ARGUMENTATION PROFILE CHARTS AS TOOLS FOR ANALYSING STUDENTS' ARGUMENTATIONS

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Traditional argument theories focus on how the structure of statements determines their contribution to an argument. Such theories are useful in analysing arguments as products, or for analysing the sub-arguments that are generated during an argumentation. This paper outlines a method for analysing an argumentation as a process, focusing on the social interactions between pairs of Year 8 students and the teacher-researcher in the context of geometric reasoning.

CONJECTURING, JUSTIFYING AND ARGUMENTATION

Over recent decades concern has been expressed that school mathematics focuses on product rather than process, with the result that many students are unable to justify or explain their reasoning. In 1991, for example, the Australian Education Council asserted that

> the systematic and formal way in which mathematics is often presented conveys an image of mathematics which is at odds with the way it actually develops. Mathematical discoveries, conjectures, generalisations, counter-examples, refutations and proofs are all part of what it means to do mathematics. School mathematics should show the intuitive and creative nature of the process, and also the false starts and blind alleys, the erroneous conceptions and errors of reasoning which tend to be a part of mathematics. (p. 14)

Mathematics curriculum statements in many countries (see, for example, National Council of Teachers of Mathematics, 2000) are now emphasising the need for students to engage in conjecturing and to justify their reasoning.

Argument and argumentation

An *argument* may be defined as a sequence of mathematical statements that aims to convince, whereas *argumentation* may be regarded as a process in which a logically connected mathematical discourse is developed. Krummheuer (1995) views an *argument* as either a specific sub-structure within a complex argumentation or the outcome of an argumentation: "The final sequence of statements accepted by all participants, which are more or less completely reconstructable by the participants or by an observer as well, will be called an *argument*" (p. 247). We can therefore distinguish between argumentation as a process and argument as a product. Krummheuer notes that argumentation traditionally relates to an individual convincing a group of listeners but may also be an internal process carried out by an individual. He uses the term 'collective argumentation' to describe an argumentation accomplished by a group of individuals.

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Some researchers, for example, Boero, Garuti, Lemut, and Mariotti (1996), assert that it is only by engaging in conjecturing and argumentation that students develop an understanding of mathematical proof. Boero et al. use the term 'cognitive unity' to signify the continuity that they assert must exist between the production of a conjecture during argumentation and the successful construction of its proof:

> During the production of the conjecture, the student progressively works out his/her statement through an intensive argumentative activity functionally intermingled with the justification of the plausibility of his/her choices. During the subsequent statement-proving stage, the student links up with this process in a coherent way, organising some of the justifications ('arguments') produced during the construction of the statement according to a logical chain. (p. 113)

Boero et al. claim that the reasoning which takes place during the argumentation plays a crucial role in the subsequent proof construction—"it allows students to consciously explore different alternatives, to progressively specify the statement [of the conjecture] and to justify the plausibility of the produced conjecture" (p. 118).

Critics of this conjecturing/argumentation approach to proof assert, however, that the natural language of students' argumentation is in conflict with the logic associated with deductive reasoning. Balacheff (1991), for example, regards argumentation in the mathematics classroom as an invitation to convince, by whatever means the students choose. He asserts that argumentation implies the freedom to convince by whatever means one chooses and hence that there is a contradiction between the natural language of students' argumentation and the logic associated with deductive reasoning:

> The aim of argumentation is to obtain the agreement of the partner in the interaction, but not in the first place to establish the truth of some statement. As a social behavior it is an open process, in other words it allows the use of any kind of means; whereas, for mathematical proofs, we have to fit the requirement for the use of some knowledge taken from a common body of knowledge on which people (mathematicians) agree. (p. 188−189)

More recently, Balacheff (1999) again makes the strong assertion that argumentation is an obstacle to the teaching of proof because of this inherent conflict between mathematical proof [démonstration], which must "exist relative to an explicit axiom system", and argumentation, which implies freedom to choose how to convince:

> The sources of argumentative competence are in natural language and in practices whose rules are frequently of a profoundly different nature from those required by mathematics, and carry a profound mark of the speakers and circumstances. (p. 3)

Responding to Balacheff's views on argumentation and proof, Boero (1999) focuses on the distinction between 'proving' as a process, that is, argumentation, and 'proof' as a product. He notes that from this perspective that the nature of arguments used by students depends on the establishment of a culture of theorems in the classroom, on the nature of the task, and the specific kinds of reasoning emphasised by the teacher.

