A STUDY OF THE GEOMETRIC CONCEPTS OF ELEMENTARY SCHOOL STUDENTS AT VAN HIELE LEVEL ONE

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This study presents partial results from the project "A Study of perceptual apprehensive, operative apprehensive, sequential apprehensive and discursive apprehensive for elementary school students (POSD)", funded by National Science Council of Taiwan (NSCTW, Grant No. NSC92-2521-S-142-004). It was undertaken to explore the geometric concepts of the elementary school students at the first level of van Hiele's geometric though. The participants were 5,581 elementary school students, randomly selected from 23 counties/cities in Taiwan. The conclusions drawn from this study were: (a) It was easier for students to identify straight and/or curved lines due to the obvious distinctions; (b) Students had difficulties in judging rotate figures because of the direction and position concepts; (c) Identifying circle was the easiest for students, triangle next; quadrilateral was the most difficult one.

INTRODUCTION

Geometry is one of the most important topics in mathematics (Ministry of Education of Taiwan (MET), 1993, 2000, 2003; National Council of Teachers of Mathematics (NCTM), 1989, 1991, 1995, 2000). Geometry curriculum is developed and designed according to the van Hiele model of geometric thought (MET, 1993, 2000, 2003).

In 1957, the van Hiele model was developed by two Dutch mathematics educators, P. M. van Hiele, and his wife (van Hiele, 1957). Several studies have been conducted to discover the implications of the theory for current K-12 geometry curricula, and to validate aspects of the van Hiele model (Burger & Shaughnessy, 1986a; Eberle, 1989; Fuys, Geddes, & Tischler, 1988; Mayberry, 1983; Molina, 1990; Senk, 1983; Usiskin, 1982, Wu, 1994, 1995). Most of researchers focus on the geometry curricula of secondary school. To discover the implications of the van Hiele theory for elementary school students. However, it is also very important. The focus of this study is at the elementary level. This research report is one of the six sessions from the project "A Study of perceptual apprehensive, operative apprehensive, sequential apprehensive and discursive apprehensive for elementary school students (POSD)", funded by National Science Council of Taiwan (NSCTW, Grand No. NSC92-2521-S-142-004).

The main objectives of this study were as follows:

1. To determine the passing rate of each geometric shape.
2. To determine the passing rate of each geometric type.

THEORICAL FRAMEWORK

There are five levels of the van Hiele's geometric thought: "visual", "descriptive", "theoretical", "formal logic", and "the nature of logical laws" (van Hiele, 1986, p. 53). These five levels have two different labels: Level 1 through Level 5 or Level 0 through Level 4. Researchers have not yet come to a conclusion of which one to use. In this study, these five levels were called Level 1 through Level 5, and the focus of this study was on Level 1, visual.

At the first level, students learned the geometry through visualization. "Figures are judged by their appearance. A child recognizes a rectangle by its form and a rectangle seems different to him than a square (Van Hiele, 1986, p. 245)." At this first level students identify and operate on shapes (e.g., squares, triangles, etc.) and other geometric parts (e.g., lines, angles, grids, etc.) according to their appearance.

METHODS AND PROCEDURES

Participants

The participants were 5,581 elementary school students who were randomly selected from 25 elementary schools in 23 counties/cities in Taiwan. There were 2,717 girls and 2,864 boys. The numbers of participants, from 1st to 6th grades, were 910, 912, 917, 909, 920, 1,013 students, respectively.

Instrument

The instrument used in this study, Wu's Geometry Test (WGT), was specifically designed for this project due to there were no suitable Chinese instruments available. This instrument was designed base on van Hiele level descriptors and sample responses identified by Fuys, Geddes, and Tischler (1988). There are 25 multiple-choose questions of the first van Hiele level (Part 1); 20 in the second (Part 2); and 25 in the third (Part 3). The test is focus on three basic geometric concepts: triangle, quadrilateral and circle. The result of the first part of WGT was used in this research report.

