4^2+\lambda_3^2)}\tilde{\mathbf{u}},$$

$$\mathbf{w}^{(1)}=-\frac{(\Delta+\lambda_2^2)(\Delta+\lambda_3^2)(\Delta-\lambda_5^2)}{(\lambda_1^2-\lambda_2^2)(\lambda_1^2-\lambda_3^2)(\lambda_1^2+\lambda_5^2)}\tilde{\mathbf{w}},\quad \mathbf{w}^{(2)}=-\frac{(\Delta+\lambda_1^2)(\Delta+\lambda_3^2)(\Delta-\lambda_5^2)}{(\lambda_2^2-\lambda_1^2)(\lambda_2^2-\lambda_3^2)(\lambda_2^2+\lambda_5^2)}\tilde{\mathbf{w}},$$

$$\mathbf{w}^{(3)}=-\frac{(\Delta+\lambda_1^2)(\Delta+\lambda_2^2)(\Delta-\lambda_5^2)}{(\lambda_3^2-\lambda_1^2)(\lambda_3^2-\lambda_2^2)(\lambda_3^2+\lambda_5^2)}\tilde{\mathbf{w}},\quad \mathbf{w}^{(5)}=\frac{(\Delta+\lambda_1^2)(\Delta+\lambda_2^2)(\Delta+\lambda_3^2)}{(\lambda_5^2+\lambda_1^2)(\lambda_5^2+\lambda_2^2)(\lambda_5^2+\lambda_3^2)}\tilde{\mathbf{w}},$$

$$\varphi_1=\frac{(\Delta+\lambda_2^2)(\Delta+\lambda_3^2)}{(\lambda_1^2-\lambda_2^2)(\lambda_1^2-\lambda_3^2)}\tilde{\theta},\quad \varphi_2=\frac{(\Delta+\lambda_1^2)(\Delta+\lambda_3^2)}{(\lambda_2^2-\lambda_1^2)(\lambda_2^2-\lambda_3^2)}\tilde{\theta},$$

$$\varphi_3=\frac{(\Delta+\lambda_1^2)(\Delta+\lambda_2^2)}{(\lambda_3^2-\lambda_1^2)(\lambda_3^2-\lambda_2^2)}\tilde{\theta}. \tag{18}$$

By virtue of (18), it follows

$$\tilde{\mathbf{u}}=\sum_{j=1}^{4}\mathbf{u}^{(j)},\quad \tilde{\mathbf{w}}=\sum_{j=1,2,3,5}\mathbf{w}^{(j)},\quad \tilde{\theta}=\sum_{j=1}^{3}\varphi_j, \tag{19}$$

$$(\Delta+\lambda_j^2)\mathbf{u}^{(j)}=0,\quad (\Delta+\lambda_l^2)\mathbf{w}^{(l)}=0,\quad (\Delta+\lambda_m^2)\varphi_m=0,\quad (\Delta-\lambda_4^2)\mathbf{u}^{(4)}=0,$$

$$(\Delta-\lambda_5^2)\mathbf{w}^{(5)}=0,\quad j=1,2,3,\quad l=1,2,3,\quad m=1,2,3. \tag{20}$$

Thus, the regular in D solution of system (2) is represented as a sum of functions $\mathbf{u}^{(j)}$, $\mathbf{w}^{(j)}$, φ_j, which satisfies Helmholtz' equations in D. □

Theorem 3.3: *In the domain of regularity the regular solution of system (2) can be represented in the form*

$$\tilde{\boldsymbol{u}}=\sum_{j=1}^{3}a_j grad\varphi_j+\boldsymbol{u}^{(4)},\qquad \tilde{\boldsymbol{w}}=\sum_{j=1}^{3}b_j grad\varphi_j+\boldsymbol{w}^{(5)},\qquad \tilde{\theta}=\sum_{j=1}^{3}\varphi_j, \tag{21}$$

where

$$\begin{aligned}&(\Delta+\lambda_j^2)\varphi_j=0,\quad j=1,2,3,\quad (\Delta-\lambda_4^2)\boldsymbol{u}^{(4)}=0,\\&(\Delta-\lambda_5^2)\boldsymbol{w}^{(5)}=0,\quad div\boldsymbol{u}^{(4)}=0,\quad div\boldsymbol{w}^{(5)}=0,\end{aligned} \tag{22}$$

a_j and b_j are constants.

Proof: Substituting $\tilde{\mathbf{u}}$, $\tilde{\mathbf{w}}$ and $\tilde{\theta}$ from (15) into (2), after some calculations we obtain

$$\begin{aligned}&(\mu\Delta-\rho\tau^2)(k_7\Delta-k_8)(\boldsymbol{u}^{(1)}+\boldsymbol{u}^{(2)}+\boldsymbol{u}^{(3)})\\&=grad\Bigg[-\frac{(\lambda+\mu)k_1k_3}{\beta_0}(\lambda_1^2\varphi_1+\lambda_2^2\varphi_2+\lambda_3^2\varphi_3)+\beta(k_7\Delta-k_8)(\varphi_1+\varphi_2+\varphi_3)\\&+\frac{(\lambda+\mu)}{\beta_0}(k\Delta-a_0)(k_7\Delta-k_8)(\varphi_1+\varphi_2+\varphi_3)\Bigg].\end{aligned} \tag{23}$$