Boero regards Balacheff's (1999) reference to "the freedom one could give oneself as a person in the play of an argument" as inappropriate, as strong teacher intervention should ensure that students' arguments are based on sound mathematical logic. Hanna (1995) also emphasises that teacher intervention must be a part of any learning methods which encourage students to interact with each other. She asserts, though, that where classroom practice is informed by constructivist theories, evidence indicates that in many cases teachers are not intervening:

> … teachers tend not to present mathematical arguments or take a substantive part in their discussion. They tend to provide only limited support to students, leaving them in large measure to make sense of arguments by themselves. (p. 44)

ANALYSING THE STRUCTURE OF ARGUMENTS

Argument theories such as those of Toulmin (1958) provide a theoretical framework for analysing the structure of written arguments, particularly deductive arguments, as well as the structure of the reasoning that occurs during a process of argumentation. Toulmin asserts that the foundation for the argument (*data*) and the *conclusion* based on this data must be bridged by a *warrant* that legitimises the inference. Toulmin describes warrants as "inference-licences", whose purpose is to show that "taking these data as a starting point, the step to the original claim or conclusion is an appropriate and legitimate one" (p. 98). Toulmin notes that his model for an argument layout is focusing on a micro-argument: "when one gets down to the level of individual sentences" (p. 94). Micro-arguments form part of the larger context of a macro-argument. Krummheuer (1995), for example, applies Toulmin's model to an argument where the conclusions from two subordinate arguments form the data for the main argument.

PROVIDING A CONTEXT FOR ARGUMENTATION

As part of a research study of the role of argumentation in supporting students' deductive reasoning in geometry (see Vincent (2005); Vincent, Chick & McCrae, 2002), 29 above-average Year 8 students at a private girls' school in Melbourne, Australia were presented with a range of conjecturing/proving tasks. Some of these tasks were pencil-and-paper proofs, some were computer-based (using Cabri Geometry II™), and others involved the investigation of the geometry of appropriate mechanical linkages. For the linkage tasks, the students worked with physical models of the linkages as well as with teacher-prepared Cabri models. During the video-recorded lessons, the students worked in pairs to formulate conjectures and to develop geometric proofs. In the context of this research, argumentation was viewed as a social process and the extent to which each participant benefited from engaging in an argumentation was influenced by the level of peer interaction.

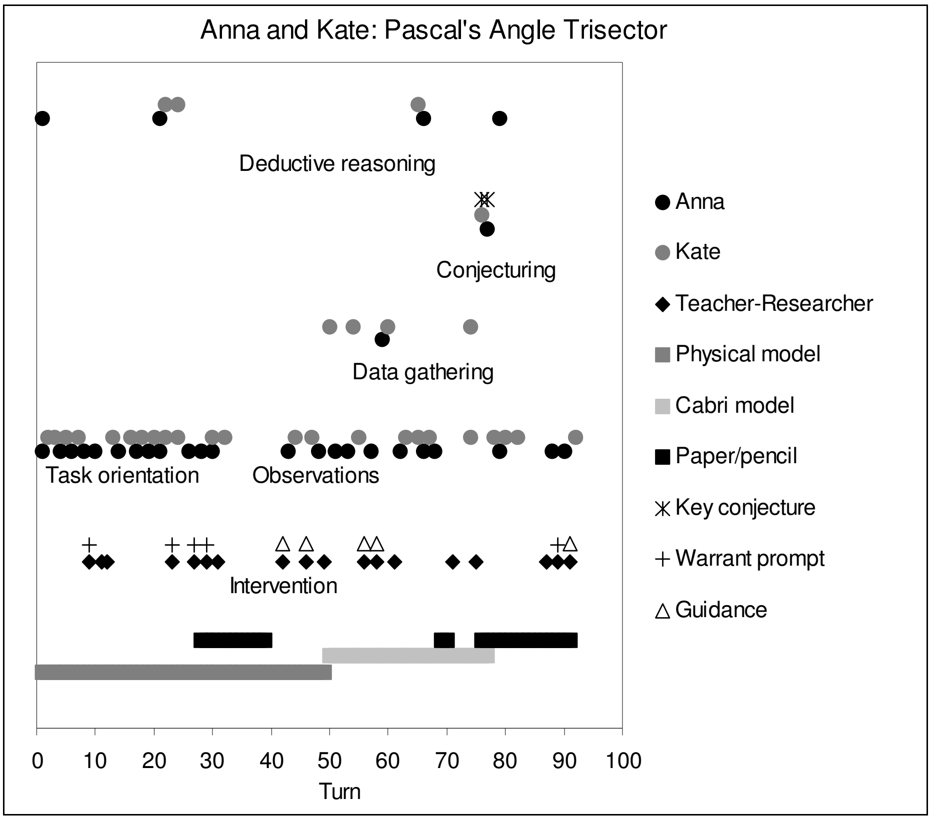
Deductive reasoning was a new experience for these students, and teacher intervention was of paramount importance in the argumentations. Some interventions were merely to clarify the content of the students' statements, answer non-geometric queries, or assist with software related difficulties. Other interventions, however, assisted the students in some way—re-directing the students' thinking if they had reached an impasse (designated *guidance*), for example, "What other things do you know about parallelograms?"; correcting false statements (*correction*), and ensuring that the students' arguments were based on sound mathematical logic. Boero (1999) notes that "the development of Toulmin-type … argumentations calls for very strong teacher mediation" (p. 1). Interventions which I termed *warrant-prompts* were intended to provoke deductive reasoning by asking the students to justify their statements. An example of a warrant-prompt is: "Why do you say that?" in response to a students' claim: "Those two angles are equal".

