

Book Reviews

Brousseau, Guy:

Theory of Didactical Situations in Mathematics

Didactique des Mathématiques 1970–1990

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1. Background of the book

Paris, June 1993. The French community of research in “didactique des mathématiques” celebrated its twenty years of research by an international conference. This event provided the opportunity of honouring the work of the French pioneers who created the community of research, shaped it and deeply influenced it, particularly Guy Brousseau and Gérard Vergnaud. Although in addition to his French papers and books, Gérard Vergnaud had published numerous texts in English, the original texts of Guy Brousseau were almost unknown in the non-French-speaking world. The present book is aimed at repairing this lack and one should acknowledge the huge enterprise of four adventurers, originating from four countries and three different continents who decided to “undertake the task of translating into English most of the works of Guy Brousseau” (Editors’ preface). The book actually presents texts written by Brousseau from the seventies until 1990. Let us thank Nicolas Balacheff (France), Martin Cooper (Australia), Rosamund Sutherland (Great Britain) and Virginia Warfield (United States of America), for their enterprise which turned out to be successful. We hope the three English native editors will be good vectors for the dissemination of this book in their own countries.

Indeed this book, entitled “Theory of Didactical Situations in Mathematics”, is of value for at least two reasons:

– its content, which is made up of both the huge theoretical framework which has been forged over many years by Guy Brousseau and key examples of his experimental work in the classroom (among them, the process for teaching decimal numbers, chapters 3 and 4, which is very large in terms of units and has been carried out

more than 10 times in several classes)
– and the fact that it is more than a collection of papers, it *is a book*. The editors created a genuine book by not only choosing texts but also recomposing some of them to avoid redundancies, fusing some distinct texts and adding preludes and interludes which clarify the links between the chapters and give some indications about the place and the role of the content of each chapter in the whole work of Guy Brousseau.

2. Contents of the book

The description of the structure of the book reflects the coherence resulting from the editorial work. It starts with an Editors’ Preface followed by a biography of Guy Brousseau which shows how deeply, since the completion of his secondary education, Guy Brousseau has been committed to mathematics education. “How do children learn mathematics?” has been a *Leitmotiv* in his professional life and he was deeply convinced that concepts elaborated within psychology would not suffice to provide answers to the question. The development of a specific field of research on the teaching of mathematics with its own concepts and methods is needed: this has been Brousseau’s claim since the mid-fifties and has constituted a driving force in his work and professional life from the time he was a primary school teacher in a small village of a vineyard region in the Southwestern France until the present where he is a professor at the University Institute for Teacher Education in Bordeaux.

Six chapters constitute the core of the book. Following the current tradition of fine restaurants offering a *mise en bouche* aimed at preparing the customer for the real meal, the editors decided to offer an introduction “Setting the scene with an example: the race to 20”. The race to 20 is a sequence of situations designed by Brousseau in the early seventies. It can be considered as a generic example on which he first built and developed elements of the theory of didactical situations (Perrin-Glorian 1994, p. 106). The aim of this introduction is to provide the reader with a joint presentation of a concrete example and its theoretical interpretation, thus allowing the reader to construct “an intuitive understanding” (editors’ own words, p. 1) of the theoretical elements fully presented in the following chapters.

Chapter 1, “Foundations and methods of didactique”, is a comprehensive presentation of the Theory of Didactical Situations, in which Brousseau synthesizes his theses.

Chapter 2 is devoted to a key constituent of this theory, the epistemological obstacles which are “inevitable and constitutive of knowledge” of the student but may hinder further learning. Chapter 2 is actually composed of two texts: the first one presents the theoretical background of the notion of epistemological obstacle as well as the notion itself and its relationships with the didactical situations; the second one provides more examples making more explicit some necessary conditions attached to an epistemological obstacle and addresses some didactical consequences of the recognition of the existence of epistemological obstacles.

Chapters 3 and 4 present a piece of research “where most of the concepts of the theory of didactical situations have been shaped and also the more obvious evidence of the power of the theory” (Chapter 3 prelude, p. 117): the teaching of rational and decimal numbers at primary school. Chapter 3 “Problems with the teaching of decimal numbers” analyses the problems that French curricula encountered in the sixties and the seventies when faced with the task of introducing pupils to decimal numbers. Brousseau demonstrates clearly in this chapter how the situations designed for fostering learning depend on the characteristics of the mathematical content and of its transformations into a content to be taught (a process called “transposition didactique” by Chevallard). Chapter 4 addresses the question of the design of a teaching sequence allowing the pupil to acquire the notion of decimal numbers under several aspects (in particular, measurement and operator aspects of decimal numbers constructed as rational numbers) and to be able to relate these different aspects. The main characteristics of the processes and situations are proposed as deriving from the epistemological analysis and from the theoretical framework. What is of great interest in this chapter is that Brousseau, after relating conceptions of decimal numbers to didactical choices in Chapter 3, proposes here real alternatives to these conceptions and makes corresponding didactical choices available for the teacher.