Equation (23) is satisfied by

$$(\mu\Delta - \rho\tau^2)(k_7\Delta - k_8)\boldsymbol{u}^{(1)}$$
$$= \left\{ \frac{(\lambda+\mu)}{\beta_0}[(a_0 + k\lambda_1^2)(k_8 + k_7\lambda_1^2) - k_1k_3\lambda_1^2] - \beta(k_8 + k_7\lambda_1^2) \right\} grad\varphi_1,$$

$$(\mu\Delta - \rho\tau^2)(k_7\Delta - k_8)\boldsymbol{u}^{(2)}$$
$$= \left\{ \frac{(\lambda+\mu)}{\beta_0}[(a_0 + k\lambda_2^2)(k_8 + k_7\lambda_2^2) - k_1k_3\lambda_2^2] - \beta(k_8 + k_7\lambda_2^2) \right\} grad\varphi_2,$$

$$(\mu\Delta - \rho\tau^2)(k_7\Delta - k_8)\boldsymbol{u}^{(3)}$$
$$= \left\{ \frac{(\lambda+\mu)}{\beta_0}[(a_0 + k\lambda_3^2)(k_8 + k_7\lambda_3^2) - k_1k_3\lambda_3^2] - \beta(k_8 + k_7\lambda_3^2) \right\} grad\varphi_3.$$

The last identity gives

$$\boldsymbol{u}^{(1)} = a_1 grad\varphi_1, \quad \boldsymbol{u}^{(2)} = a_2 grad\varphi_2, \quad \boldsymbol{u}^{(3)} = a_3 grad\varphi_3, \tag{24}$$

where

$$a_j = \frac{-\beta}{\mu\lambda_4^2 + \mu_0\lambda_j^2}, \quad j = 1,2,3.$$

Quite similarly we obtain

$$\boldsymbol{w}^{(1)} = b_1 grad\varphi_1, \quad \boldsymbol{w}^{(2)} = b_2 grad\varphi_2, \quad \boldsymbol{w}^{(3)} = b_3 grad\varphi_3,$$

where

$$b_j = \frac{-k_3}{k_6\lambda_5^2 + k_7\lambda_j^2}, \quad j = 1,2,3.$$

Finally, the general solution of the equation (2) takes the form

$$\begin{aligned}
&\tilde{\mathbf{u}} = \sum_{j=1}^{3} a_j grad\varphi_j + \boldsymbol{u}^{(4)}, \qquad \tilde{\mathbf{w}} = \sum_{j=1}^{3} b_j grad\varphi_j + \boldsymbol{w}^{(5)}, \qquad \tilde{\theta} = \sum_{j=1}^{3} \varphi_j, \\
&(\Delta + \lambda_j^2)\varphi_j = 0, \quad j = 1,2,3, \quad (\Delta - \lambda_4^2)\boldsymbol{u}^{(4)} = 0, \\
&(\Delta - \lambda_5^2)\boldsymbol{w}^{(5)} = 0, \quad div\boldsymbol{u}^{(4)} = 0, \quad div\boldsymbol{w}^{(5)} = 0.
\end{aligned} \tag{25}$$

□

4. Matrix of fundamental solutions

System (2) may be written as

$$\mathbf{A}(\partial\mathbf{x},\tau)\mathbf{U}=0, \tag{26}$$

where $\mathbf{A}(\partial\mathbf{x},\tau)$ is a matrix differential operator corresponding left-hand side of the equation(2)

$$\mathbf{A}(\partial\mathbf{x},\tau)=\parallel A_{lj}(\partial\mathbf{x})\parallel_{5x5},$$

$$A_{\alpha\gamma}=\mu\delta_{\alpha\gamma}(\Delta-\rho\tau^2)+(\lambda+\mu)\frac{\partial^2}{\partial x_\alpha\partial x_\gamma},$$

$$A_{\alpha+2;\gamma+2}=\delta_{\alpha\gamma}(k_6\Delta-k_8)+(k_4+k_5)\frac{\partial^2}{\partial x_\alpha\partial x_\gamma},$$

$$A_{\alpha,\gamma+2}=A_{\alpha+2,\gamma}=0,\quad A_{\alpha5}=-\beta\frac{\partial}{\partial x_\alpha},\quad A_{\alpha+2;5}=-k_3\frac{\partial}{\partial x_\alpha},$$

$$A_{5\gamma}=-\beta_0\frac{\partial}{\partial x_\gamma},\quad A_{5;\gamma+2}=k_1\frac{\partial}{\partial x_\gamma},\quad A_{55}=k\Delta-a_0,\quad \alpha,\gamma=1,2,$$

$\delta_{\alpha\gamma}$ is the Kronecker delta, $\mathbf{U}=(\tilde{\mathbf{u}},\tilde{\mathbf{w}},\tilde{\theta})$.

The matrix $\widetilde{\mathbf{A}}(\partial\mathbf{x},\tau)=\parallel\widetilde{A}_{lj}(\partial\mathbf{x},\tau)\parallel_{5x5}=\mathbf{A}^T(-\partial\mathbf{x},\tau)$, where $\widetilde{A}_{lj}(\partial\mathbf{x},\tau):=A_{jl}(-\partial\mathbf{x},\tau)$, will be called the associated operator to the differential operator $\mathbf{A}(\partial\mathbf{x},\tau)$.

We assume that $\mu(\lambda+2\mu)kk_6k_7\neq0$, where $k_7=k_4+k_5+k_6$. Obviously, if the latter condition is satisfied, then $\mathbf{A}(\partial\mathbf{x},\tau)$ is the elliptic differential operator.

