SOLVING ADDITIVE PROBLEMS AT PRE-ELEMENTARY SCHOOL LEVEL WITH THE SUPPORT OF GRAPHICAL REPRESENTATION

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This research offers empirical evidence of the importance of supplying diverse symbolic representations in order to support concept development in mathematics. Graphical representation can be a helpful symbolic tool for concept development in the conceptual field of additive structures. Nevertheless, this symbolic tool has specific difficulties that are better dealt with when graphics are combined with symbolic-manipulative tools like building blocks. This combination showed to be effective in the context of a didactic sequence addressed to students in the beginning of elementary school level and aimed to support conceptual development in the domain of additive structures. It provides a theoretical backing for the proposal of using diverse symbolic representations in concept development in mathematics.

The availability of symbolic representations is considered very helpful in conceptual building, since each particular representation (or symbolic model) allows different approaches of conceptual properties (Vergnaud, 1997; Nunes, 1997). In mathematical education, symbolic representation based on concrete artifacts has been considered specially beneficial, since these artifacts are supposed to allow a concrete-metaphorical approach to abstract principles (Selva, 2003; Da Rocha Falcão, 1995; Gravemeijer, 1994; Bonotto, 2003; Bills, Ainley & Wilson, 2003). In fact, the representational power of concrete devices used as didactic tools is not inherent to these devices per se, but is construed in a social and meaningful context of use (Vygotsky, 2001; Meira, 1998). According to this theoretical approach, the "epistemic fidelity" of representational devices is an *essentialist* idea to overcome.

Cartesian graphics are another symbolic representational support for dealing with quantities and their relations. This specific tool has a widespread use in and out of school context; it allows comparisons, demonstration of tendencies in a serial set of data, with the support of visual-cognitive schemas, like "bigger/taller/longer/ is more". Even though the use of graphics is supported by these perceptual schemas, this representational tool is not easy to be used by children at elementary school level (Selva & Da Rocha Falcão, 2002; Bell e Janvier, 1981; Ainley, 2000; Guimarães, 2002). The present research tried then to propose a didactic approach of graphics at pre-elementary school level, in the general context of additive structures. This didactic effort covered two studies, as described below.

The first study was a clinical-exploratory enquiry about the use of building bricks (like those developed by Lego™ - see illustration 1 below) as manipulative auxiliary

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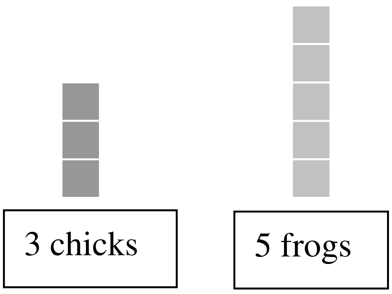
tools for graphics comprehension, these graphics being used afterwards as auxiliary tools in solving additive problems.

Twelve pairs of six to seven year-old students (pre-elementary Brazilian public school level) were presented to a set of situations in which they were asked to use the building bricks by organizing them in piles to represent quantities. Each pair of children worked under the supervision of a teacher-researcher in a working-room at school, in a clinical basis, the complete set of activities being covered in seven sixty minutes long meetings. These activities are summarized below:

1. Familiarization with the building blocks: free manipulation; counting of blocks, comparison of piles of blocks.

2. Representing quantities using blocks and solving additive problems:



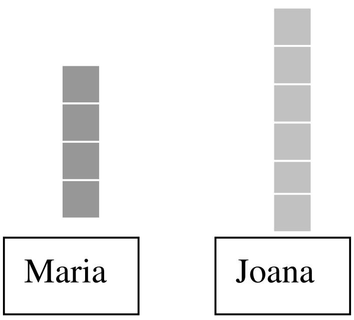
Are there more chicks or frogs?

How many more chicks do we need to have the same amount of chicks and frogs?

How many animals are there in total?

I was told that there were also some beetles in this set of 12 animals. How many beetles were there?

3. Solving comparison problems:

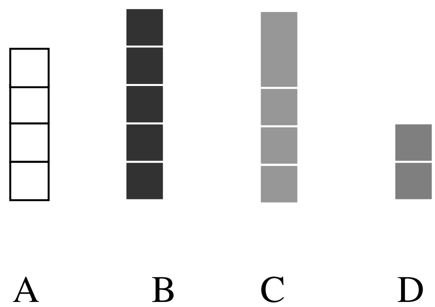


These piles represent the number of school days lost by Maria (4 days) and Joana (6 days).

Who lost more school days? How many school days has Joana lost more than Maria?

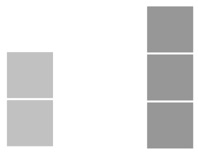
Patrícia, another girl, has had 6 absences. We know she had 2 absences more than Luíza. How many absences has Luíza had?