Twenty-five questions at level one were characterized into nine types based on its geometric attributions. They are Type 1: identification of open and closed figures, Type 2: identification of convex and concave figures, Type 3: identification of straight line and curve line, Type 4: identification of rotate figure, Type 5: identification of figures of different sizes, Type 6: identification of extremely obtuse figures, Type 7: identification of wide and narrow figures, Type 8: identification on the width of contour line, Type 9: identification on filled and hollow figures.

The scoring criteria were based on the van Hiele Geometry Test (VHG), developed by Usiskin, in the project "van Hiele Levels and Achievement in Secondary School Geometry" (CDASSG Project). In the VHG test, each level has five questions. If the student answers four or five the first level questions correctly, he/she has reached the first level. If the students (a) answered 4 questions or more correctly from the second level; (b) reached the criteria of the first level; and (c) did not correctly answer 4 or more questions, from level 3, level 4, and level 5, they were classified as in second level. Therefore, using the same criteria set by Usiskin (1982), the passing rate of this study was set at 80%.

Validity and Reliability of the Instrument

The attempt to validate the instrument (WGT) involved the critiques of a validating team. The members of this team included elementary school teachers, graduate students majored in mathematics education, and professors from Mathematics Education Departments at several preservice teacher preparation institutes. The team members were given this instrument, and provide feedback regarding whether each test item was suitable or not. They also gave suggestions about how to make this test better.

In order to measure the reliability of the WGT, 289 elementary school students (Grades 1-6) were selected to take the WGT. These students were not participants in this study. The alpha reliability coefficient of the first part of WGT was .6754 ($p <$ .001) using SPSS® for Windows® Version 10.0.

Procedure

The one-time WGT was given during April 2004. The class teachers of the participants administered the test in one mathematics class. The tests were graded by the project directors.

The distribution of the questions is in Table 1.

	Triangle	Quadrilateral	Circle
Type 1: open and closed figure	Q 1	Q2	Q3
Type 2: convex and concave figures	Q 4	Q5	Q6
Type 3: straight line and curve line	Q 7	Q8	Q9
Type 4: rotate figure	Q10	Q11	Q12
Type 5: figures of different sizes	Q13	Q14	Q15
Type 6: extremely obtuse figures	Q16	Q17	
Type 7: wide and narrow figures	Q18	Q19	
Type 8: identification on width of the contour	Q20	Q21	Q22
Type 9: identification on filled and hollow	Q23	Q24	Q25

Table 1: The type and distribution of questions in level one

RESULTS

The passing numbers and passing rate for each type and each geometric shape at level 1 were reported in Table 2.

Overall performance on basic figures

From the data of Table2, the total passing rate was 77.5%. The overall passing rates of the triangle concept were 75.88%, 71.49% for quadrilateral, and 85.14% for circle. It seemed that the circle concept is the easiest one for students, followed by triangle concept, and quadrilateral concept.

	Total		Triangle N=5581		Quadrilateral N=5581		Circle N=5581	
Type 1	12289	73.40%	4072	72.96%	3976	71.24%	4241	75.99%
Type 2	14308	85.46%	4750	85.11%	4181	74.91%	5377	96.34%
Type 3	15642	93.42%	5307	95.09%	4932	88.37%	5403	96.81%
Type 4	13213	78.92%	4522	81.02%	3723	66.71%	4968	89.02%
Type 5	11401	68.09%	3047	54.60%	3713	66.53%	4641	83.16%
Type 6	6122	54.85%	3675	65.85%	2447	43.85%		
Type 7	6537	58.56%	3232	57.91%	3305	59.22%		
Type 8	14088	84.14%	4940	88.51%	4706	84.32%	4442	79.59%
Type 9	13686	81.74%	4570	81.88%	4928	88.30%	4188	75.04%
Total	107286	77.50%	38115	75.88%	35911	71.49%	33260	85.14%

Table2: The numbers passed and passing rate of each type and shape

Overall performance on each type

The overall passing rates, from Type 1 to Type 9, were 73.40%, 85.46%, 93.42%, 78.92%, 68.09%, 54.85%, 58.56%, 84.14%, and 81.74% respectively. It seemed that Type 3 is the easiest one for students, followed by Type 8, and Type 9. Type 6 was the most difficult one, followed by Type 7, and Type 2.