In general, four different phases of activity could be identified in the argumentations. An initial *observation* phase generally commenced with task orientation, where the students familiarised themselves with the task by referring, for example, to the given data or noting how the mechanical linkage moved. Following this *observation* phase, or sometimes associated with it, was a *data gathering* that led into *conjecturing* and *proving* phases. The phases were not always distinct, and observations and data gathering often continued throughout the conjecturing phase, and statements of deductive reasoning occasionally occurred in the task orientation phase.

ARGUMENTATION PROFILE CHARTS

Toulmin's model was used to analyse the structure of the students' arguments. In order to provide a visual display of the features of each argumentation, however, I devised an argumentation profile chart (for example, see Figure 1). The charts were constructed as X-Y scatter graphs, with speaking turns on the x-axis. Each characteristic to be displayed—the two students and the teacher-researcher, the phases of the argumentation associated with each statement (task orientation, data gathering, conjecturing, proving), and the medium in which the students were working (computer environment, pencil-and-paper, or a physical model of a linkage)—was given a unique y-value.

Figure 1 depicts the argumentations of two pairs of students, Jane and Sara, and Anna and Kate, during their first conjecturing-proving task—an investigation of Pascal's angle trisector (referred to as Pascal's mathematical machine to avoid disclosing its geometric function). The students had access to a physical model of the linkage as well as to a Cabri model, where they were able make accurate measurements and drag the linkage to simulate its operation. Although Anna and Kate's argumentation is more condensed, the structure is similar in each case, with both pairs of students engaging in a large number of observations and requiring substantial guidance. In both argumentations, initial tentative steps of deductive reasoning were supported by further data gathering and conjecturing. Jane and Sara, however, made many unproductive observations and incorrect statements, for example, "… so these two [two angles which formed a straight line] added together would have to equal 90 or something like that" (Sara, turn 58). Both pairs of students moved between the physical model, the Cabri model, and their pencil-and-paper drawings. Anna and Kate, however, did not return to the physical model once they began exploring the Cabri model.