4. Using different units of measure:



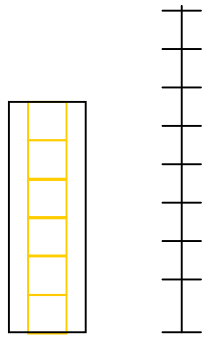
These piles of bricks represent the quantities of absences of four students during the school-year. In the case of C, we used a double-brick that is equivalent to two single bricks. Can we say that B and C had the same amount of absences?

5. Attributing different values to each brick in a pile:

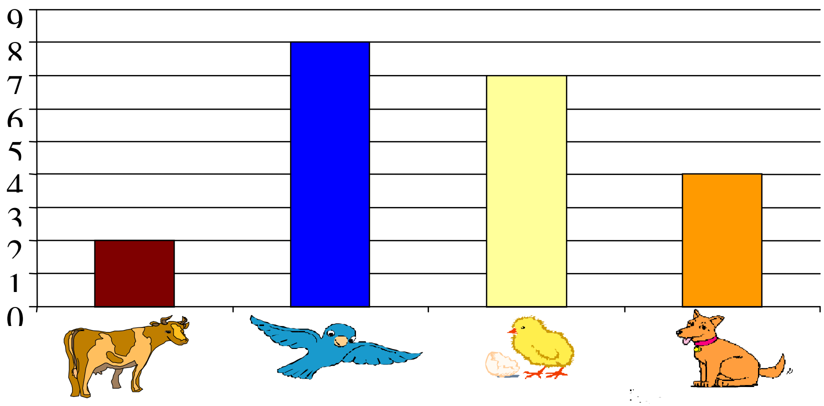


Each brick in the piles representing the pencils owned by Augusto and Pedro stands for 2 units. How many pencils does each boy have?

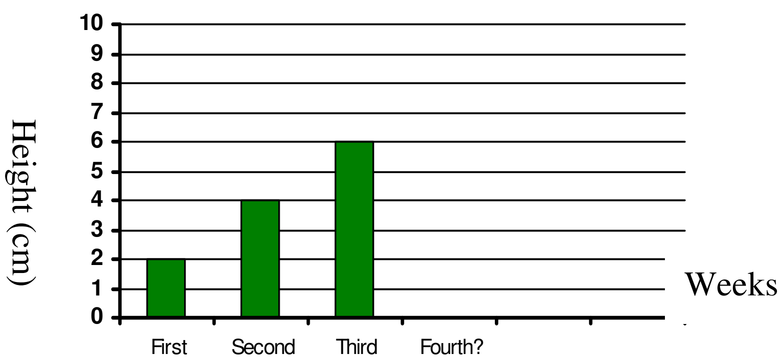
6. Using piles of bricks covered by opaque paper, in order to avoid direct visual inspection of the number of bricks per pile. On the other hand, subjects were introduced to the use of a paper-device for measuring the number of bricks per pile, as shown in the illustration on the right:



7. Using bar graphics in the place of piles of bricks: how many cows are there? What about birds? How many animals in total are there? How many chicks are there more than dogs?



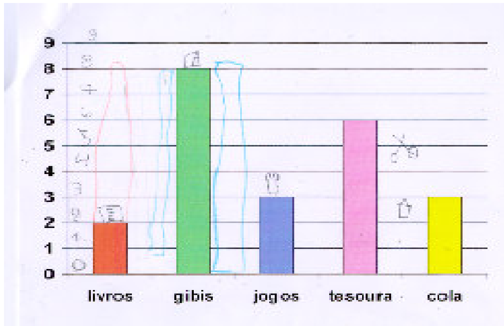
8. Thinking about tendencies in a bar graphic: the growing of a quantity (height in centimeters) during a period (weeks). What is probably going to happen in the fourth week?



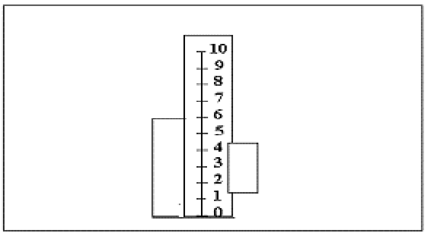
SUMMARY OF RESULTS OF STUDY 1

Clinical analysis of the protocols produced by the pairs in cooperation with the researcher showed that they were able to perform all the activities proposed. Nevertheless, three aspects concerning the use of graphics were sources of difficulty:

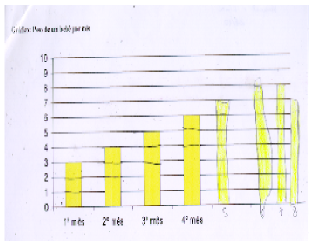
1. Summing up quantities represented by different columns of the graphic: in the protocol on the right, when asked about the sum of books ("livros") and magazines ("gibis"), this pair decided to move the column of magazines to the top of the column of books, in order to see the total amount of books and magazines, but they represented the column of magazines with 6 units (instead of 8), in order to equalize the heights of the original column of magazines and the height of the transported column:



2. Considering different baselines (starting points) to compare columns representing different quantities: this pair of children puts two columns to be compared in different starting points, what makes this comparison task inaccurate.