We introduce the matrix differential operator $\mathbf{B}(\partial\mathbf{x},\tau)$ consisting of cofactors of elements of the transposed matrix $\mathbf{A}^T$ divided on $\mu(\lambda+2\mu)kk_6k_7\neq0:$

$$\mathbf{B}(\partial\mathbf{x},\tau):=\parallel B_{lj}(\partial\mathbf{x})\parallel_{5x5},$$

where

$$B_{\alpha\gamma}=B^*_{11}\delta_{\alpha\gamma}-B^*_{12}\xi_\alpha\xi_\gamma,\quad B_{\alpha+2,\gamma+2}=B^*_{33}\delta_{\alpha\gamma}-B^*_{34}\xi_\alpha\xi_\gamma,$$

$$B_{1\gamma+2}=B^*_{13}\xi_1\xi_\gamma,\quad B_{2\gamma+2}=B^*_{13}\xi_2\xi_\gamma,\quad B_{\alpha5}=B^*_{15}\xi_\alpha,\quad B_{5\alpha}=B^*_{51}\xi_\alpha,$$

$$B_{5\gamma+2}=B^*_{53}\xi_\gamma,\quad \xi_\alpha:=\frac{\partial}{\partial x_\alpha},\quad \alpha,\gamma=1,2,\quad B_{55}=B^*_{55},$$

$$B_{3\gamma}=B^*_{31}\xi_1\xi_\gamma,\quad B_{4\gamma}=B^*_{31}\xi_2\xi_\gamma,\quad B_{2+\gamma,5}=B^*_{35}\xi_\gamma,$$

$$B^*_{11}=\frac{1}{\mu}(\Delta+\lambda_1^2)(\Delta+\lambda_2^2)(\Delta+\lambda_3^2)(\Delta-\lambda_5^2),$$

$$B^*_{12} = \frac{(\Delta - \lambda_5^2)}{kk_7\mu\mu_0}\left\{-\beta\beta_0(k_7\Delta - k_8) + (\lambda+\mu)[(k\Delta - a_0)(k_7\Delta - k_8) + k_1k_3\Delta]\right\},$$

$$B^*_{13} = -\frac{\beta k_1}{\mu_0 kk_7}((\Delta - \lambda_4^2)(\Delta - \lambda_5^2), \quad B^*_{15} = \frac{\beta}{\mu_0 kk_7}(\Delta - \lambda_4^2)(\Delta - \lambda_5^2)(k_7\Delta - k_8),$$

$$B^*_{51} = \frac{h\beta_0}{\mu_0 kk_7}(\Delta - \lambda_4^2)(\Delta - \lambda_5^2)(k_7\Delta - k_8), \quad j = 1,2, \quad \mu_0 := \lambda + 2\mu,$$

$$B^*_{53} = -\frac{hk_1}{\mu_0 kk_7}(\Delta - \lambda_4^2)(\Delta - \lambda_5^2)(\mu_0\Delta - \rho\tau^2),$$

$$B^*_{55} = \frac{1}{\mu_0 kk_7}(\Delta - \lambda_4^2)(\Delta - \lambda_5^2)(\mu_0\Delta - \rho\tau^2)(k_7\Delta - k_8),$$

$$B^*_{31} = \frac{k_3\beta_0}{\mu_0 kk_7}((\Delta - \lambda_4^2)(\Delta - \lambda_5^2) \quad B^*_{35} = \frac{k_3}{\mu_0 kk_7}(\Delta - \lambda_4^2)(\Delta - \lambda_5^2)(\mu_0\Delta - \rho\tau^2),$$

$$B^*_{33} = \frac{1}{k_6}(\Delta + \lambda_1^2)(\Delta + \lambda_2^2)(\Delta + \lambda_3^2)(\Delta - \lambda_4^2),$$

$$B^*_{34} = \frac{(\Delta - \lambda_4^2)}{\mu_0 kk_6k_7}\{k_1k_3(\mu_0\Delta - \rho\tau^2) + (k_4 + k_5)[(\mu_0\Delta - \rho\tau^2)(k\Delta - a_0) - \beta\beta_0\Delta]\}.$$

Substituting the vector $\mathbf{U}(\mathbf{x}) = \mathbf{B}(\partial\mathbf{x}, \tau)\mathbf{\Psi}$ into (26), where $\mathbf{\Psi} = \mathbf{E}\psi$, is a five-component vector function, $\mathbf{E}$ is an unit matrix, ψ is scalar function, we get

$$(\Delta + \lambda_1^2)(\Delta + \lambda_2^2)(\Delta + \lambda_3^2)(\Delta - \lambda_4^2)(\Delta - \lambda_5^2)\psi = 0.$$

Whence, applying the method developed in [16,17], after some calculations, the function ψ can be represented as

$$\psi = \sum_{j=1}^{3} d_j H_0^{(1)}(\lambda_j r) + \sum_{j=4}^{5} d_j H_0^{(1)}(i\lambda_j r), \tag{27}$$

where

$$\sum_{j=1}^{5} d_j = 0, \quad \sum_{j=1}^{3} d_j(\lambda_4^2 + \lambda_j^2)(\lambda_5^2 + \lambda_j^2) = 0,$$

$$d_1^{-1} = (\lambda_1^2 - \lambda_2^2)(\lambda_1^2 - \lambda_3^2)(\lambda_1^2 + \lambda_4^2)(\lambda_1^2 + \lambda_5^2),$$

$$d_2^{-1} = (\lambda_2^2 - \lambda_1^2)(\lambda_2^2 - \lambda_3^2)(\lambda_2^2 + \lambda_4^2)(\lambda_2^2 + \lambda_5^2),$$

$$d_3^{-1} = (\lambda_3^2 - \lambda_1^2)(\lambda_3^2 - \lambda_2^2)(\lambda_3^2 + \lambda_4^2)(\lambda_3^2 + \lambda_5^2),$$

$$d_4^{-1} = (\lambda_4^2 + \lambda_1^2)(\lambda_4^2 + \lambda_2^2)(\lambda_4^2 + \lambda_3^2)(\lambda_4^2 - \lambda_5^2),$$

$$d_5^{-1} = (\lambda_5^2 + \lambda_1^2)(\lambda_5^2 + \lambda_2^2)(\lambda_5^2 + \lambda_3^2)(\lambda_5^2 - \lambda_4^2),$$