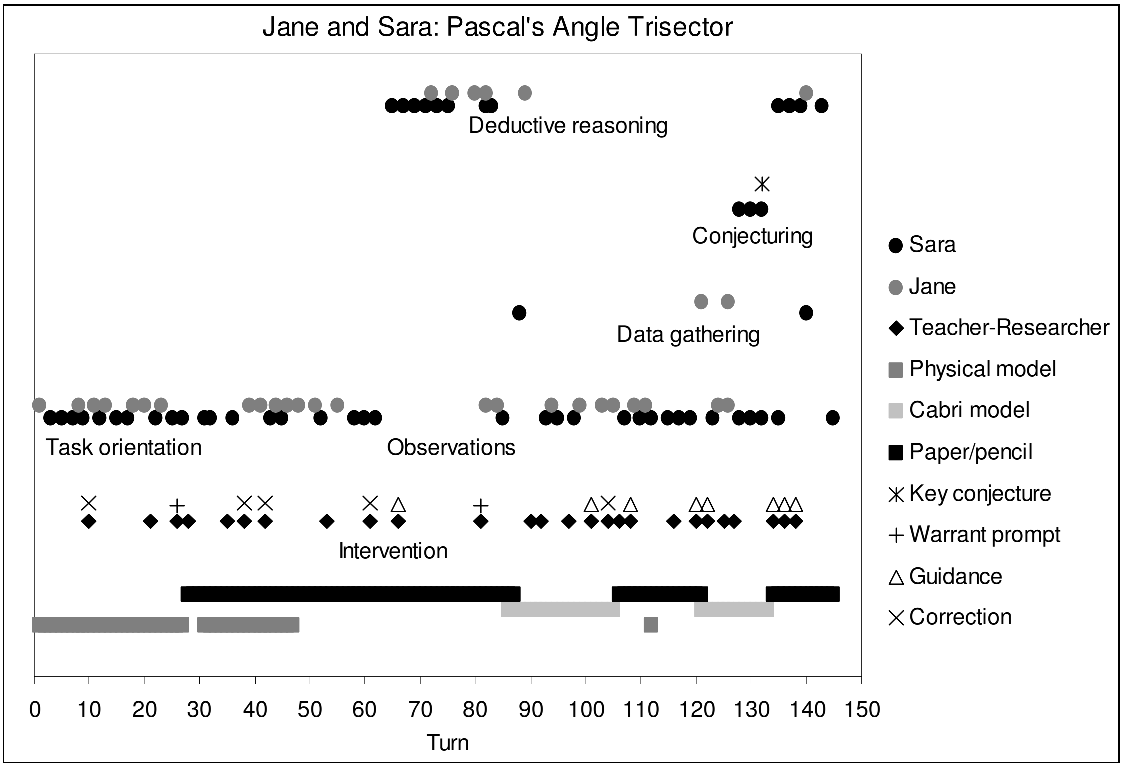


Figure 1

Figure 2 shows the argumentation profiles for the students' fourth conjecturing-proving task—a Cabri-based task in which the students investigated the joining of the midpoints of the sides of a quadrilateral.