3. Representing tendencies properly: this pair of children understood that the weight of a baby represented by the graphic on the right was increasing, but could not represent properly the continuation of the tendency (see the two last columns on the right) :

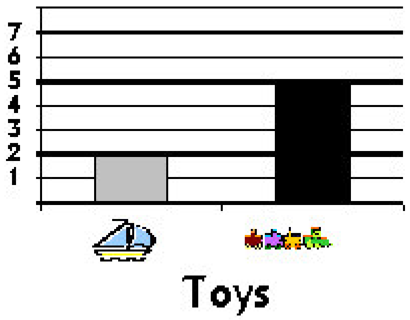


On the other hand, subjects have shown to be able to move from representing quantities through building bricks to doing it through graphics, as suggested by their good performance in activity 7 (see description below). We decided then to test more effectively the didactic importance of building bricks combined to graphics as representational tools for problem solving in the conceptual field of additive structures. In this second study, the research question was the following: is the combination in a didactic sequence between concrete-manipulative representational tools (building bricks) and graphics really helpful in concept development in the conceptual field of additive structures? Or would the proposition of a set of activities concerning the use of graphics (without activities with bricks) allow students to reach an equivalent level of conceptual development? In order to answer this question, twenty-seven children at pre-elementary school level and thirty children in the first year of elementary level, with ages varying between 6 and 8 years, from a private elementary school in Recife (Brazil) took part in this second study. These children were divided, in a controlled way, in three groups: Experimental group 1, submitted to learning activities covering building bricks and the use of graphics, as firstly explored in study 1; Experimental group 2, submitted to activities concerning only the use of graphics, without any exploration of manipulative tools like building bricks; finally, a Control group, submitted only to algorithmic activities involving the same numbers and operations explored by the experimental groups, but without any offer of didactic activity explicitly aimed at conceptual development (for ethical reasons children of control group and experimental group 2 were submitted to the same activity of experimental group 1 at the end of the research). The three groups were submitted to a same pre-test, post-test and delayed post-test (eight weeks after the teaching intervention). Pre and post-tests consisted of a set of thirty problems of combination and comparison, concerning the conceptual field of additive structures. These problems were presented under two forms: verbal-pictorial and graphic. Both forms and structure of problems were randomly presented. Examples of structure and form of representation of the problems are given below:

Structure of problem: comparison

Form of presentation: verbal-pictorial

Problem: A toy-shop has six teddy-bears and two teddy-rabbits. How many more teddy-bears are there than teddy-rabbits?



Structure of problem: combination

Form of presentation: graphic

Problem: A boy has little boats and trains. How many toys has this boy in all?

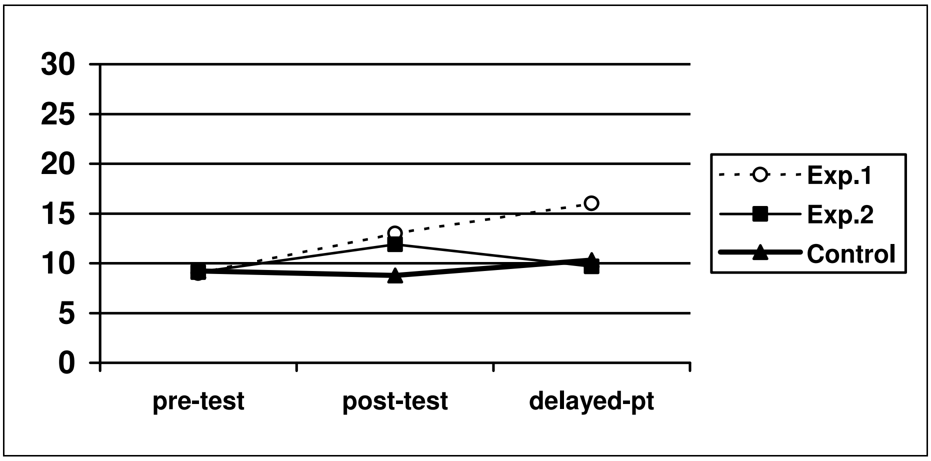
Both pre and post tests were presented collectively, in the classroom. Problems were displayed with the aid of a data-show apparatus, and the children didn't have access to any aid during tests. Each child had a booklet with reproductions of all the questions displayed, where he/she could write their answers.

