CHINESE WHISPERS – ALGEBRA STYLE: GRAMMATICAL, NOTATIONAL, MATHEMATICAL AND ACTIVITY TENSIONS

Dave Hewitt

School of Education, University of Birmingham, UK

This paper analyses students' written work from an activity based on two well known games in the UK: Chinese Whispers and Consequences. Within this activity students were asked to translate formal algebraic equations into word statements and vice versa. Using the framework of affordances and constraints to offer an account for what the students' wrote, I have identified some tensions between four different aspects of the activity: the grammar of the word statements; notational conventions; mathematical sense-making; and the rules of the activity itself. Through an increased awareness of these tensions I surmise that such tensions are not special to this activity and may be taking place during daily mathematical activity in classrooms.

INTRODUCTION

I have argued elsewhere (Hewitt, 2001) for a distinction between students' ability to work algebraically and students' competence and confidence with formal algebraic notation. It is the latter what often presents a barrier for students, both when being asked to write formal notation and when they have to interpret it. Radford (2000), for example, said that students tended to state generalities in words rather than in formal notation and seemed to lose a sense of the original figures from which the generalisations were made when trying to write in formal notation. Zazkis and Liljedahl (2002) and Neria and Amit (2004) also report students finding it difficult to express themselves in algebraic notation and that it is still not natural for high achievers to use formal notation even after years of instruction. The reverse process of interpreting formal algebraic notation is also problematic for most learners (Sáenz-Ludlow and Walgamuth, 1998). Formal notation has a particular structure determined not by students themselves but by mathematicians long in the past. The reasons behind decisions made are not transparent and so the conventions of formal notation can appear quite arbitrary for students who are then left only with the surface features of the notation. These surface features rarely point to the underlying systemic structure (Kieran, 1989) such as order of operations or commutativity. Zazkis and Sirotic (2004) have used the terms *transparent* and *opaque* to indicate whether a certain property can be seen or derived from a given representation or not. Of course, this is dependent upon the awareness of the individual looking at a representation. For example, certain properties are transparent in the expression $2(3+4)$ for those who know about brackets and implied multiplication and opaque for others. Linchevski and Livneh (1999) have looked at whether students have a structure sense when working with arithmetic statements and Hoch and Dreyfus (2004) have looked at this with respect to algebraic statements and manipulations. The paper I present here looks at ways in which students structure their reading and writing of algebraic equations and written word statements of such equations. In particular the tensions which can exist when asked to translate from one form to the another.

METHODOLOGY

The original interest behind this study was to look at the ways in which students translate word statements into formal algebraic notation and vice versa. The vehicle I used for this was based on two well-known games in the UK. The first is called Chinese Whispers, where someone whispers a message to another person, who in turn whispers what they heard to a third person, etc., until the message goes round a loop and returns to the original person. This person then states the original message and the message at the end of the loop (these usually being different). The second game is called Consequences. This starts off with a group of people (again in a form of a loop) who all write down on their individual piece of paper a boy’s name. Then the paper is folded over to hide what has just been written and passed onto the next person, who writes down a girl’s name. Again the paper is folded and passed on. This continues with the following being written: (i) a boy’s name; (ii) a girl’s name; (iii) where they met; (iv) what he said to her; (v) what she said to him; and (vi) what the consequences were. The paper is then unfolded and each person reads out the whole sequence on their sheet.

For my activity each student received a unique sheet where there was an original algebraic or arithmetic equation (different for each student) given at the top of the sheet. Students were then asked to translate this equation into a word statement (not using numerals or mathematical signs, except for letters such as x), fold over the sheet so that only their word statement could be seen and pass it on to the next person in the loop. This continued with a sheet going round the loop with a different person each time translating from equation to word statement, then back to equation, then word statement, etc., for up to ten translations. Each time only the last person’s writing could be seen.

The equations at the top of each of the sheets were chosen to address particular issues which are not relevant to this paper and so I will not go into detail about these here. Before having one of these issues in mind, I looked through the students’ sheets and noted anything which struck me to be of interest. This was working within a Discipline of Noticing (Mason, 2002) where what I noticed related to the awareness I had at the time of reading and that the reading informed that awareness. In Mason’s language, I not only noticed but also marked and recorded and began to form links between the separate noticings with strong links being given labels. One of those links was the issue of students appearing to stress the visual symbols or words they saw on paper, rather than the mathematical operations which they represented or which were implied through those symbols/words. For example, whether a bracket in $2(3+4)=14$ was seen as merely a bracket, or as an indication of order of operations and that a multiplication was taking place. Since this concerns two different aspects, notation and mathematical structure, and since the some of the students' written equations became notationally incorrect, I did not find either the framework of surface and systemic structure (Kieran, 1989), nor structure sense (Linchevski and Livneh, 1999) particularly helpful for this situation. Instead I began to analyse the students' sheets making use of Greeno's (1994) identification of affordances, constraints and attunements which has been developed recently in a mathematical context by Watson (2004). The grammar of word statements has certain affordances which allow possible readings and parsing of a given statement. There are also constraints due to the rules of grammar, and any particular individual may have regular patterns in the way in which they respond to certain grammatical forms, which are examples of attunements. Of course, mathematical language will have its own grammar, for example it might not have a verb. So the grammar of these word statements will be different to the grammar of a novel.