$H_0^{(1)}(\lambda_m r)$ is Hankel's function of the first kind with the index 0

$$H_0^{(1)}(\lambda_m r) = \frac{2i}{\pi} J_0(\lambda_m r)\ln r + \frac{2i}{\pi}\left(\ln\frac{\lambda_m}{2} + C - \frac{i\pi}{2}\right) J_0(\lambda_m r),$$

$$-\frac{2i}{\pi}\sum_{k=1}^{\infty}\frac{(-1)^k}{(k!)^2}\left(\frac{\lambda_m r}{2}\right)^{2k}\left(\frac{1}{k} + \frac{1}{k-1} + ... + 1\right),$$

$$J_0(\lambda_m r) = \sum_{k=0}^{\infty}\tfrac{(-1)^k}{(k!)^2}\left(\frac{\lambda_m r}{2}\right)^{2k}, \quad r^2 = (x_1 - y_1)^2 + (x_2 - y_2)^2.$$

Substituting (27) into $\mathbf{U} = \mathbf{B\Psi}$, we obtain the matrix of fundamental solutions of Eq. (26) which we denote by $\mathbf{\Gamma}(\mathbf{x}\text{-}\mathbf{y}, \tau)$

$$\mathbf{\Gamma}(\mathbf{x}\text{-}\mathbf{y}, \tau) = \| \Gamma_{kj}(\mathbf{x}\text{-}\mathbf{y}) \|_{5x5},$$

where

$$\Gamma_{\alpha\gamma}(\mathbf{x}\text{-}\mathbf{y}) = \delta_{\alpha\gamma}\frac{H_0^{(1)}(i\lambda_4 r)}{\mu} - \frac{\partial^2\Psi_{11}}{\partial x_\alpha \partial x_\gamma},$$

$$\Psi_{11} = \frac{H_0^{(1)}(i\lambda_4 r)}{\mu\lambda_4^2} + \sum_{m=1}^{3}\frac{l_m}{\lambda_m^2\mu_0 k k_7}[(k_8 + k_7\lambda_m^2)(a_0 + k\lambda_m^2) - k_1 k_3\lambda_m^2] H_0^{(1)}(\lambda_m r),$$

$$\Gamma_{\alpha+2,\gamma+2}(\mathbf{x}\text{-}\mathbf{y}) = \delta_{\alpha\gamma}\frac{H_0^{(1)}(i\lambda_5 r)}{k_6} - \frac{\partial^2\Psi_{33}}{\partial x_\alpha \partial x_\gamma},$$

$$\Psi_{33} = \frac{H_0^{(1)}(i\lambda_5 r)}{k_6\lambda_5^2} + \sum_{m=1}^{3}\frac{l_m}{\lambda_m^2\mu_0 k k_7}[(a_0 + k\lambda_m^2)(\rho\tau^2 + \mu_0\lambda_m^2) + \beta\beta_0\lambda_m^2] H_0^{(1)}(\lambda_m r),$$

$$\Gamma_{55}(\mathbf{x}\text{-}\mathbf{y}) = \frac{1}{k k_7 \mu_0}\sum_{m=1}^{3} l_m(\rho\tau^2 + \mu_0\lambda_m^2)(k_8 + k_7\lambda_m^2)\} H_0^{(1)}(\lambda_m r),$$

$$\Gamma_{\alpha 5}(\mathbf{x}\text{-}\mathbf{y}) = -\beta\frac{\partial\psi_{15}}{\partial x_\alpha}, \quad \psi_{15} = \frac{1}{k k_7\mu_0}\sum_{m=1}^{3} l_m(k_8 + k_7\lambda_m^2) H_0^{(1)}(\lambda_m r),$$

$$\Gamma_{2+\alpha,5}(\mathbf{x}\text{-}\mathbf{y}) = -k_3\frac{\partial\psi_{51}}{\partial x_\alpha}, \quad \psi_{51} = \frac{1}{k k_7\mu_0}\sum_{m=1}^{3} l_m(\rho\tau^2 + \mu_0\lambda_m^2) H_0^{(1)}(\lambda_m r),$$

$$\Gamma_{5\gamma}(\mathbf{x}\text{-}\mathbf{y}) = -\beta_0\frac{\partial\psi_{15}}{\partial x_\alpha}, \quad \Gamma_{5,2+\gamma}(\mathbf{x}\text{-}\mathbf{y}) = k_1\frac{\partial\psi_{51}}{\partial x_\alpha}, \quad \alpha,\gamma = 1,2,$$

$$\Gamma_{\alpha,2+\gamma}(\mathbf{x}\text{-}\mathbf{y}) = -k_1\beta\frac{\partial^2\psi_{13}}{\partial x_\alpha\partial x_\gamma}, \quad \Gamma_{\alpha+2,\gamma}(\mathbf{x}\text{-}\mathbf{y}) = k_3\beta_0\frac{\partial^2\psi_{13}}{\partial x_\alpha\partial x_\gamma},$$

$$\psi_{13} = \frac{1}{kk_7\mu_0}\sum_{m=1}^{3} l_m H_0^{(1)}(\lambda_m r), \quad l_m = d_m(\lambda_4^2+\lambda_m^2)(\lambda_5^2+\lambda_m^2),$$

$$\sum_{m=1}^{3} l_m = 0, \quad \sum_{m=1}^{3} l_m\lambda_m^2 = 0, \quad \sum_{m=1}^{3} l_m\lambda_m^4 = 1.$$

