Burton, Leone (Ed.):

Learning Mathematics
From Hierarchies to Networks

London: Falmer Press, 1999. – 265 p.
(Studies in Mathematics Education Series; 13)
ISBN 0-7507-1009 8 (pbk)

Leslie P. Steffe, Athens, GA (USA)

The book began at a symposium on the joint day of the two conferences, The Growing Mind and Piaget – Vygotsky, held in Geneva, Switzerland in September, 1996. It is organized into three main sections titled Abandoning Hierarchies, Abandoning Dichotomies; Mathematics as a Socio-Cultural Artefact; and Teaching and Learning of Mathematics. Each section consists of four or five papers followed by a commentary. In the review, I maintain this structure of the book primarily because each individual paper makes a special contribution. My main strategy in the review is to identify and discuss issues and themes arising from the papers both within and across the three sections.

1. Abandoning hierarchies, abandoning dichotomies
Given the context of the origins of the book, it was appropriate that Jere Confrey's paper on applying and modifying Piagetian theory in mathematics education was chosen as the lead paper. Her main thrust is to examine the changes in Piagetian theory that she envisions as necessary when taking the stance of equity of educational opportunity in multicultural societies. Her argument is that although radical constructivism permits one to recognize the need for stance as a result of multiple possibilities, it does not guide one in the selection of a particular stance. Additional theory is necessary to guide the selection of stance, and thus obliges one to clearly articulate one's stance. This is a theme that I discuss in the last section.

Articulation of the individual and the social is one of the changes advocated by Confrey, and I discuss it in the second section. Another issue of importance to Confrey concerns universality of mathematical structure. She points to the elitism of mathematics in the Eurocentric tradition and the need to uncover elitist practices and to document historically how these practices have been used to secure mathematics its privileged and protected epistemological status. Finally, she points to the changes that new technological tools bring forth in the meaning of mathematical competency and technological fluency and makes an impassioned plea for theories of how individuals learn to move effectively in complex spaces, and how they gradually construct reflective and critical tools to model complexity.

Leone Burton in the second of the papers apparently agrees with Confrey's assumption of the fundamentality of the individual. In Confrey's framework, "Voice according to an equity stance is more than the expression of a student's point of view. It suggests that students have a fundamental right to express their own understanding of an idea" (p. 15). Burton extends the idea of student voice to her concept of agency: "In a constructivist context, the knowing agent is unquestionably the individualized learner who is, however, seen in a socio-cultural relationship both with other members of the community who influence the authorship process (coming to know) and with the socially validated knowledge 'objects', authorship of which is always external" (p. 25).

Burton points to what she regards as a dualism in some constructivist writing between agency and mathematical knowledge as externally authored socially accepted knowledge. Because this issue of dualism is a theme that runs throughout the book, it is somewhat unfortunate that both Confrey and Burton bring the issue forth in the context of Piaget's constructivism without further comment. Toward this end, von Glasersfeld (2000) commented that: "Piaget said dozens of times that, in his theory, 'to know' does not mean to construct a picture of the real world ..." (p. 4). So, even though an observer may consider the authorship of socially validated knowledge "objects" as external to students, this does not mean that such knowledge should be regarded by the observer as serving as an endpoint in students' knowledge construction although such a possibility should be kept open.

Burton uses the idea of a narrative approach to the learning of mathematics in an attempt to alleviate the dualism that she sees in educational practice in mathematics. Narrative – constructing and telling our own stories – is used as a means of fostering "A reflective process of coming to know within a learning community where discourse is paramount" (p. 31).