Chapter 5 is entitled “The didactical contract: the teacher, the student and the milieu”. When trying to model the construction of meaning by students faced with tasks given by the teacher with the intention of fostering learning of a specific content, the researcher necessarily encounters the problem of interpreting how the students understand the tasks, information and constraints given by the teacher in the context of the culture of the mathematics classroom. The specific habits of the teacher expected by the student and the behaviour of the student expected by the teacher constitute the didactical contract underlying all behaviours of teacher and students in the classroom. Although the notion of didactical contract is present in previous chapters, Chapter 5 analyses the articulation of the didactical contract and the concept of “devolution” and “adidactical” situation.

Chapter 6 concludes the book with an answer given by Brousseau to the question often asked by teachers: “Didactique: what use is it to a teacher?”

An appendix describes the primary school Jules Michelet at Talence, which is a very original instrument of obser-

vation of students and teachers *in vivo*. This is neither an experimental school, nor a pilot school supposed to have advanced pedagogy presented as a model. Outside observation phases there is no imposition of methods or techniques. This appendix is a text written by Brousseau in 1975 in which he explains the concrete organization of research and observation in this school. A bibliography of texts published by Brousseau between 1970 and 1990 is provided and followed by references to papers and books used by Brousseau in the six chapters of the book or added by the editors with the intention of giving more accessible publications. Two helpful indexes are given at the end of the book: an index of names and an index of subjects.

3. Finally, what is it about?

When finishing reading a book and closing it, one is often tempted to find keywords for describing it. In the case of this book, finding keywords in English would constitute a double challenge. The ambition of the purpose, the richness of the content, be it of theoretical or experimental nature, makes the task difficult on the one hand. The very French essence of the work and its originality which reflects the highly creative personality of its author provides an additional factor of difficulty.

Some years ago I enjoyed reading “A Year in Provence” a book reporting humorously the experience of a British couple who decided to buy a house and live in Lubéron. The author could not give an account of the environment they were living in, without using French words. The English native editors of Brousseau’s book must have experienced a similar adventure and shared the decision of keeping in French some words belonging to Brousseau’s terminology (and adopted by the French community). *Didactique*, *problématique*, *milieu* are examples of such directly imported words. The funny detail is that *milieu* is an existing English word and that the editors decided to use it although its original meaning is not exactly the specific one of Brousseau’s theory.

Two of these words at least, *didactique* and *milieu*, have known an international life. At the international level, it is known that the French word *didactique* cannot be translated into other languages without losing its flavour of unique ambition to build a global theoretical model of the communication of mathematical knowledge in teaching. This is the first crucial feature of the work of Brousseau which deeply oriented the French *didactique*. The process of learning itself is not the object of the study but the relations between teaching and learning: what kind of learning does a given teaching produce? Brousseau proposed to model this through a systemic approach allowing thus controlled actions on teaching.

Chapters 1 and 5 illustrate very clearly this audacious intention of identifying the phenomena of didactique, and possibly of modelling the interactions between the student and a situation (a problem) in terms of a formal game. “Modelling a teaching situation consists of producing a game specific to the target knowledge, among different subsystems: the educational system, the student system, the milieu, etc.” (p. 47). In the constructed model, the “milieu” is the system opposed to the student in the game, “a

law of nature, that in some ways stops her from the desired result at every move" (p. 53). The notion of milieu is a product of the modelling enterprise of Brousseau which he and French researchers, such as Margolinas (1993) refined over the years, eventually proposing in the late eighties a structure of the milieu in layers similar to the structure of an onion (p. 248). The milieu plays a key role in the theory and cannot be detached from the notion of situation. Situation and milieu are intrinsically linked and cannot be defined independently.

