# A FRAMEWORK FOR THE COMPARISON OF PME RESEARCH INTO MULTILINGUAL MATHEMATICS EDUCATION IN DIFFERENT SOCIOLINGUISTIC SETTINGS 

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The effects of multilingualism have been an explicit focus of a number of PME research reports in recent years. These reports, however, are located in a wide range of socio-linguistic circumstances, making it difficult to compare findings and develop a clearer understanding of the relationship between the teaching, learning or understanding of mathematics. In this paper, I describe a framework that organises the different socio-linguistic settings in which multilingual mathematics classrooms are commonly found. I use this framework to analyse recent PME research reports that focus on multilingualism in mathematics education. My analysis shows that, although the English language has a strong influence in a range of settings, the manifestation of this influence varies.

## RESEARCH INTO MULTILINGUAL MATHEMATICS EDUCATION

The prevalence of multilingualism (including bilingualism) in mathematics classrooms around the world is increasingly reflected in research in mathematics education. Research reports at PME meetings in the past 10 years include several concerned with different aspects of the relationship between multilingualism and psychological dimensions of the teaching and learning of mathematics. These papers report research from many parts of the world and with a range of foci, including, for example:

- Clarkson's (1996; Clarkson and Dawe, 1997) research into how multilingual learners from non-English-speaking backgrounds make use of their different languages in solving mathematics problems in Australia;
- Hofmannová et al.'s (2001) research in the Czech Republic into the development and implementation of a curriculum in which mathematics is studied using a language from outside the country;
- Khisty's (2001) ethnographic study of how different languages are used in English/Spanish bilingual classrooms in the United States;
- Setati's (2003) investigation into the relationship between the politics of language and language use in mathematics classrooms in South Africa.
As these examples suggest, PME research in the area of multilingual mathematics education is highly diverse. In this paper, I will focus, in particular, on sociolinguistic setting, that is, the constellation of languages available and used within different parts of a society, and the different power and values associated with each of these languages. It is clear that PME research in this area has been conducted in a wide range of sociolinguistic settings. Such settings include, for example, classrooms in
which many languages are used (e.g., South Africa), and, in contrast, classrooms in which only one language is used, despite the presence of multilingual students (e.g., Australia). This diversity can be both a strength and a weakness. Diversity can be a strength, in that the dangers of generalising from particular situations, or of privileging particular languages or issues are avoided. Research conducted in a range of settings potentially provides a broader picture of the role of multilingualism in the teaching, learning and understanding of mathematics. Diversity can be a weakness, however, if it becomes difficult to build up such a picture, particularly when the number of studies reported remains low. Much of the research, moreover, is concerned with particular issues arising from particular settings. Findings are likely to be highly circumscribed by the particular setting in which the research was conducted. Cummins (2000, pp. 43-44) has argued, for example, that broad social factors, such as sociolinguistic setting, are implicated in patterns of classroom interaction. A current problem for research within mathematics education, however, is that there is no way of comparing, contrasting or otherwise analysing different studies on the basis of sociolinguistic setting. In the rest of this paper, I propose a framework which makes such comparison possible and offer some initial analysis of PME research in this area.


## FRAMING SOCIOLINGUISTIC SETTINGS

In applied linguistics, a number of ways of classifying sociolinguistic settings of multilingual education have been proposed (e.g., Skutnab-Kangas, 1988; Baker, 2001, p. 194), many of which are focused on the different institutional approaches to the teaching and learning of second or additional languages (L2), such as second language immersion, for example. This approach does not easily transfer to consideration of classrooms where the focus is on the teaching and learning of mathematics, rather than language. An alternative approach, based on Siegel (2003, p. 179) is to focus on the role of the learner's L2 in the society in which the classroom is situated. Siegel describes 5 different settings using this approach:

Dominant L2: The main classroom language is the dominant or majority language in wider society. Multilingual students are speakers of minority languages, such as many immigrants or indigenous peoples. E.g. Turks learning German in Germany; Native Americans learning Spanish in Peru.
External L2: The main classroom language is a foreign or distant language. Multilingual students are speakers of the dominant language. E.g. Japanese learning English in Japan; English speakers in Western Canada learning French.

Coexisting L2: The main classroom language is a nearby language spoken by a large proportion of the population. Students are from a broadly multilingual environment. E.g. German speakers learning French in Switzerland.
Institutional L2: The main classroom language is an indigenous or imported language with a wide range of official uses. Students speak several local languages and inhabit highly multilingual environments. E.g. learning English in India; Swahili in Tanzania.

Minority L2: The main classroom language is that of a minority group (indigenous or immigrant). Students are speakers of the dominant or majority language. E.g English speakers learning Welsh or Panjabi in the UK.