In the third paper of the section Merrilyn Goos, Peter Galbraith, and Peter Renshaw discuss their work with es-

tablishing a community of practice in a secondary school mathematics class from within a Vygotskian framework, and it can be profitably interpreted in terms of Burton's idea of narrative. Through a classroom teaching episode involving the students' finding the inverse of a 2×2 matrix, however, Goos et al. make clear that they consider the teacher indispensable in facilitating vigorous mathematical debate and in ensuring that the substantive arguments of students are tested against disciplinary knowledge. Although they thereby maintain the dualism explained by Burton in her paper, their point about including the teacher's knowledge in studies of mathematical learning deserves special attention because how a teacher uses his or her knowledge in fostering students' learning is a major issue. Walter Secada, in his commentary on the papers, speaks to this issue in the context of commenting on the paper by Shirley Booth, Inger Wistedt, Ola Halldén, Mats Martinsson, and Ference Marton: "Intentional analysis begins with the assumption that a student's actions are reasonable and, in some sense, correct ... A phenomenographic stance is, on the other hand, more interested in what can be said about a population of students, as opposed to an individual student. ... Dichotomies of the sort they outline are really matters of stance, not of metaphysical reality .... one's stance shapes what one looks for and, therefore, what one sees" (p. 87). By analogy, the particular stance of teachers are critical in how they use their knowledge in fostering student learning, and it is an issue which points to the necessity of teachers understanding models of knowing in the way that researchers understand them. In fact, the paper by Booth et al. is notable for the self-reflexive attitude of the authors.[1]

2. Mathematics as a socio-cultural artefact

It was difficult to make a sharp distinction between the papers of this section and the papers of the first section. In fact, Restivo commented that: "It is clear that Smith's sociological perspective is consistent with the theme of this book, which moves us away from thinking and seeing in terms of hierarchies instead of social networks" (p. 123). Restivo's paper is based on what he calls The Social Construction Conjecture: "The basic claim relevant here is that all knowledge is socially constructed. This should be simple enough. For how else is it – how else could it be – that we humans come to know things, come to formulate words and sentences about things, except through our interactions with others? How else, indeed, is it that we are ourselves constructed?" (p. 121) Restivo's understanding of thinking – the experience of "inner thought" – as internal conversation demonstrates just how strong his claim is: "individuals are vehicles for expressing the thoughts of social worlds or 'thought-collectives'. Or, to put it another way, minds are social structures" (p. 128).

In contrast to the paper by Booth et al. in the first section, Restivo did not seem sensitive to the principle of self-reflexivity. Given his rejection of the individual, I pause

to consider what it might mean for Restivo to apply the principles of his sociology to himself in the construction of the sociology. As I read his chapter, I was surprised that I felt compelled to interpret it as Restivo speaking as an individual. For how else could he say, "I am not going to defend the very idea of a sociology of mathematics but rather (having set the foundations) assume it" (p. 130). If he were speaking from a sociological stance, it would seem that his sociology of mathematics would need to be socially warranted, not defended nor assumed, but debated and critiqued. Such critique sets the stage, not for contentious debate, attack and counter-attack, but rather for a scientific discussion of how we view the learner. Restivo's rejection of the individual notwithstanding, I find much of what he says compatible with my reading of Piaget as well as Vygotsky although neither of these authors rejected the individual in spite of attempts to cast them as irreconcilable opponents (van der Veer , Valsiner, 1994, p. 6). So, in the service of scientific discussion, it would be very helpful if Restivo would set forth conditions under which he would consider his understanding of inner thought falsified in Lakatos' (1970) sense of sophisticated falsificationism. Knowing these conditions would be useful not only in actually testing hypotheses concerning inner thought, but also in understanding a model which would include inner thought as internal conversation as well as other possibilities. My request should not be regarded as a rejection of internal conversation. Rather, it is more of a request to include the observer in a sociology of mathematics. In discussing inner thought as internal conversation, Ackermann (1995) elevated the designer of educational materials to a second-order observer: "They (designers) switch roles from being the producers to being critics of their own production. Every now and then they 'stop and think,' ... and they look at what they have done through their own and another person's eyes. ... I suggest that while interacting with their own forms, designers necessarily start a dialogue with a whole chorus of interlocutors, imaginary or real, to whom them dedicate their work" (p. 349).

Following Restivo's advice, I looked to the other authors of the section to explore their view of mathematical minds as well as how the social might determine the mathematical experiences from which we generate our mathematical knowledge. In Restivo's opinion, "the 'powerful computation contexts' to which Kathryn Crawford draws attention ... are simply underlining something that is true of the best forms of human learning: that 'networked environments', in the most general sense, are more constructive than hierarchical ones" (p. 123). In her elaboration of such a mathematical environment, Crawford emphasized self-directed learning, orienting students' to take responsibility for their own learning, and individual experience. Her emphasis is compatible with the work of Goos, et al. in the first section, and it certainly runs counter to Restivo's rejection of all Piagetian influence. Suzanne Damarin pointed out in her commentary that the issue of individual development and knowledge is quite unsettled among social constructivist theories and I agree with her. Outstanding scientific work that offers penetrating insights into the issue is of the greatest importance.