There will also be affordances, constraints and attunements relating to the syntactic conventions of formal algebraic notation, and again for any mathematical sense-making developed when looking at an equation or a word statement. The notion of affordances, constraints and attunements provided a framework within which I have tried to account for some of the students' writings. In this paper I am not reporting the tracking of individual students through their writing across several sheets, which is where the attunements of individual students to aspects of this activity were to be found. So for this paper I have used a framework for my analysis of affordances and constraints only. Through working with this analysis I re-articulated my area of interest as concerning the apparent tension between four aspects (each of which has its own related affordances and constraints): the grammar of the word statements; the conventions of formal algebraic notation; mathematical sense-making for these statements/equations; and the rules of the Chinese Whispers-type activity. Of course, the extent to which someone will explore the affordances which something offers is down to the awareness they have, likewise the extent to which someone feels constrained will be down to the awareness they have of possibilities going against those constraints. So, although I will talk about affordances and constraints of certain aspects of the activity, the extent to which they were explored or experienced by students will have varied.

Teachers from 17 schools gave one of their classes the activity outlined above. The teachers were following a taught Masters course on the teaching and learning of algebra and were invited to carry out the activity on a class of students with whom some algebra had been taught including the conventions of mathematical notation. The schools were mainly from the West Midlands area and included two selective schools. The classes varied between year 7 and year 10 with just one class being year 11. The National Curriculum levels of the students ranged from four to eight.

DISCUSSION

In reporting some of the detail of what students from a particular school wrote, I will use the convention of putting in brackets after the name of the first person from that school with a letter to stand for the school they were from. I will also include the year of the class and a number which represents a guide to the national curriculum (NC) level to which their teacher felt they were performing (e.g. Dave (C, Yr7, NC6)). Future references to students will assume the same school, year and NC level until I indicate otherwise by including a bracket with the new school letter, year and NC level. Then further students will be assumed to be from this new school, etc.

I will offer a few examples of situations where there appears to be tensions between the grammar, the notational conventions, and mathematical sense-making along with the rules of the activity itself. With nearly half of all the sheets ending with an equation different to the original equations on the sheet, these are not isolated examples of the points being made. However, my interest here is in analysing certain examples rather than making more general quantitative statements and in doing so educate my own awareness as to the tensions which can exist within different aspects of any activity.

Stressing the activity

As a generality, the rules of the activity (to translate from words to equation and vice versa) seemed to dominate over whether the equation students were writing was mathematically correct or not. Eight of the 28 sheets had starting equations which were purely numeric and of those, five were mathematically incorrect equations. After students had completed the activity there was nearly the same percentage of sheets which had their last equation the same as the original one on the sheet, whether those equations were initially mathematically correct or not (52% for correct equations and 54% for incorrect equations). So generally the students appeared to engage with the task of faithfully translating an equation irrespective of its mathematical correctness.

I offer here some examples of when the rules of the activity may have been stressed. Tim (D, Yr9, NC4) read *eight minus brackets two + three equals three* and translated this into $8-(2+3=3$. Here he appears to have carried out a one-to-one translation with each word being translated into a symbol:

eight	*minus*	*brackets*	*two*	+	*three*	*equals*	*three*
↓	↓	↓	↓	↓	↓	↓	↓
8	–	(	2	+	3	=	3

Affordances of the activity allowed students to choose their own way of expressing whilst a stated constraint is that students were to write what they saw (in a different notational form) and not just anything. Tim was successful in that he wrote what he saw and this appears to have overridden a global view of what role a bracket plays within a mathematical expression and the mathematical meaning of the equation as a whole. This is not a perfect one-to-one translation however, as the word *brackets* is plural but a single bracket was written. There is a potential tension here, irrespective of whether Tim felt the tension or not, that plurality is stated yet a pair of brackets ‘()’ are not normally written next to one another in a mathematical expression. Also the words *bracket* or *brackets* do not appear again in the word statement and so multiple left-brackets cannot be closed later on by associated right-brackets. So to follow the grammar of the word statement (more than one bracket) conflicts with the conventional constraints of formal notation and so Tim is in a ‘no-win’ situation.

The next person in the loop, Emily, also appears to have performed a one-to-one translation by writing *open brackets* for the ‘(’ symbol in her word statement of *eight take away open brackets two add three equals three* and did not include a written *closed brackets*. Again, Emily obeyed the constraints of the activity and in doing so broke the constraints of notational convention.