In Siegel's framework, the five settings describe most situations in which school students may use or learn an L2. The term L2 should be seen as referring to any additional language: the framework does not preclude the use of more than two languages. The framework offers a way of analysing research in mathematics classrooms in different sociolinguistic settings. It is not, however, a precise description of interaction in a classroom. Many classrooms in South Africa, for example, officially use English as the medium of instruction and would be classified as 'Institutional L2' but this does not mean that other languages are not used by students or teachers during mathematics lessons. Finally, different settings may apply within the same geographical area. In the UK, for example, there are examples of mathematics classrooms within the dominant L2 (e.g., with immigrant communities), minority L2 (e.g., English speakers learning Welsh) and external L2 (e.g., French immersion) settings. The framework is, therefore, probably best used at the level of individual classrooms, rather than whole communities or schools.

## COMPARING PME RESEARCH ON MULTILINGUAL MATHEMATICS EDUCATION

I have located examples of relevant PME research reports within Siegel's framework (see table, below). I have included all research reports with a clear focus on the role of multilingualism in different aspects of the psychology of mathematics education presented at PME conferences in the past 10 years. I have not included reports in which multilingual issues were tangentially noted or referred to. Nor have I include reports in which the focus was on the relationship between the structure of a language and students' mathematical learning. This survey resulted in the inclusion of 13 research reports.
In applying the framework, I have modified one of the categories. I have divided dominant L2 settings into 'monolingual' and 'bilingual' forms. The former refers to dominant L2 settings in which English is the main language of the curriculum and of classroom interaction, as in the UK, for example. Bilingual dominant L 2 settings are those in which both learners' L1 and L2 are legitimately used in the mathematics classroom (a scenario that does not generally occur in language-focused classrooms). Examples include Spanish-English bilingual mathematics classrooms in the USA, where both English and Spanish are used.

| Mathematics classroom setting | PME research reports | Location | Focus |
| :---: | :---: | :---: | :---: |
| Dominant L2 (monolingual) | $\begin{aligned} & \text { Barwell (2001; } \\ & \text { 2003) } \end{aligned}$ | UK (mainstream classrooms) | How learners of English make sense of word problems when the classroom language is English |
|  |  <br> Prabhu (2000) | USA (ESL classrooms) | Relationship between learning algebra and learning English as a second language (ESL) |
|  | Clarkson (1996) <br>  <br> Dawe (1997) | Australia (mainstream classrooms) | Comparing attainment of bilingual and monolingual students; languages learners use 'privately'; reasons for 'switching' between languages in problem solving |
| Dominant L2 (bilingual) | Khisty et al. (2003) | USA <br> (Spanish/English bilingual classrooms) | Role of multimodality in a bilingual mathematics lesson |
|  | Khisty (2001) |  | Features of effective teaching in bilingual mathematics |
|  | Moschkovich, J. (1996) |  | 'Discontinuity' and 'situated' models of bilingualism in mathematics classrooms |
| External L2 | Hofmannová et al. (2003) | Czech Republic (English-medium classrooms) | Emotional barriers of students training to teach mathematics in English in the Czech Republic |
| Coexisting L2 | NO REPORTS |  |  |
| Institutional L2 | Adler (1995) | South Africa (multilingual classrooms) | Multilingual learners' ability to express themselves; how a teacher supports them to do this; challenges for teaching |
|  | Prins (1997) |  | Multilingualism, problem solving and problem readability |
|  | $\begin{aligned} & \text { Setati (1999, } \\ & 2003) \end{aligned}$ |  | Politics of language and teachers' use of different languages and language practices in mathematics lessons |
| Minority L2 | NO REPORTS |  |  |

Table 1: PME research into multilingual mathematics education and sociolinguistic setting