---

[1] A self-reflexive observer explicitly applies the principle and constructs of a model of knowing first and foremost to him- or herself (Steier, 1995).

The view of mathematics as "a social object, produced through social relations and available for use by the community in the production of further social constructs" (Damarin, p. 143) undergirds the problem that Lerman addresses in his chapter on transfer of mathematical knowledge among communities. Damarin commented that "one way of reading his concern with transfer is as a discussion of the difficulty of importing or interpolating into one community of knowers a truth constructed in another" (p. 141). Lerman's concern with transfer of mathematical knowledge among communities is appropriate and worthy of considerable investigation and articulation: "In particular," he wrote, "I want to ask what Piagetian and Vygotskian psychological frameworks might offer for working with the problem of transfer" (p. 94).

Lerman's task seemingly would be to investigate whatever meanings neo-Piagetians and neo-Vygotskians ascribe to mathematics and then how these meanings might be "imported or interpolated" into the other community. However, after an inspired discussion of the Piagetian framework, Lerman essentially concluded that: "for Piaget the individual's social interactions are the only place of entry of the social and it is inadequate to account for the strength and range of socio-cultural milieux on people" (p. 101). This dismissal of the Piagetian framework essentially closes off any scientific investigations that arise because of tensions created by comparing and contrasting these psychological frameworks. In fact, it could be interpreted to mean that Lerman doesn't consider it possible for neo-Piagetians to analyze the "strength and range of socio-cultural milieux on people". According to Ackermann (1995), however, there need not be a rejection of "preexisting culture" within the Piagetian framework if that phrase is regarded as pointing to records and artifacts of previous interactions and their results. "Along with many constructivists, I believe that the very act of endowing objects with an existence independent of people's current interactions with them is, indeed, a mental construct. It is an act of constructing invariants in Piaget's sense. However, such a mental construct is of great use to anyone engaged in teaching and designing tools and places for learning" (p. 350).

Lerman's rejection of the Piagetian framework introduces an ambiguity into his chapter because his analysis of the two frameworks was at a quite different level than his analysis of various communities of knowers from within the Vygotskian framework. In fact, his analysis of the two frameworks was carried out at the same level as the self-reflexive analysis of Booth et al. It is interesting that these latter authors did not reject either framework in which they worked, whereas Lerman did reject the Piagetian framework. One reason for his rejection might be his view that: "For a psychological theory to account for constitutive relations between social factors and cognition the process of the construction of the individual's social plane of consciousness must be 'open' to social life" (p. 101). Ackermann's understanding of "pre-existing culture" does relax the assumption of the relation between social factors and cognition being constitutive, but it also opens the possibility of formulating an experientially grounded and functional model of a sense in which social factors might determine the experiences from which students generate mathematical knowledge. In such a program of research, Janet Kaahwa points to the necessity of not regarding "pre-existing culture" as fixed and immutable by her account of the plight of girls in Ugandan mathematics classrooms: "A colleague told me a story of what his teacher in primary school used to do. The teacher would put a mathematics question to girls before boys. Whenever a girl failed to answer, he would say: 'You see, you are just seated there punishing the stool, you should be at home cooking'" (pp. 137–138). Shouldn't one of the goals in mathematics education be to modify pre-existing culture? Regarding culture as produced "through living interaction among human beings" (Bauersfeld, 1995, pp. 156–157) does have certain advantages because it opens the way for mutually reciprocal influences.