Clearly

$$\frac{\pi}{2i}H_0^{(1)}(\lambda r) = \ln|\mathbf{x}-\mathbf{y}| - \frac{\lambda^2}{4}|\mathbf{x}-\mathbf{y}|^2\ln|\mathbf{x}-\mathbf{y}| + const + O(|\mathbf{x}-\mathbf{y}|^2)$$

and we can directly prove the following

Theorem 4.1 : *The elements of the matrix* $\boldsymbol{\Gamma}(\boldsymbol{x}\text{-}\boldsymbol{y},\tau)$ *have a logarithmic singularity as* $\boldsymbol{x}\to\boldsymbol{y}$ *and each column of the matrix* $\boldsymbol{\Gamma}(\boldsymbol{x}\text{-}\boldsymbol{y},\tau)$, *considered as a vector, is a solution of the system* $\boldsymbol{A}(\partial\boldsymbol{x},\tau)\boldsymbol{U}=0$ *at every point* $\boldsymbol{x}$ *in the space* E_2 *except* $\boldsymbol{x}=\boldsymbol{y}$.

Note that the matrix $\boldsymbol{\Gamma}(\mathbf{x}\text{-}\mathbf{y},\tau)$, is unsymmetrical and its rows considered as vectors do not satisfy (26).

According to the method developed in [16], we construct the matrix $\widetilde{\boldsymbol{\Gamma}}(\mathbf{x},\tau) := \boldsymbol{\Gamma}^T(-\mathbf{x},\tau)$ and the following basic properties of $\widetilde{\boldsymbol{\Gamma}}(\mathbf{x},\tau)$ may be easily verified

Theorem 4.2 : *Each column of the matrix* $\widetilde{\boldsymbol{\Gamma}}(\boldsymbol{x}\text{-}\boldsymbol{y},\tau)$, *considered as a vector, satisfies the associated system* $\widetilde{\boldsymbol{A}}(\partial\boldsymbol{x},\tau)\boldsymbol{V}=0$, *at every point* $\boldsymbol{x}$ *except* $\boldsymbol{x}=\boldsymbol{y}$ *and the elements of the matrix* $\widetilde{\boldsymbol{\Gamma}}(\boldsymbol{x}\text{-}\boldsymbol{y},\tau)$ *have a logarithmic singularity as* $\boldsymbol{x}\to\boldsymbol{y}$.

5. Matrix of singular solutions

In solving BVPs of the theory of thermoelasticity with microtemperatures by Boundary Integral Method, besides the matrix of fundamental solutions, some other matrices of singular solutions to system (2) are of great importance. Using the fundamental solutions, we can construct new so-called singular solutions of the system (26) by means of elementary functions, playing an important role in the theory of boundary-value problems.

We introduce the special generalized stress vector $\boldsymbol{R}^\epsilon(\partial\mathbf{x},\mathbf{n})\mathbf{U}$, which acts on the element of the arc with the unit normal $\mathbf{n}=(n_1,n_2)$ where

$$\boldsymbol{R}^\epsilon(\partial\mathbf{x},\mathbf{n}) = \| R^\epsilon_{lj} \|_{5x5},$$

$$R^\epsilon_{\alpha\gamma} = \delta_{\alpha\gamma}\mu\frac{\partial}{\partial\mathbf{n}} + (\lambda+\mu)n_\alpha\frac{\partial}{\partial x_\gamma} + \epsilon_1\mathcal{M}_{\alpha\gamma},$$

$$R^\epsilon_{\alpha,\gamma+2} \equiv R^\epsilon_{\alpha+2,\gamma} \equiv R^\epsilon_{\alpha+2,5} \equiv R^\epsilon_{5\gamma} \equiv 0, \quad R^\epsilon_{\alpha 5} := -\beta n_\alpha, \tag{28}$$

$$R^\epsilon_{\alpha+2;\gamma+2} = \delta_{\alpha\gamma}k_6\frac{\partial}{\partial\mathbf{n}} + (k_4+k_5)n_\alpha\frac{\partial}{\partial x_\gamma} + \epsilon_2\mathcal{M}_{\alpha\gamma},$$

$$R^{\epsilon}_{5,\gamma+2} = k_1 n_\gamma, \quad R^{\epsilon}_{55} = k\frac{\partial}{\partial \mathbf{n}}, \quad \mathcal{M}_{\alpha\gamma} = n_\gamma \frac{\partial}{\partial x_\alpha} - n_\alpha \frac{\partial}{\partial x_\gamma},$$

here $\boldsymbol{\epsilon} = (\epsilon_1, \epsilon_2)$, ϵ_α, $\alpha = 1,2$, are the arbitrary numbers. If $\epsilon_1 = \mu$, $\epsilon_2 = k_5$, we have the stress vector and we denote it by $\P(\partial\mathbf{x},\mathbf{n})\mathbf{U}$. The operator, which we get from $(\boldsymbol{R})^{\boldsymbol{\epsilon}}(\partial\mathbf{x},\mathbf{n})$ for $\epsilon_1 = \dfrac{\mu(\lambda+\mu)}{\lambda+3\mu}$, $\epsilon_2 = \dfrac{k_6(k_4+k_5)}{k_4+k_5+2k_6}$, we denote by $\mathbf{N}(\partial\mathbf{x},\mathbf{n})$ and the vector $\mathbf{N}(\partial\mathbf{x},\mathbf{n})\mathbf{U}$ will be called the pseudostress vector.

