# WILL "THE WAY THEY TEACH" BE "THE WAY THEY HAVE LEARNED"? <br> Pre-service teachers' beliefs concerning computer embedding in math teaching. 

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Embedding computers in math teaching is not a totally new issue, but the dilemmas related to it multiply more rapidly than the answers are being supplied.
One of the dilemmas that we refer to is related to the success of those who are being taught using a certain teaching approach versus their attitude towards the subject they learn. E.g., Funkhouser (2002-2003) found in his research that students who receive geometry instruction using a constructivist approach by means of computeraugmented activities do achieve stronger gains in knowledge of geometry concepts than students who receive more traditional geometry instruction but they do not develop more positive attitude toward mathematics than students who receive a more traditional approach. Actually, for the group that studied math in a traditional way it was one of the preferred subjects, unlike the group that studied it following the constructivist approach with computerized tools. Similar phenomenon was described by Poohkay \& Szabo (1995) who studied the achievements and attitudes of preservice teachers in a primary school program in three groups. One of the groups was taught using animation, the second one was taught using computer, the third one used texts only. The first two groups gave the same grade to the form of instruction they obtained which was lower than the grade the participants of the third group gave, though the achievements of the third group were the lowest of the three.
Norton, McRobbie \& Cooper (2000) say that in spite of availability of technology the secondary school teachers rarely used computers in their teaching. They investigated the reasons for this phenomenon. The results indicate that individual teachers' resistance was related to their beliefs concerning math teaching and learning and the existing pedagogies, including their views on examinations, concerns about time constraints, and preferences for particular text resources. It was also found that teachers with transmission/absorption images of teaching and learning and teachercentered, content-focused pedagogy had a restricted image of the potential of computers in mathematics teaching and learning. By contrast, one teacher with images of teaching consistent with social constructivist learning theory and a learnerfocused pedagogy had a broader image of the potential of computers in mathematics teaching. Further, staff discourse was also found to be important in determining whether computers would be used by students to facilitate their conceptualization of mathematics. These findings have implications for professional development related to the integrated use of computers in mathematics teaching.

Hazzan (2002-2003) related to the attitudes of prospective high school mathematics teachers toward integrating computers into their future classroom teaching. She found that many of the prospective teachers have added a remark in the following spirit "It is worth integrating learning with computers together with learning and teaching without computers". Following Hazzan, a declaration like this indicates that the teachers-to-be don't expect that the computers will resolve all the problems related to the math teaching and learning. They rather tend to consider seriously combining computer in math teaching. Moreover, the students who have already been exposed to the school reality, point at the fact that more experienced teachers hardly encourage their younger colleagues to introduce teaching novelties.
We shall base our reference to the situation on findings both of previous research and on our own one. Our research population consists of pre-service and in-service teachers of primary and secondary school programs. We would like to point out that the link between purposes of a math teaching program and courses constructed in view of these purposes are not always reflected in students' views on these links, see e.g Patronis (1999).

In our opinion, the role of computerized tools in math learning and in developing the math insight of the students is related to numerous factors:

- The rapidly developing computerized environments demand appropriate changes in didactic approaches to be adopted by the teachers or even to be developed by them anew "in real time".
- Variety of math assignments implies a teacher's ability to fit properly a tool to an assignment. E.g., there are several geometric tools of different levels of complexity, those more complex demand more skills to operate them effectively, and the less complex are also less efficient. There are also several computerized algebraic environments etc., some specific topic-oriented computerized tools etc.
- The teacher's professional knowledge must include mastering the variety of computerized tools, mathematical knowledge that would render him flexible enough in his attitudes and responses to the outcomes of the student's activities, and versatility in combining math tools with computerized tools in and optimal way, the optimality itself being a versatile concept.

Keeping all these in mind, we have been asking ourselves for some time: "What is the most appropriate way to prepare instrumentally and mentally the future math teachers to the reality demanding permanent competent adjustment to rapidly developing computerized environment in math teaching?"
In order to try to refer to this question at least partially, we designed a research project in which we studied the performances of several groups of students, studying several courses at different levels of mathematical knowledge and embedding a variety of computerized tools. Moreover, we induced all of the students to experience at least two-three different computerized tools in different courses during three years of their main education program. The embedding of computerized tools occurred in courses in mathematical subject matter courses and in courses in didactics and
pedagogy of math teaching. In addition to these, the students took advanced courses in embedding computers in math teaching.

## Our research questions were:

- How effective are our computer-equipped courses in providing the future math teachers with skills and professional qualifications in appropriate computer usage at least for the beginning of their professional career?
- How our students assess their mastering skills in computerized tools developed as a result of theses courses?
- To what extent has their attitude towards computer embedding changed during their years in the college?
- To what extent do they expect themselves to use computers in their future teaching practice?

