THE DOTS PROBLEM: THIRD GRADERS WORKING WITH FUNCTIONS¹

Darrell	S.	Earnest
TERC		

Analúcia D. Schliemann Tufts University

In a lesson popularized in a Japanese professional development demonstration (Bass,

Usiskin, & Burrill, 2002), sixthgrade students worked through a problem involving a series of dots increasing in number over time. We gave this problem to third



graders (8 to 9 year-olds) in four classrooms in a Greater Boston Public School, midway through a longitudinal Early Algebra study from grades 2 through 4 (see earlyalgebra.terc.edu and Schliemann et al., 2003). The children participated in six to eight 90-minute Early Algebra lessons during each semester. The intervention involved linear functions, tables, graphs, and algebraic notation and focused on algebra as a generalized arithmetic of numbers and quantities. We highlighted the shift from thinking about relations between particular numbers and measures towards thinking about relations among sets of numbers and measures, from computing numerical answers to describing and representing relations between variables, and aimed at the understanding of arithmetic operations as functions. In our presentation we will describe, on the basis of videotape analysis, children's discussions and representations of the dots problem in the penultimate lesson of their third grade year. Students' visualizations and discussions led to meaningful verbal representations and to use of drawings, tables, and algebraic notation that demonstrated their understanding of the function at play. Their work across multiple representational systems provides evidence that, when given the opportunity to work with algebraic concepts and representations, third graders can develop a rich understanding of functions and can represent and solve problems usually taught to be accessible only to older children. Our results support proposals that algebra should become a central part of the elementary school mathematics curriculum.

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References:

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