Applying the operator $\boldsymbol{R}^{\epsilon}(\partial\mathbf{x},\mathbf{n})$ to the matrix $\boldsymbol{\Gamma}(\mathbf{x}\text{-}\mathbf{y},\tau)$, we construct the so-called singular matrix of solutions

$$\boldsymbol{R}\epsilon(\partial\mathbf{x},\mathbf{n})\boldsymbol{\Gamma}(\mathbf{x}\text{-}\mathbf{y},\tau) = \| M^{\epsilon}_{lj}(\partial\mathbf{x}) \|_{5\times5},$$

where

$$M^{\epsilon}_{\gamma\gamma}(\partial\mathbf{x}) = \frac{\partial H_0^{(1)}(i\lambda_4 r)}{\partial n} + (-1)^{\gamma}(\varepsilon_1+\mu)\frac{\partial}{\partial s}\frac{\partial^2\Psi_{11}}{\partial x_1\partial x_2} - n_\gamma\rho\tau^2\frac{\partial\Psi_{11}}{\partial x_\gamma},$$

$$M^{\epsilon}_{12}(\partial\mathbf{x}) = \frac{\tau_1}{\mu}\frac{\partial}{\partial s}H_0^{(1)}(i\lambda_4 r) - (\epsilon_1+\mu)\frac{\partial}{\partial s}\frac{\partial^2\Psi_{11}}{\partial x_2^2} - \rho\tau^2 n_1\frac{\partial\Psi_{11}}{\partial x_2},$$

$$M^{\epsilon}_{21}(\partial\mathbf{x}) = -\frac{\tau_1}{\mu}\frac{\partial}{\partial s}H_0^{(1)}(i\lambda_4 r) + (\epsilon_1+\mu)\frac{\partial}{\partial s}\frac{\partial^2\Psi_{11}}{\partial x_1^2} - \rho\tau^2 n_2\frac{\partial\Psi_{11}}{\partial x_1},$$

$$M^{\epsilon}_{1,\gamma+2}(\partial\mathbf{x}) = -k_1\beta\left[n_1\varrho\tau^2\frac{\partial\psi_{13}}{\partial x_\gamma} + (\mu+\epsilon_1)\frac{\partial}{\partial s}\frac{\partial^2\psi_{13}}{\partial x_\gamma\partial x_2}\right],$$

$$M^{\epsilon}_{15}(\partial\mathbf{x}) = -\beta\left[(\epsilon_1+\mu)\frac{\partial}{\partial x_2}\frac{\partial}{\partial s} + \rho\tau^2 n_1\right]\psi_{15},$$

$$M^{\epsilon}_{2,\gamma+2}(\partial\mathbf{x}) = -k_1\beta\left[n_2\varrho\tau^2\frac{\partial\psi_{13}}{\partial x_\gamma} - (\mu+\epsilon_1)\frac{\partial}{\partial s}\frac{\partial^2\psi_{13}}{\partial x_\gamma\partial x_1}\right],$$

$$M^{\epsilon}_{25}(\partial\mathbf{x}) = \beta\left[(\epsilon_1+\mu)\frac{\partial}{\partial x_1}\frac{\partial}{\partial s} - \rho\tau^2 n_2\right]\psi_{15}, \quad \frac{\partial}{\partial s} = n_2\frac{\partial}{\partial x_1} - n_1\frac{\partial}{\partial x_2},$$

$$M^{\epsilon}_{3\alpha}(\partial\mathbf{x}) = k_3\beta_0\left[-\frac{n_1}{k\mu_0}\sum_{m=1}^{3} l_m\lambda_m^2\frac{\partial}{\partial x_\alpha}H_0^{(1)}(\lambda_m r) + (\epsilon_2+k_6)\frac{\partial}{\partial s}\frac{\partial^2\psi_{13}}{\partial x_2\partial x_\alpha}\right],$$

$$M^{\epsilon}_{2+\gamma,2+\gamma}(\partial\mathbf{x}) = \frac{\partial H_0^{(1)}(i\lambda_5 r)}{\partial n} + (-1)^{\gamma}(\epsilon_2+k_6)\frac{\partial}{\partial s}\frac{\partial^2\Psi_{33}}{\partial x_1\partial x_2} + n_\gamma\frac{\partial}{\partial x_\gamma}[k_1k_3\psi_{51} - k_8\Psi_{33}],$$

$$M^{\epsilon}_{34}(\partial\mathbf{x}) = \frac{\epsilon_2}{k_6}\frac{\partial H_0^{(1)}(i\lambda_5 r)}{\partial s} - (\epsilon_2+k_6)\frac{\partial^2}{\partial x_2^2}\frac{\partial\Psi_{33}}{\partial s} + n_1\frac{\partial}{\partial x_2}[k_1k_3\psi_{51} - k_8\Psi_{33}],$$

$$M^{\epsilon}_{35}(\partial\mathbf{x}) = \frac{k_3}{k\mu_0}\sum_{m=1}^{3} l_m(\rho\tau^2+\mu_0\lambda_m^2)\left[n_1\lambda_m^2 - \frac{\epsilon_2+k_6}{k_7}\frac{\partial}{\partial x_2}\frac{\partial}{\partial s}\right]H_0^{(1)}(\lambda_m r),$$

$$M^\epsilon_{4\alpha}(\partial\mathbf{x}) = -k_3\beta_0\left[\frac{n_2}{k\mu_0}\sum_{m=1}^{3} l_m\lambda_m^2\frac{\partial}{\partial x_\alpha}H_0^{(1)}(\lambda_m r) + (\epsilon_2 + k_6)\frac{\partial}{\partial s}\frac{\partial^2\psi_{13}}{\partial x_1\partial x_\alpha}\right],$$