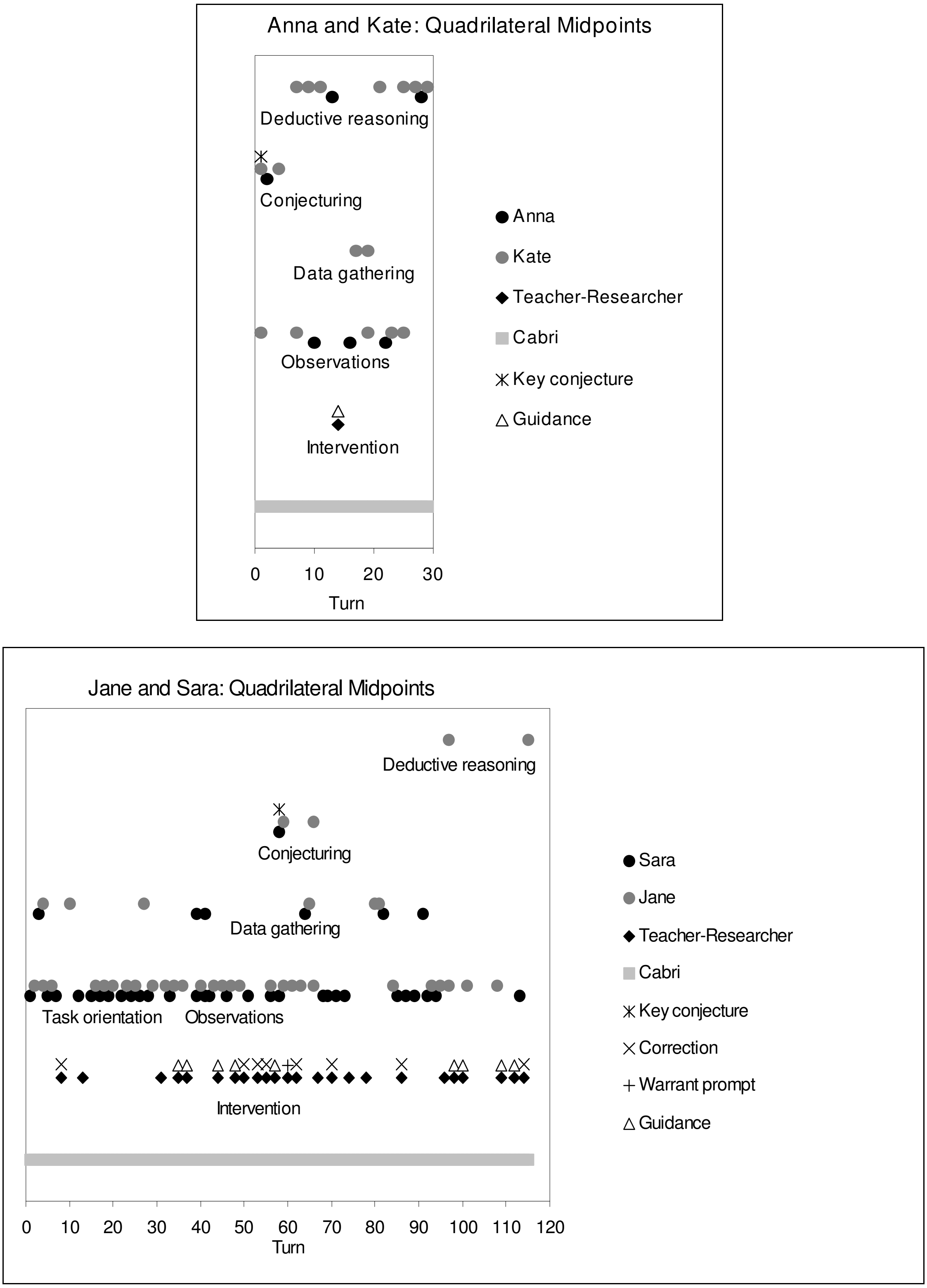


Figure 2

Jane and Sara's argumentation contrasts sharply with that of Anna and Kate, who immediately recognised the parallelogram formed when the midpoints were joined. Anna and Kate completed their proof without the need for teacher guidance, although it was Kate who dominated the deductive reasoning. Jane and Sara, however, focused on other features of the figure and failed to notice the parallelogram until their attention was drawn to it by intervention at turns 37 and 44. They were also handicapped in their conjecturing and arguing by frequent incorrect observations and their lack of confidence with quadrilateral properties and relationships:

69	Sara:	I think it's … um … because the midpoint always stays the same and if the angles of the triangle are always joined to the shape …
70	TR:	Which triangle?
71	Sara:	I mean of the square … sorry … of this … the parallelogram … this parallelogram is always … it's centred … it's in the very centre of the whole shape because of the lines … therefore it stays there.

DISCUSSION

A comparison of Anna and Kate's argumentation profile charts for their first conjecturing-proving task (Pascal's angle trisector) and for their fourth task (Quadrilateral midpoints) demonstrates the development of the deductive reasoning ability of these two students. Further evidence for this development was provided by an analysis of their argumentations and written proofs for other tasks which they completed. The ability of Anna and Kate to engage in argumentation was largely due to their facility with the language of geometry and their understanding of basic properties of triangles and quadrilaterals. It was, however, the process of argumentation which provided these two students with a sense of ownership of their proof. During the argumentation, deductive reasoning statements became ordered so that production of the written proof followed naturally, supporting the claims of cognitive unity by Boero et al. (1996).

By contrast, Jane and Sara were hindered by a poor knowledge of geometric language and properties and substantial teacher intervention was required. However, the process of argumentation did create an environment in which Jane and Sara were able to develop some understanding of the nature of deductive reasoning and to gain a sense of satisfaction from their proof construction.

CONCLUSION

Argumentation profile charts facilitate comparisons of the extent of collaboration between students during the argumentation; the efficiency of the students' data collection, conjecturing and deductive reasoning; and the level of intervention required by different pairs of students, or by the same pair of students in different tasks. By focusing on interactions and the overall structure of an argumentation, that is, on conjecturing and proving as a *process*, the argumentation profile chart can provide valuable insight into how students approach problem-solving tasks.

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