## APPLYING THE FRAMEWORK

My first observation is that two settings are not represented in PME research. The minority $L 2$ setting involves mathematics classrooms in which the main language used is a minority language within wider society. Whilst there has been research in such settings, such work has not been reported at PME meetings. This absence may be because such research draws more on sociological, anthropological or sociopolitical frameworks, rather than the explicitly psychological perspectives seen to be favoured at PME. This does not mean, however, that there are not important issues relevant to PME research. How, for example, is mathematical understanding influenced by the use of what are generally less widely-used languages? How are learners' motivations to study mathematics related to the use of such languages? The co-existing $L 2$ setting is also not represented, perhaps reflecting its geographical confinement to one or two locations (e.g. Switzerland, Québec). The research reports I have identified are fairly evenly distributed between the remaining 4 settings. In the rest of this paper, I critically compare the research reported from the three settings represented by more than one report: monolingual dominant L2, bilingual dominant L2 and institutional L2.
The dominance of English in the monolingual dominant L2 setting is reflected in the research reports. Clarkson (1996) compares the performance of bi/multilingual students with monolingual English-speakers, setting the latter as the norm. The students' home languages, such as Vietnamese are portrayed as 'other'. Clarkson seeks to show how these 'other' languages are used by students in solving arithmetic problems. Indeed, his research implies that these languages are largely used covertly. Czarnocha \& Prabhu (2000) are interested in how students' mathematical learning can contribute to their learning of English. Similarly, the research reported in my own papers reflects the general absence of languages other than English in the mathematics classrooms reported, despite the students being speakers of one or more languages other than English. It is apparent that both questions and findings in these research reports are closely related to the setting in which they are located.
The three papers from the bilingual dominant L2 setting are all from the USA, where the use of two languages such as Spanish and English to teach mathematics has been common. Again, the dominance of English is apparent. There is a concern, for example, that students should appropriate mathematical ways of talking, that is, mathematical ways of talking in English. Khisty (2001), for example, gives an example of how an effective mathematics teacher introduces the English word 'congruent'. As Moschkovich (1996) discusses in her paper, in most of the research in Spanish-English settings, the relationship between language and learning is described in terms of 'discontinuities'. In particular, the relationship between English in Spanish is seen as a discontinuity. This approach is problematic in several ways, such as its connection to deficit models of bilingual students, who may be penalised for using 'incorrect' mathematical English. Moschkovich does not speculate on the
origins or persistence of this approach, but it is arguably related to the nature of the bilingual dominant L2 setting, in which Spanish would be seen as an obstacle that must be overcome on the way to learning to do mathematics in English.

The concerns of the papers in the institutional L2 setting are recognizably different from those located in dominant L2 settings. All the contributions come from South Africa, a nation of 11 official languages, with English as the main language of education in most schools. Firstly, it is clear that multilingualism is a clear feature of the research. Indeed, in the case of Prins (1997), the research is a comparison between students who have English as L1, L2 or L3, showing that L3 learners were more likely to score badly on written test items, and that this trend is related to the readability of the items, an essentially linguistic issue. On the other hand, Prins's study, like Clarkson's, treats English as the main language. There was no attempt to use test items in Afrikaans or Xhosa, for example, reflecting the institutional importance of English. This institutional position is also apparent in Adler's (1995) paper, in which she explores how a student struggles to explain his thinking due to a lack of familiarity with mathematical language (concerning triangles) in English. Thus, the influence of English is apparent in both dominant L2 and institutional L2 settings. This influence is due to the power and opportunities associated with English in both settings. Indeed, Setati (2003) shows how the status of English in society can be related to language use in South African mathematics classrooms. The institutional dominance of English manifests itself in its use for more formal and procedural mathematical talk, such as talking through a standard algorithm. African languages, on the other hand, tend to be used in informal talk, including, for example, conceptual discussions of the mathematics involved in a problem.
The power and opportunities of English arise in different ways in the different settings. In the South African institutional L2 setting, although English has some institutional prestige, other languages are widely used. In the monolingual dominant L2 settings of the UK and Australia, languages other than English are silenced. It is notable, for example, that Clarkson (1997) had to ask students if they used languages other than English in working on mathematics; such usage was not generally easily observable. In the bilingual dominant L2 setting represented by Spanish-English classrooms in the USA, Spanish has some institutional recognition and is used in classrooms - a position in between monolingual dominant and Institutional L2 settings. The difference is that Spanish is seen as a stepping stone to English in the US; English is the norm. Dual language mathematics classrooms are part of a system designed to turn students into competent speakers of English.

The differences in the manifestation of the influence of English identified in the above analysis raise questions concerning the teaching and learning of mathematics. What effect does covert L2 use have on students' understanding of mathematics, their relationship with the subject, their motivation and engagement? If a Spanish-speaking students struggles to express their mathematical thinking in English, in a setting in
which Spanish is seen as a stepping-stone to English, how do they then value their mathematical understanding?

## CONCLUSION

Siegel's framework provides a useful starting point from which to develop a more nuanced understanding of the relationship between multilingualism and the teaching and learning of mathematics. The framework facilitates the comparison of research in different parts of the world. Through such comparisons, it becomes possible, for example, to identify phenomena that are specific to one or more setting and those that arise more widely. Recent research reports at PME have been fairly evenly spread around four different settings, although nothing has been reported from two settings.

Finally, most mathematics classrooms around the world are multilingual, in the sense that most classrooms include teachers or students who are speakers of two or more languages in their day-to-day lives. This multilingualism is rarely acknowledged in PME research reports, perhaps because of the difficulty of concisely describing complex settings when these settings do not form part of the focus of the research. Siegel's framework offers a way in which multilingualism can be acknowledged whenever and wherever it occurs.

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