Stressing grammar and notation

Sue (A, Yr8, NC4/5) inherited $3(4+2=80$, after a sequence which had gone: $3(4+2)=18$ → *three brackets fore add two equs eightty* [sic] → $3(4+2=80$. She wrote *three bracket four plus two bracket equals eighty* demonstrating an awareness of the notational need for a second bracket and where it would be placed. In contrast to Tim and Emily, she broke the constraints of the activity – that she was to write in words what she saw in symbolic notation – rather than breaking the constraints of the notation.

The fact that this was a paper activity with no-one checking what was written meant that one of the affordances of the activity was that the rules could be broken! This would be different if, for example, this activity was carried out on a computer with a program that would not accept a statement which included anything extra to what was already there on the line above. So although the activity had stated constraints, as laid out by the rules, the medium through which it was played meant that these theoretical constraints could be ignored. In some isolated examples this was taken to an extreme with one student writing down his own made-up equation which was unconnected to what had come before. In Sue’s case, the mathematical sense was also ignored with no attempt to make the equation mathematically correct, only notationally correct.

Continuing this sequence, Joe inherited Sue’s statement of *three bracket four plus two bracket equals eighty* and wrote $(3)4+2(=80)$ showing little awareness of the notational conventions or the mathematical role that brackets play in expressions. Once again, the affordances of the activity, by being played on paper without some form of checking, meant that the constraints of formal notation could be broken. However, the sequence was then finished in the following way:

Joe: $(3)4+2(=80)$ → Alan: *three in brackets four add two brackets equals eighty* → Nigel: $3(4+2)=80$.

There are many ways in which Alan's statement could be read, three of which are:

three in brackets four add two brackets equals eighty; or
three in brackets four add two brackets equals eighty; or
three in brackets four add two brackets equals eighty

The first was the way in which Alan had interpreted the statement and the second was the way in which Nigel interpreted the statement. I assume that Nigel didn't choose to interpret the statement the third way because this did not fit with notational conventions. The constraints of notation provide a context which can influence the way a word statement is read. The affordances of grammar are quite different to that of formal notation. The latter offers a strength in clarifying a particular way of reading and interpreting. The former offers a strength in allowing exploration of poetry and humour through the fact that there can be different ways of reading and interpreting text. For example, a Christmas cracker joke is as follows:

Why couldn't the skeleton go to the Christmas Party?
He had no body to go with!

The joke would not be possible without two ways of reading *no body*, one where they are associated together to form a single word and one where they are separate.

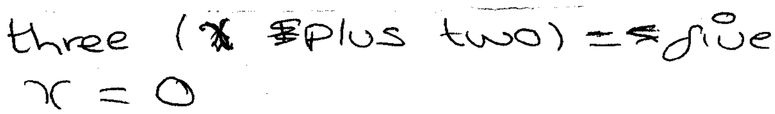
Pimm (1995, p.xii) noted that with plane spotting and car number plates the actual symbols are often the focus of interest rather than the referents and here notation was usually the focus rather than a mathematical operation or property implied by those symbols. For example, consider the following sequence from School C (Yr8, NC6). Note this is after the initial equation of $6x-1=y-2+3$ had already been changed into just an expression:

$6x-y-(2+3)$ → *six x minus y minus 2 add three in brackets* → $6x-y(-2+3)$ → *six x take away y bracket minus two plus three bracket*

Six x is stated rather than *six times x*. The notation is described rather than the underlying mathematics. The use of the words *bracket/brackets* describe a notational sign rather than what it means in terms of mathematical operations. The ambiguity in the word statement of exactly where the brackets are placed has led to an interpretation which changes a mathematical operation and the sign of one of the numbers. An alternative for the last word statement could have stressed the mathematics by saying something like: *six times x minus y multiplied by minus two and three added together*. This is also ambiguous and I am not suggesting it as a preferred option, only as an example of a case where the mathematics is stressed rather than the notation. This was rarely done by any student.

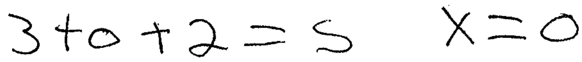
Stressing mathematics

20 of the 28 sheets had starting equations which involved letters. This meant that there was not such an issue about whether these were mathematically correct or not. It was more a matter of whether they were notationally correct. Notation can be considered as 'looking right' even without someone carrying out mathematical calculations in making that judgement. However, even within the equations containing letters there were still examples of students stressing the mathematics. In one sequence the start equation was $3(x+2)=5$ and Bridget (A, Yr8, NC4/5) translated this as follows:



She had broken some of the rules of the activity by using symbols for brackets and the equals sign. Then, having written out the word statement, she appears to have then tried to solve it, which was certainly not within the stated constraints of the activity. Looking at her word statement, if the strange curly symbols of *x* and brackets were ignored then the statement is mathematically correct: three plus two does equal five. So, I interpret that Bridget wanted to ignore the *x* and so she implied it must equal zero. Irrespective of this incorrectness, Bridget considered and tried to address the mathematics of the statement and shown she has made mathematical sense of the equation. An affordance of working mathematically is that new statements can be deduced from other statements and this is what Bridget has done.