$$M^\epsilon_{43}(\partial\mathbf{x}) = -\frac{\tau_2}{k_6}\frac{\partial H_0^{(1)}(i\lambda_5 r)}{\partial s} + (\epsilon_2 + k_6)\frac{\partial^2}{\partial x_1^2}\frac{\partial\Psi_{33}}{\partial s} + n_2\frac{\boxplus}{\partial}\partial x_1[k_1k_3\psi_{51} - k_8\Psi_{33}],$$

$$M^\epsilon_{45}(\partial\mathbf{x}) = \frac{k_3}{k\mu_0}\sum_{m=1}^{3} l_m(\rho\tau^2 + \mu_0\lambda_m^2)\left[n_2\lambda_m^2 + \frac{\epsilon_2 + k_6}{k_7}\frac{\partial}{\partial x_1}\frac{\partial}{\partial s}\right]H_0^{(1)}(\lambda_m r),$$

$$M^\epsilon_{5\gamma}(\partial\mathbf{x}) = \frac{\beta_0}{k_7\mu_0}\sum_{m=1}^{3} l_m\left[\frac{k_1k_3}{k} - k_8 - k_7\lambda_m^2\right]\frac{\partial^2 H_0^{(1)}(\lambda_m r)}{\partial x_\gamma\partial n},$$

$$M^\epsilon_{5,\gamma+2}(\partial\mathbf{x}) = k_1\left[\frac{n_\gamma}{k_6}H_0^{(1)}(i\lambda_5 r) - \frac{\partial^2(\Psi_{33} + k\psi_{51})}{\partial x_\gamma\partial n}\right],$$

$$M^\epsilon_{55}(\partial\mathbf{x}) = \frac{1}{\mu_0 k_7}\sum_{m=1}^{3} l_m(\rho\tau^2 + \mu_0\lambda_m^2)\left[-\frac{k_1k_3}{k} + k_8 + k_7\lambda_m^2\right]\frac{\partial H_0^{(1)}(\lambda_m r)}{\partial n},$$

$$l_m = d_m(\lambda_4^2 + \lambda_m^2)(\lambda_5^2 + \lambda_m^2), \quad m = 1,2,3, \quad \alpha,\gamma = 1,2.$$

The following theorem may be easily verified

Theorem 5.1: *Every column of the matrix* $[\boldsymbol{R}^\epsilon(\partial\boldsymbol{y},\boldsymbol{n})\boldsymbol{\Gamma}(\boldsymbol{y}\text{-}\boldsymbol{x},\tau)]^T$, *considered as a vector, is a solution with respect to the point* $\boldsymbol{x}$ *of the system* $\widetilde{\boldsymbol{A}}(\partial\boldsymbol{x},\tau)V = 0$ *everywhere in* E_2, *except* $\boldsymbol{x} = \boldsymbol{y}$.

Let

$$\tilde{\boldsymbol{R}}^\epsilon(\partial\mathbf{x},\mathbf{n}) = \begin{pmatrix} R^\epsilon_{11} & R^\epsilon_{12} & 0 & 0 & \beta_0 n_1 \\ R^\epsilon_{21} & R^\epsilon_{22} & 0 & 0 & \beta_0 n_2 \\ 0 & 0 & R^\epsilon_{33} & _{34} & 0 \\ 0 & 0 & R^\epsilon_{43} & R^\epsilon_{44} & 0 \\ 0 & 0 & k_3 n_1 & k_3 n_2 & R^\epsilon_{55} \end{pmatrix},$$

where $R^\epsilon_{\alpha\gamma}$, $R^\epsilon_{\alpha+2,\gamma+2}$, R^ϵ_{55}, $\alpha,\gamma = 1,2$, are given by (28), then

$$\tilde{\boldsymbol{R}}^\epsilon(\partial\mathbf{x},\mathbf{n})\tilde{\mathbf{\Gamma}}(\mathbf{x}\text{-}\mathbf{y},\tau) = \|\widetilde{M}^\epsilon_{lj}(\partial\mathbf{x})\|_{5\times5},$$

Here

$$\widetilde{M}^\epsilon_{\alpha\gamma}(\partial\mathbf{x}) = M^\epsilon_{\alpha\gamma}(\partial\mathbf{x}), \quad \widetilde{M}^\epsilon_{\alpha+2,\gamma+2}(\partial\mathbf{x}) = M^\epsilon_{\alpha+2,\gamma+2}(\partial\mathbf{x}), \quad \widetilde{M}^\epsilon_{55}(\partial\mathbf{x}) = M^\epsilon_{55}(\partial\mathbf{x}),$$

$$\widetilde{M}^\epsilon_{1,\gamma+2}(\partial\mathbf{x}) = k_3\beta_0\left[n_1\rho\tau^2\frac{\partial\psi_{13}}{\partial x_\gamma} + (\epsilon_1 + \mu)\frac{\partial}{\partial s}\frac{\partial^2\psi_{13}}{\partial x_2\partial x_\gamma}\right],$$

$$\widetilde{M}^{\epsilon}_{2,\gamma+2}(\partial\mathbf{x}) = k_3\beta_0\left[n_2\rho\tau^2\frac{\partial\psi_{13}}{\partial x_\gamma} - (\epsilon_1+\mu)\frac{\partial}{\partial s}\frac{\partial^2\psi_{13}}{\partial x_1\partial x_\gamma}\right],$$

$$\widetilde{M}^{\epsilon}_{15}(\partial\mathbf{x}) = \beta_0\left[n_1\rho\tau^2\psi_{15} + (\epsilon_1+\mu)\frac{\partial}{\partial s}\frac{\partial\psi_{15}}{\partial x_2}\right],$$

