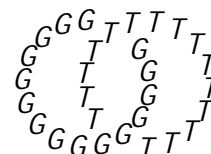


Geometry & Topology

Volume 7 (2003) 641{643

Published: 22 October 2003



Splitting the concordance group of algebraically slice knots

Charles Livingston

*Department of Mathematics, Indiana University
Bloomington, IN 47405, USA*

Email: livingst@indiana.edu

Abstract

As a corollary of work of Ozsvath and Szabo [8], it is shown that the classical concordance group of algebraically slice knots has an infinite cyclic summand and in particular is not a divisible group.

AMS Classification numbers Primary: 57M25

Secondary: 57Q60

Keywords: Knot concordance, algebraically slice

Proposed: Robion Kirby

Seconded: Tomasz Mrowka, Cameron Gordon

Received: 1 June 2003

Accepted: 21 September 2003

Let A denote the concordance group of algebraically slice knots, the kernel of Levine's homomorphism $\lambda : C \rightarrow G$, where C is the classical knot concordance group and G is Levine's algebraic concordance group [6]. Little is known about the algebraic structure of A : it is countable and abelian, Casson and Gordon [2] proved that A is nontrivial, Jiang [5] showed it contains a subgroup isomorphic to \mathbf{Z}^1 , and the author [7] proved that it contains a subgroup isomorphic to \mathbf{Z}_2^1 . We add the following theorem, a quick corollary of recent work of Ozsvath and Szabo [8].

Theorem 1 *The group A contains a summand isomorphic to \mathbf{Z} and in particular A is not divisible.*

Proof In [8] a homomorphism $\lambda : C \rightarrow \mathbf{Z}$ is constructed. We prove that λ is nontrivial on A . The theorem follows since, because $\text{Im}(\lambda)$ is free, there is the induced splitting, $A = \text{Im}(\lambda) \oplus \text{Ker}(\lambda)$. No element representing a generator of $\text{Im}(\lambda)$ is divisible.

According to [8], $j(K) \leq g_4(K)$, where g_4 is the 4-ball genus of a knot, and there is the example of the $(4,5)$ -torus knot T for which $j(T) = 6$. We will show that there is a knot T' algebraically concordant to T with $g_4(T') < 6$. Hence, $T \# -T'$ is an algebraically slice knot with nontrivial λ , as desired.

Recall that T is a braid knot with braid F of genus $(4-1)(5-1)=2=6$. Let V be the 12×12 Seifert matrix for T with respect to some basis for $H_1(F)$. The quadratic form $q(x) = xVx^t$ on \mathbf{Z}^{12} is equal to the form given by $(V + V^t)/2$. Using [3] the signature of this symmetric bilinear form can be computed to be 8, so q is indefinite, and thus by Meyer's theorem [4] there is a nontrivial primitive element z with $q(z) = 0$. Since z is primitive, it is a member of a symplectic basis for $H_1(F)$. Let V' be the Seifert matrix for T with respect to that basis. The canonical construction of a Seifert surface with Seifert matrix V' ([9], or see [1]) yields a surface F' such that z is represented by a simple closed curve on F' that is unknotted in S^3 . Hence, F' can be surgered in the 4-ball to show that its boundary T' satisfies $g_4(T') < 6$. Since T and T' have the same Seifert form, they are algebraically concordant. \square

Addendum An alternative proof of Theorem 1 follows from the construction of knots with trivial Alexander polynomial for which λ is nontrivial, to appear in a forthcoming paper.

References

- [1] **G Burde, H Zieschang**, *Knots*, de Gruyter Studies in Mathematics, 5, Walter de Gruyter & Co., Berlin (1985)
- [2] **A Casson, C McA Gordon**, *Cobordism of classical knots*, from: "À la recherche de la Topologie perdue", (Guillou and Marin, editors), Progress in Mathematics, Volume 62 (1986), originally published as an Orsay Preprint (1975)
- [3] **C McA Gordon, R Litherland, K Murasugi**, *Signatures of covering links*, Canad. J. Math. 33 (1981) 381{394
- [4] **D Husemoller, J Milnor**, *Symmetric Bilinear Forms*. Ergebnisse der Mathematik und ihrer Grenzgebiete, Band 73, Springer-Verlag, New York-Heidelberg (1973)
- [5] **B Jiang**, *A simple proof that the concordance group of algebraically slice knots is finitely generated*, Proc. Amer. Math. Soc. 83 (1981) 189{192
- [6] **J Levine** *Knot cobordism groups in codimension two*, Comment. Math. Helv. 44 (1969) 229{244
- [7] **C Livingston**, *Order 2 algebraically slice knots*, from Proceedings of the Kirbyfest (Berkeley, CA, 1998) 335{342, Geom. Topol. Monogr., 2, Geom. Topol. Publ., Coventry (1999)
- [8] **P Ozsvath, Z Szabo**, *Knot Floer homology and the four-ball genus*, Geometry and Topology 7 (2003) 615-639, arXiv: math.GT/0301149
- [9] **H Seifert**, *Über das Geschlecht von Knoten*, Math. Ann. 110 (1934) 571{592