The students who participated in the experiment, belong to several categories:

- 31 freshmen of pre-service primary school and secondary school programs.
- 27 sophomores of pre-service primary school and secondary school programs
- 12 freshmen and sophomores of in-service secondary school program, who have had previous experience with computer usage in teaching (not necessarily mathematics).
Students of different categories usually studied together in all the courses, thus we could compare their performances and the impact of the courses on their professional beliefs, and follow the changes that these beliefs underwent as a result of the activities.
Materials and methods:
We used three computerized environments in geometry:
- The Geometric Supposer, The Geometer's Sketchpad, The Word drawing tool;
And three computerized tools for algebraic- analytic courses:
- MATLAB, No-Limits, MathematiX.

The students were presented with assignment sheets which included questions of two types: what we regarded "routine" problems and what we regarded "non-routine" problems. The teaching settings were also of two types: separate computer-usage courses in which the students were being trained to use specific mathematical programs, and math courses in which the computerized tools were embedded accounting for the context of the lesson.
The students got an assignment sheet for about 40-45 minutes without an access to a computer; after that, they were encouraged to use the computerized tools familiar to them to try to solve the problems they had not succeeded to solve, and to substantiate the solutions they had found.
The students were asked to answer whether the computerized tools were used for a better understanding of the problems or/and in order to find a solution.

Referring to the routine problems vs. non-routine ones, we decided not to confine ourselves to open-ended problems as a non-routine type in the spirit e.g. of Takahashi
(2000), though his findings seem to be rather conclusive. Keeping in mind the future vocation of our students, we intended to equip them with approaches that would serve both themselves in coping with problems or mathematical concepts they had never come across, and their future teaching activities, providing them with a precious experience of evolving a concept from the very first steps of acquaintance with it.

Hence, we decided to regard as a non-routine any problem that is not familiar to a student, never mind how routine it may be after future sufficient teaching and exercising. The routine problem is accordingly a problem in a familiar topic and the one the solution to which the student can construct on his own, needing no help either from an instructor or from the computer. In this classification of problem, we account for an important aspect of the Van Hiele theory, which is the development of the insight in the students see e.g. Hoffer (1983). Following Van Hiele, Hoffer defines insight as a merge of three main abilities: a) to perform in a possibly unfamiliar situation; b) to perform competently (correctly and adequately) the acts required by the situation; c) to perform intentionally (deliberately and consciously) a method that resolves the situation. Applying newly learned computerized tools both to routine and to non-routine problems creates a situation in which the mathematical insight is invoked and developed, even if the mathematical problem is originally familiar (routine).

In addition to the tests, the students were asked several questions. As we have mentioned earlier one of these questions was: "Do you think you will use these or other computerized tools in your future teaching activities?"

## Results and observations

In attempt to examine the effectiveness of our computer-equipped courses in the future professional activity of our math students we first studied the way they used the provided computerized environment in a variety of courses and assignments see Gurevich et al (2003).

The students' responses were analyzed and classified according to the group category and the problem kind. Here we refer to the students' answers only concerning the solution of the non-routine problems. The results show that in the groups that studied various mathematical courses combined with the intensive computer usage in about $69 \%$ of cases the students answered that they have used the computerized tools for the better understanding of the problem and in about $93 \%$ of the cases they used computer in order to find the solution. On the other hand in one group where the students were only briefly acquainted with the appropriat computerized tools only about $12 \%$ of the participants admitted that they used the computers for the better understanding and only in $3 \%$ of the cases the computer was used for the solution finding.

We present the selection of typical answers reflecting the common atmosphere and the opinions of the majority of students:
"I had only basic knowledge in computers before the college. Now I can use NoLimits, MathematiX, The Geometer's Sketchpad. I shall use the computers in teaching if it will be technically possible."
"My proficiency in computers did not change, since I was a practical engineer in computers before I came to the college. My attitude towards computers usage in teaching math also did not change, since I have always enjoyed it, and I shall use them if it is possible."
"Before the college, I was acquainted only with basic computer tools. Now I learned to work with No-Limits, MathematiX, The Geometer's Sketchpad, MATLAB, The Geometric Supposer. I master best The Geometer's Sketchpad, MATLAB, and No-Limits. The computer may be very useful for "visual" learners. For example, we have talked and learned about functions, but I actually understood the concept of a limit at a point only when I studied it with the computer. I shall combine computer and the chalk-and-blackboard methods in my future teaching."
" In spite of my positive experience with computers (for the reasons similar to those of the previous student), I am not sure I shall use it in my teaching, since too much technical problems are involved: there are no enough computers for all the pupils, no spare equipped rooms etc."
" The computer shows things that it is difficult to imagine: e.g., logarithmic function, or the sum of angles in a triangle that does not change."
" The computer opened new world for me. But still, when it comes to teaching, I doubt if I shall use it. It is too messy to use with the pupils".
" I am good at computers, especially in MATLAB, No-Limits, MathematiX. These packages demonstrate beautifully the things we have learned, e.g. graphs of functions, geometric constructions etc. But should understand that it does not prove things but rather presents them in an unexpected aspect and thus sometimes facilitates the search for the proof. I do think that one should use the computers in presenting the mathematical concepts, but this should fit the system and the class".