$$\widetilde{M}^{\epsilon}_{25}(\partial\mathbf{x}) = \beta_0\left[n_2\rho\tau^2\psi_{15} - (\epsilon_1+\mu)\frac{\partial}{\partial s}\frac{\partial\psi_{15}}{\partial x_1}\right],$$

$$\widetilde{M}^{\epsilon}_{3\gamma}(\partial\mathbf{x}) = k_1\beta\left[\frac{n_1}{k\mu_0}\sum_{m=1}^{3} l_m\lambda_m^2\frac{\partial H_0^{(1)}(\lambda_m r)}{\partial x_\gamma} - (\epsilon_2+k_6)\frac{\partial}{\partial s}\frac{\partial^2\psi_{13}}{\partial x_2\partial x_\gamma}\right],$$

$$\widetilde{M}^{\epsilon}_{4\gamma}(\partial\mathbf{x}) = k_1\beta\left[\frac{n_2}{k\mu_0}\sum_{m=1}^{3} l_m\lambda_m^2\frac{\partial H_0^{(1)}(\lambda_m r)}{\partial x_\gamma} + (\epsilon_2+k_6)\frac{\partial}{\partial s}\frac{\partial^2\psi_{13}}{\partial x_1\partial x_\gamma}\right],$$

$$\widetilde{M}^{\epsilon}_{35}(\partial\mathbf{x}) = k_1\left[\frac{n_1}{k\mu_0}\sum_{m=1}^{3} l_m\lambda_m^2(\rho\tau^2+\mu_0\lambda_m^2)H_0^{(1)}(\lambda_m r) - (\epsilon_2+k_6)\frac{\partial}{\partial s}\frac{\partial\psi_{51}}{\partial x_2}\right],$$

$$\widetilde{M}^{\epsilon}_{45}(\partial\mathbf{x}) = k_1\left[\frac{n_2}{k\mu_0}\sum_{m=1}^{3} l_m\lambda_m^2(\rho\tau^2+\mu_0\lambda_m^2)H_0^{(1)}(\lambda_m r) + (\epsilon_2+k_6)\frac{\partial}{\partial s}\frac{\partial\psi_{51}}{\partial x_1}\right],$$

$$\widetilde{M}^{\epsilon}_{5\gamma}(\partial\mathbf{x}) = \frac{\beta}{k_7\mu_0}\sum_{m=1}^{3} l_m\left[k_8 + k_7\lambda_m^2 - \frac{k_1k_3}{k}\right],\frac{\partial^2 H_0^{(1)}(\lambda_m r)}{\partial x_\gamma\partial n},$$

$$\widetilde{M}^{\epsilon}_{5,\gamma+2}(\partial\mathbf{x}) = k_3\left[\frac{n_\gamma}{k_6}H_0^{(1)}(i\lambda_5 r) + \frac{\partial^2(-\psi_{33}+k\psi_{51})}{\partial x_\gamma\partial n}\right].$$

Let $[\tilde{\boldsymbol{R}}^{\epsilon}(\partial\mathbf{y},\mathbf{n})\widetilde{\mathbf{\Gamma}}(\mathbf{y}\text{-}\mathbf{x},\tau)]^T$, be the matrix which we get from $\tilde{\boldsymbol{R}}^{\epsilon}(\partial\mathbf{x},\mathbf{n})\widetilde{\mathbf{\Gamma}}(\mathbf{x}\text{-}\mathbf{y})$ by transposition of the columns and rows and the variables $\mathbf{x}$ and $\mathbf{y}$. The superscript "T" denotes transposition.

Let $\mathbf{g}$ and $\boldsymbol{\phi}$ be continuous (or Hölder continuous) five-component vectors and let S be a closed Lyapunov curve.

We introduce the potential

$$\mathbf{Z}^{(1)}(\mathbf{x},\mathbf{g}) = \int_S \mathbf{\Gamma}(\mathbf{x}-\mathbf{y},\tau)\mathbf{g}(\mathbf{y})ds$$

of a single-layer, the potential

$$\mathbf{Z}^{(2)}(\mathbf{x},\mathbf{g}) = \int_S [\tilde{\boldsymbol{R}}^{\epsilon}(\partial\mathbf{y},\mathbf{n})\mathbf{\Gamma}^T(\mathbf{y}\text{-}\mathbf{x},\tau)]^T\mathbf{g}(\mathbf{y})ds$$

of double-layer and the potential

$$\mathbf{Z}^{(3)}(\mathbf{x},\boldsymbol{\phi}) = \int\limits_{D^{\pm}} \boldsymbol{\Gamma}(\mathbf{x}-\mathbf{y},\tau)\boldsymbol{\phi}(\mathbf{y})ds$$

of volume.

We may show as in the classical theory of thermoelasticity, that the following theorem is valid

Theorem 5.2: *The vectors* $\boldsymbol{Z}^{(j)}$ $(j = 1, 2,)$ *are solutions of equation* $\boldsymbol{A}(\partial \boldsymbol{x}, \tau)\boldsymbol{U} = 0$ *in both the domains* D^+ *and* D^- *and the elements of the matrix* $\left[\tilde{\boldsymbol{R}}^{\epsilon}(\partial \boldsymbol{y},\boldsymbol{n})\boldsymbol{\Gamma}^{\boldsymbol{T}}(\boldsymbol{y}\text{-}\boldsymbol{x},\tau)\right]^T$, *contain a singular part, which is integrable in the sense of the Cauchy principal value. The vector* $\boldsymbol{Z}^{(3)}(\boldsymbol{x},\boldsymbol{\phi})$ *is solution of the system* $\boldsymbol{A}(\partial \boldsymbol{x},\tau)\boldsymbol{Z}^{(3)} = \boldsymbol{\phi}$.

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