The didactic intervention proposed for the two experimental groups consisted of the assisted resolution of nine combination problems and eighteen comparison problems. Forms of presentation and structure of problems were randomized, the whole set of twenty-seven problems being presented in three subsets of nine problems in a daily session. Control group, as mentioned before, was invited to solve 27 additive operations in a session (making this calculation activity was a familiar task for them). Children of the three groups were assisted by teachers-researchers during the intervention session, their roles consisting mainly in explaining the activities and encouraging debate and argumentation inside the group.

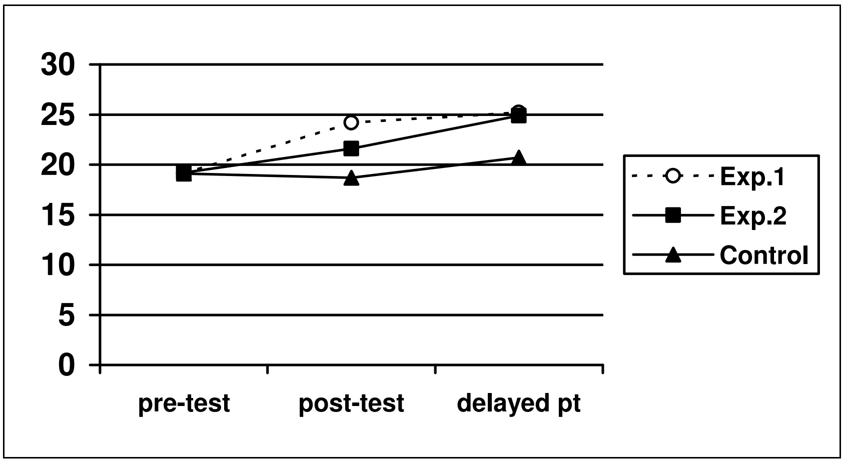
SUMMARY OF RESULTS OF STUDY 2

Performance of children in the post-test was submitted to an analysis of variance having as sources of effect the *school level* (pre-elementary versus first elementary level) and the *group* (experimental 1 or 2 or control). A significant isolated effect of both school level (F=4.61, 1 d.f., p=.037) and group (F=9.552, 2 d.f., p=.000) was observed. Interaction between these two sources of interaction was not observed (F_{inter}= .229, 2 d.f., p> .05). Children from the first elementary level performed significantly better than pre-elementary (difference confirmed by U Mann-Whitney test, U=78.5, one-tailed, p<.000). Children from experimental group 1 (bricks+graphic) and experimental group 2 (graphic) performed significantly better than children from control group (Bonferroni test, p=.000 and p=.033, respectively), but these two experimental groups did not show significant difference when their performance in post-test was compared.

A second analysis of variance was performed having the same factors of previous ANOVA as sources of effect, and performance in a delayed post-test as dependent variable. Results of this analysis was quite similar to those from previous analysis, since isolated effect of school level remains in the same way detected for post-test (U_{M-W} = 99.5 , one-tailed, p<.000), as well as isolated effect of group, this time with a slight difference: significant difference was noticed only between experimental group 1 and control group (Bonferroni test, p=.014). Interaction effects of both sources of variance analyzed were equally non-significant. A closer analysis of data showed that children from pre-elementary school level, experimental group 2 have had their performance lowered from post-test to delayed post-test, which was not the case for children from first elementary level, as suggested by the graphics below:



Graphic 1: Mean level of right answers in pre, post and delayed post-test, pre-elementary level group.



Graphic 2: Mean level of right answers in pre, post and delayed post-test, first-elementary level group.

CONCLUSIONS AND FINAL REMARKS

Empirical evidences gathered here support the general theoretical hypothesis that symbolic representations are relevant in concept development in mathematics. More specifically, the combination of symbolic tools, including concrete-manipulative tools (like building bricks) as precursors of graphics showed to be effective in conceptual development in the conceptual field of additive structures. Nevertheless, representational aids are not completely effective by themselves, since previous development allows different outcomes for the same didactic tools, as shown by decreasing performance of pre-elementary students from post-test to eight-weeks later delayed post-test. As shown by data from study 2, younger students are those for whom the use of concrete-manipulative representational aids are specially relevant for concept development.

Good didactic effects of the combination of symbolic representations, as shown by both experimental groups when performances at pre and post-tests are compared, do not allow theoretical interpretations in terms of the developmental precedence of concrete, more primitive representations over abstract, more developed ones. Representations allow ways of thinking about information, relations and models; diverse availability of representations can be helpful in concept development, as shown by these data. It does not mean that abstract is based upon concrete because of a "natural" order of acquisition *concrete first, then abstract* (as criticized by Vygotsky, 2001). On the other hand, the interest of combining familiar, practical knowledge with incoming new and formal knowledge, in a *metaphorical* way (Lakoff & Núñez, 2000) receives empirical support by the data presented here.

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