3. Teaching and learning mathematics

In her commentary on the paper by Barbara Jaworski, Christine Keitel said: "terms like social constructivism and cognitive theories are used lavishly without substantiation and critical reflection, so that for the reader as well as for the teachers the theoretical background has the effect of jargon ..." (p. 250). I found this to be a very interesting comment because it is indicative of the vast gulf that can be created between mathematics teaching and models of knowing. In my reading of Jaworski's paper, however, I found her to be quite sensitive to the notion that constructivism is not about pedagogy: "What is actually needed is parallel explorations in both theory and practice, so that a genuine theory-practice dialectic might result" (p. 159).

A question recurred throughout Jaworski's paper concerning what a general model of knowing can contribute to the construction of a model of practice. Four rather important points were made concerning this question in other papers in the section. Hilary Povey and Leone Burton commented that: "Such an epistemological perspective takes the concrete and the personal as the starting point for meaning-making ..." (pp. 243–244). Moreover, Thomas Kieren, Susan Pirie, and Lynn Gordon Calvert went to great lengths in their narrative of Kara's understanding of fractions to establish the nature and quality of her current knowledge of fractions. Both sets of authors explicitly acknowledge the necessity of understanding the current mathematical knowledge of students throughout the teaching-learning process. But the general models they offer do not include specification of such knowledge. Rather, the examples they provide were personally generated examples that were used to provide meaning to more general constructs within their models. In fact, both papers should be interpreted as parallel explorations in both theory and practice called for by Jaworski.

Second, in their account of Kara's progress, Kieren et al. made a very explicit differentiation between Kara's meaning of paper folding and their meaning of paper folding: "It is easy ... to think that at this point Kara has 'our' image of halves or sixteenths. But her image of fractions, like so many of her peers, was related to the action sequence that brought it about and not the fractional part or

piece that was the product" (p. 214). So, while it may be an expectation of a teacher that the mathematical knowledge of students and teachers substantially overlap, the teacher must be open to the distinct possibility that there may be little or no overlap or that the overlaps may play out at very different levels of abstraction. Piaget's reflective abstraction certainly oriented the authors to look for such levels, but for reflective abstraction to be useful to teachers, it must be observable in the way that Kieren et al. explained.

The third point, already made by Burton in the first section and again made by Hilary Povey and Leone Burton in their paper, is that the students should be considered as authors of what transpires in the classroom. From the perspective of the teacher, Kieren et al. obviously considered Kara's and her peers' images of fractions as constraints. Their sensitivity to the fractional knowledge of the children permitted them to function as co-authors of their knowledge. The researchers fit their language and actions as teachers within these constraints and proceeded in harmony with the children. Their continual experimentation concerning what might be appropriate was exemplified by the 'half-fraction kits' they described and by the comment concerning their students: "With only a couple of such image-making sessions, Kara and her classmates showed that they had extended their image of fractions" (p. 215). A major dilemma of teachers reported by Jaworski that they "wanted to encourage students' autonomy in knowledge construction, but the construction had to result in particular knowledge" (p. 161) simply did not arise for Kieren et al. because in their model of knowing, they see students as rational, knowing human beings and they attribute mathematical realities to them that are distinct from their own.

The fourth point concerning Jaworski's question originated in an example she made. It arose because of the difficulties the teachers' with whom she worked had in interpreting a written comment by von Glasersfeld (1987) that: "The teacher will realize that knowledge cannot be transferred to the student by linguistic communication but that language can be used as a tool in the process of guiding the student's construction" (p. 163). Jaworski commented that: "Ben, along with the other teachers, seemed to agree almost literally with the first part of this statement, but without having a clear rationalization of the second part. Here is an example where links between theory and practice seem particularly fragile" (p. 165). What it illustrates in my opinion is the difficulty of simply applying a general model to practice. A general model of knowing can be used in building models of mathematics teaching, but these latter models are not to be confused with the general model used in building them.

The papers by Povey et al., Kieren et al., and Jaworski clearly point to the necessity of teachers constructing their own models of educational practice that are grounded in their experiences, being guided and constrained by their general models in the constitution of their experiences. This necessity is caught by Keitel's admonition that: "Practising teachers need to be involved (in research), as equal partners, in mathematics education research projects, and the theoretical assumptions and practical approaches

in projects should not be predetermined by outside 'experts'" (p. 246). In other words, it is the teachers who must be the "experts". Although there was no paper devoted explicitly to the issue of teachers as "experts" in the sense meant by Keitel, the paper by Terry Wood and Tammy Turner-Vorbeck does indicate that the involved transformation will not be an easy one: "In our findings, dilemmas were seen to arise for these teachers as they began to conceptualize and practise teaching as a complex activity" (p. 184).