## Some conclusions

Referring to this selection of answers, we observe several common features:

- We observed a qualitative difference in socio-mathematical aspects of the computer usage between math courses and special courses for computer usage in math teaching. The lessons in the first setting were lessons in mathematics, they were centered about mathematical issues and concepts, and the students regarded computers to be another tool, in addition to the chalk-and-blackboard, sometimes a very useful one. The lessons in the second setting were regarded as lessons in computers, and the mathematics seemed to be of minor importance. Thus, the students learned to work with computerized tools, but remembered very little of what it was all about. This led us to the conclusion that all the computerized tools are to be learned and taught in context - only in such a way this is the meaningful way of study.
- As a result of the previous conclusion concerning the need to merge the two forms of courses, the role of the teacher in such a course will also have to change: The teacher must become a mediator between two types of knowledge: disciplinary knowledge in mathematics, and mastering skills of ever developing media. The more the computerized tools develop, the more they may tempt one to replace the rigorous mathematical concept or reasoning by a more or less precise and very pretty picture. Hence the teacher is to be able to lead a enlightened mathematical discussion delimiting and defining the abilities of the tool.
- The teacher-to-be who has undergone the process of coping with non-familiar mathematical situation being aware of the limitations of computer usage in mathematical learning, is apt to search and find appropriate ways of embedding the computer in his future teaching practice.
- The students appreciate the visual contribution to their learning process. In this aspect, it seems appropriate to relate to vast research literature on the concept of preformal proof and pre-formal approach in general to the teaching of mathematics for the students who are at the visual level or are at the transition stage from the visual towards more formal levels see e.g. Blum \& Kirsch (1991), Straesser (2001), De Villiers (1996), Pinto \& Tall (2002). It is also known from the literature (see e.g. Senk (1989), Mayberry (1983)), that an essential part of freshmen in teaching education programs is at the visual level of perception of e.g. of geometrical notions.
- The students claimed that mastering several mathematical packages was essential in their success and thus they supported the embedding of computers in their own learning process. We assert that among the students who participated in the experiment, those who were at lower levels of mathematical (in particular, geometrical) thinking ${ }^{1}$, developed some pre-formal reasoning and proof skills, similar to those they may expect to come across in their future pupils. This has led them to rather positive attitude towards the role of computer in teaching/learning procedures. Among the students who appeared to be at upper levels, we discovered in several of them a gap at the visual geometrical level. These students did not use the computer for their own visualization purposes, but appreciated its potential contribution to the learning of their future pupils who are supposed to use it this way.
- The technical problems related to the computer usage are not negligible. So much so that they may persuade not to apply computer at the classroom even those students who would otherwise be quite enthusiastic about the idea.
- It is important that the lessons in didactics of math teaching include discussions on socio-mathematical norms accounting for contemporary research e.g.Yackel, 2001, Doerr \& Zangor, 2000, Goldenberg, 1999. The students who are also future teachers experience the approach to computer embedding that does not contradict the

[^0]traditional mathematics see e.g. Yerushalmy 1991, Galindo, 1998, Hanna, 1998, but rather enhance some of inductive options on the way to a solution or to a proof.

- Another important aspect is the adjustment of computerized tools used in the math course to the level of the students' mastering the computer. As some of the our students' responds indicate, an inappropriate choice of a tool may obscure the idea of the lesson which is sometimes very elementary and accessible for a relatively simple tool, like Excel or Word drawing tool. On the other hand, the students must become aware of the vast range of opportunities that more sophisticated tools bring with them, and be able to utilize these opportunities to the maximal extent. Hence, the teacher who is to lead the calls to use the computerized environment is to master a wide range of tools. This equally refers to the teachers in the college and to the teachers-to-be who are students at present.
The absolute majority of students tend to use the computers in their future practice if the technical side enables that. One of the students pointed out that she will be cautious not to appear too innovative, in fear of not being accepted in the math team of the school. Others feared that they might loose the control of the class and prefer to use the computer for teacher-provided demonstration alone. No one emphasized the teacher's high abilities needed for this kind of teaching. This may indicate at the lacking self-image of the students as future teachers and hence their inability to place themselves where they are to be in a more or less near future.


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[^0]:    ${ }^{1}$ We presented all the participants of the experiment group with the Van Hiele tests to appraise their level of geometrical thinking, in particular, in order to relate it to their usage of geometrical packages.