Coming back to the challenge of finding keywords for this book, I would myself propose as a first round the unbreakable pair "situation milieu" and the word "knowledge". A critical feature of the work of Brousseau is certainly the association between situation and knowledge. I would like to make a bet with any reader of ZDM, who has access to the computer files of the book that the proportion of sentences containing jointly situation/milieu and knowledge is significantly high. A central place is given to the situations allowing a specific functioning of knowledge and/or the construction of knowledge by the learner. Brousseau considers teaching "as the devolution of a learning situation from the teacher to the student" (p. 40), the teacher as a player faced with a system made of two subsystems, the student and the milieu (the environment, with which the student interacts). In the student interaction with the milieu, "knowledge is the means of understanding the ground rules and strategies and later the means of elaborating winning strategies and obtaining the result being sought" (p. 40). One of the most important hypotheses of the theory of didactical situations is that "to each piece of knowledge, and perhaps to each function of a piece of knowledge, there must correspond specific situations" (p. 41). One of the key problems *didactique* is faced with is the determination of the critical features of these situations and of the organization of the milieu that will allow the expected functioning of knowledge. Ten or more years later, the notion of situated cognition successfully became widespread. Actually Brousseau not only recognized how the setting or the context structures the way of using knowledge or the construction of new knowledge, but attempted to elicit and distinguish different patterns of the situation or of the milieu with respect to different forms of knowledge (patterns for action, formulation and validation, p. 65 and ff.). Whereas situated cognition is aimed at analyzing various existing situations (often outside an institutional setting), Brousseau's final aim is the design of relevant situations allowing the construction of knowledge by the learner. Brousseau also investigated historical creation of knowledge in relation with the economy it brought in given situations in which it was used. As an illustration of this kind of investigation, the exclusive use of unit fractions in ancient Egypt is developed in Chapter 2 as a good candidate for being an epistemological obstacle. Brousseau concludes (p. 107) that "this example shows that an obstacle is made neither by clumsiness nor by really false explanations; it is a legitimate adaptation to precise conditions".

Again this dialectical link between knowledge and situation is part of the well known notion of didactical contract

(Chapter 1, p. 21 and ff., Chapter 5 and Chapter 6, p. 263 and ff.): the constraints of the teaching context modify the meanings of the taught knowledge and of the students' answers to the teacher's questions. The rules of this didactical contract affect knowledge to be taught and are knowledge specific. They become apparent when the implicit rules of this contract are broken, for example, when students give a correct but unexpected answer to a task. Factoring $\sqrt{2}$ in $4x + 2$ is mathematically correct but will not be accepted by the teacher. Solving a construction task in geometry by eye may also produce a correct drawing but not be accepted by the teacher because it is not based on geometrical properties. One of the tasks of research is to elicit the rules of this contract in their aspect specific to knowledge.

In a second round I would propose a further pair of keywords, theory/experimentation, which again is inseparable as the sides of a coin in the work of Brousseau. The impressive theoretical framework may appear very French (and why not sharing a similar ambition to the one of Bourbaki for mathematics?). But actually it has been developed by Brousseau in very deep interaction with experimentation in the classroom. Theory originates from the long teaching practice of Brousseau and again its elaboration calls for experimentation in school.

The whole work of Brousseau results from a competition between a growing theory and a careful and long experimentation in school (Jules Michelet at Talence at first but also other schools when needed). The reader will certainly make sense of the theoretical reflections through the introduction and the two chapters about didactic processes on decimal and rational numbers. Theoretical considerations of Chapter 5 are also illustrated by a teaching situation on subtraction.

4. And what is it for?

Two kinds of books are never read a second time. Books of the first kind are often those bought just for a short time, the time of a travel for instance, and left like newspapers in the room of a hotel or in the train. Books of the second kind are those considered to be of value and carefully stored on a shelf where they may have a long and quiet existence ...

The present book does not belong to either of these categories. I view it not only as a reference book for the large theoretical framework it offers but also as a book which is used in the practice of research. This is a working book. The thorough editorial work contributes to facilitating the use of this book by supplying indexes by subject or name, by adding notes referring to other works and extending the references made by Brousseau himself. One can only regret that the list of references is too eclectic, mixing episodic references and works in *didactique* deeply related to the content of the texts. It could have been categorized into several sub-lists according to the function of these references, to assist the work of the book user.

The theory of didactical situations has certainly been deeply affected by the context of primary school in which it originates experimentally. Some researchers working on more advanced levels (university or college) claim diffi-

culty in finding appropriate situations for the emergence of some concepts like limit of a sequence or for linear algebra. The complexity of the concepts at this level and especially the fact that they often have not been created for solving problems but for generalizing and unifying mathematics is at the origin of this difficulty in use of the theory of didactical situations. Nevertheless numerous aspects of the work of Brousseau, such as the didactical contract, the double process of contextualization/decontextualization, and the notion of devolution have been used in that type of research.

“Theory of didactical situations in mathematics” is a book to stay on the desk and not on the shelf, a book which your colleague in the office next to yours has just borrowed when you are looking for it.

5. References

- Margolinas, C. (1993): De l'importance du vrai et du faux dans la classe de mathématiques. – Grenoble: La Pensée Sauvage
- Perrin Glorian, M.-J. (1994): Théorie des situations didactiques: naissance, développement, perspectives. – In: M. Artigue et al. (Eds.), Vingt ans de didactique des mathématiques en France. Grenoble: La Pensée Sauvage, p. 96–147

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