John Mason provided a thumb-nail sketch of the essence of an expert teacher which I find very insightful. Mason commented on his agreement with Kieren et al. that understanding is not a thing that is acquired, but rather is most usefully thought of as understanding-in-action in the following way: "We are in agreement, though, that teacher expertise is not a static knowing-that or even knowing-about, for it is much more than passing tests and writing essays. It is knowing-to act in the moment. It is understanding-in-action, it is acting-in-the-moment that marks the expert teacher from the novice" (p. 188).

Mason rejects the thesis that understanding is the action and this opens consideration of the expert teacher beyond acting-in-the-moment. Unfortunately, he does not explicitly explain what he means by understanding. Nevertheless, his emphasis on labeling should be interpreted as opening a way for teachers to become experts. His question, "What can we learn from what happens naturally in order to be more systematic and disciplined, more effective in provoking communities into developing a richly meaningful shared vocabulary, which enables them to probe beneath the surface of phenomena of interest, rather than remaining with surface jargon?" (p. 189) lies beneath his emphasis on labeling and he takes great care in making it clear that he regards labels as "the constituents of theories" (p. 190).

Mason captures the essence of Jaworski's question concerning what a general model of knowing can contribute to the construction of a model of practice in his consideration of labels. He doesn't leave the community of educational practitioners at the level of what he considers as phenomena[2] as important as that level is: "Discussions of mathematics education are often confounded by the lack of agreed technical terms, and by confusing phenomena being explained with the explanation of those phenomena. ... Frameworks are useful both in discussing teaching and learning, and in theorizing about teaching and learning" (p. 197). In other words, a general model of knowing can be used in the explanation of phenomena. This understanding of the function of a general model of knowing alleviates Keitel's criticism that the terms of a general model have the effect of jargon. Of course, I can only agree with Keitel if those terms are not deeply understood and used in explanation however they might be transformed in their use.

---

[2] According to Mason, a particular incident becomes a phenomena when generality has been detected, when the particular has been seen as a generic in some way by connections to past experience.

4. References

Ackermann, E. (1995): Construction and transference of meaning through form. – In: L. P. Steffe; J. Gale (Eds.), Constructivism in education (pp. 341–354). Hillsdale, NJ: Lawrence Erlbaum Associates

Bauersfeld, H. (1995): The structuring of the structures: Development and function of mathematizing as a social practice. – In: L. P. Steffe; J. Gale (Eds.), Constructivism in education (pp. 137–158). Hillsdale, NJ: Lawrence Erlbaum Associates

Lakatos, I. (1970): Falsification and the methodology of scientific research programs. – In: I. Lakatos; A. Musgrave (Eds.), Criticism and the growth of knowledge (pp. 91–196). Cambridge: Cambridge University Press

Steier, F. (1995): From universing to conversing: An ecological constructionist approach to learning and multiple description. – In: L. P. Steffe; J. Gale (Eds.), Constructivism in education (pp. 67–84). Hillsdale, NJ: Lawrence Erlbaum Associates

van der Veer, R.; Valsiner, J. (1994): Introduction. – In: R. van der Veer; J. Valsiner (Eds.), The Vygotsky reader (pp. 1–9). Cambridge, MA: Blackwell

von Glasersfeld, E. (1987): Constructivism in education. – In: T. Husen; N. Postlethwaite (Eds.), International encyclopedia of education, (Supplementary Volume, pp. 162–163). Oxford: Pergamon

von Glasersfeld, E. (2000): Problems of constructivism. – In: L. P. Steffe; P. Thompson (Eds.), Radical constructivism in action: Building on the pioneering work of Ernst von Glasersfeld (pp. 3–9). London: Falmer Press

Author

Steffe, Leslie P., Dr., University of Georgia, Mathematics Education Department, 105 Aderhold Hall, Athens, GA 30602-7124, USA. E-mail: lsteffe@coe.uga.edu