Later in the same sequence, Martin translated *three add x add two equals five. x equals zero* to the following:



Here, I suggest that Martin has not solved the equation but substituted in the given value of $x=0$. Again, he has attended to the mathematics of the situation and stressed this above the stated constraints for the activity.

SUMMARY AND REFLECTIONS

Retrospectively I would like to have spoken with the students about their writing and to explore tensions they may have experienced. Partly due to the organic process through which this particular focus appeared, it was too late to go back and arrange for those conversations to take place. However, the existing analysis of the written sheets has raised for me an increased awareness of the tensions between the grammatical, notational and mathematical aspects of this activity. The affordances of one of these gives opportunities which can conflict with the constraints of others. Although this has been an artificially created activity, it has brought to my attention the potential tensions between different aspects of any classroom activity. How often have I heard a teacher (and myself) say to a student "You were not supposed to do that". Yet a student may just have been stressing one aspect of the given activity and exploring the affordances it offers and this has ended up going against the constraints of a different aspect of the activity. It is sometimes difficult for students to find a way of meeting the constraints of each aspect of the activity at the same time. Even if it is possible it may be at the expense of exploring affordances. So a tension arises for us as educators: do we want students to explore affordances? If so, we have to accept that such exploration may lead to conflicts with other aspects of the activity.

References

Greeno, J. G. (1994), 'Gibson's affordances', in *Psychological Review,* 101(2)**,** 336-342.

Hewitt, D. (2001), 'On learning to adopt formal algebraic notation', in H. Chick, K. Stacey, J. Vincent and J. Vincent (Eds), *Proceedings of the 12th Study conference of the International Commission on Mathematical Instruction: The Future of the Teaching and Learning of Algebra*, Vol. 1, Melbourne, Australia, The University of Melbourne, pp. 305-312.

Hoch, M. and Dreyfus, T. (2004), 'Structure sense in high school algebra: the effect of brackets', in M. J. Høines and A. B. Fuglestad (Eds), *Proceedings of the 28th Conference of the International Group for the Psychology of Mathematics Education*, Vol. 3, Bergan, Norway, Bergan University College, pp. 49-56.

Kieran, C. (1989), 'The early learning of algebra: a structural perspective', in S. Wagner and C. Kieran (Eds), *Research Issues in the Learning and Teaching of Algebra*, Reston, Virginia, USA: National Council of Teachers of Mathematics, Lawrence Erlbaum Associates, pp. p33-56.

Linchevski, L. and Livneh, D. (1999), 'Structure sense: the relationship between algebraic and numerical contexts', in *Educational Studies in Mathematics,* 40(2)**,** 173-196.

Mason, J. (2002) *Researching your own practice: the Discipline of Noticing,* London: RoutledgeFalmer.

Neria, D. and Amit, M. (2004), 'Students preference of non-algebraic representations in mathematical communication', in M. J. Høines and A. B. Fuglestad (Eds), *Proceedings of the 28th Conference of the International Group for the Psychology of Mathematics Education*, Vol. 3, Bergen, Norway, Bergen University College, pp. 409-416.

Pimm, D. (1995) *Symbols and Meaning in School Mathematics,* London: Routledge.

Radford, L. (2000), 'Signs and meanings in students' emergent algebraic thinking: a semiotic analysis', in *Educational Studies in Mathematics,* 42**,** 237-268.

Sáenz-Ludlow, A. and Walgamuth, C. (1998), 'Third graders' interpretations or equality and the equal symbol', in *Educational Studies in Mathematics,* 35**,** 153-187.

Watson, A. (2004), 'Affordances, constraints and attunements in mathematical activity', in O. McNamara and R. Barwell (Eds), *Research in Mathematics Education*, Vol. 6, 23-34.

Zazkis, R. and Liljedahl, P. (2002), 'Generalization of patterns: the tension between algebraic thinking and algebraic notation ', in *Educational Studies in Mathematics,* 49(3)**,** 379 - 402.

Zazkis, R. and Sirotic, N. (2004), 'Making sense of irrational numbers: focusing on representation', in M. J. Høines and A. B. Fuglestad (Eds), *Proceedings of the 28th Conference of the International Group for the Psychology of Mathematics Education*, Vol. 4, Bergen, Norway, Bergen University College, pp. 497-504.