Type 1 (Identification of open and closed figure)

The example of Type 1 questions is shown in Fig. 1. The passing rates of the triangle concept were 72.96%, 71.24% for quadrilateral, and 75.99% for circle. It showed that students could easily identify the open and closed figures in circular concept and have difficulties on quadrilateral.

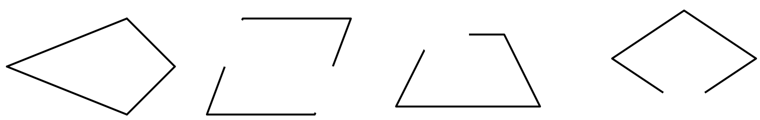

Fig. 1: The identification of open and closed figure

Type 2 (identification of convex and concave figures)

The example of Type 2 questions is shown in Fig. 2. The passing rates of the triangle concept were 85.11%, 74.91% for quadrilateral, and 96.34% for circle. It showed that students could easily identify the convex and concave figures in circular concept and have difficulties on quadrilateral.

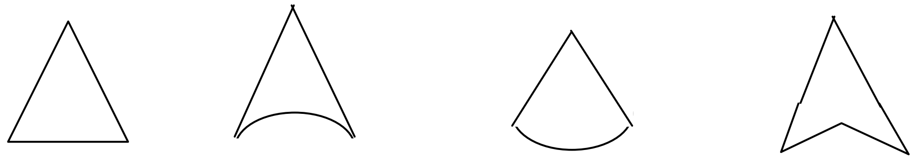

Fig. 2: The identification of convex and concave figure

Type 3 (identification of straight line and curve line)

The example of Type 3 questions is shown in Fig. 3. The passing rates of the triangle concept were 95.09%, 88.37% for quadrilateral, and 96.81% for circle. It showed that students could easily identify the straight line and curve lines in circular concept and have difficulties on quadrilateral.

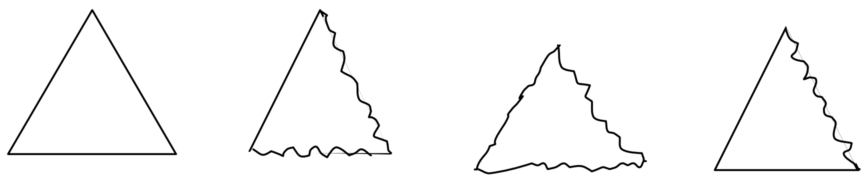

Fig. 3: The identification of straight line and curve line

Type 4 (identification of rotate figure)

The example of Type 4 questions is shown in Fig. 4. The passing rates of the triangle concept were 81.02%, 66.71% for quadrilateral, and 89.02% for circle. It showed that students could easily identify the rotate figures in circular concept and have difficulties on quadrilateral.

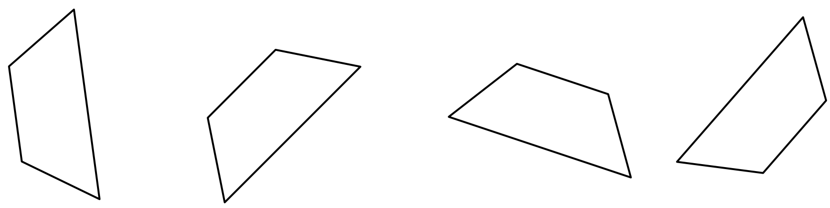

Fig. 4: The identification of rotate figures

Type 5 (identification of figures of different sizes)

The example of Type 5 questions is shown in Fig. 5. The passing rates of the triangle concept were 54.60%, 66.53% for quadrilateral, and 83.16% for circle. It showed that students could easily identify the figures of different sizes in circular concept and have difficulties on quadrilateral.

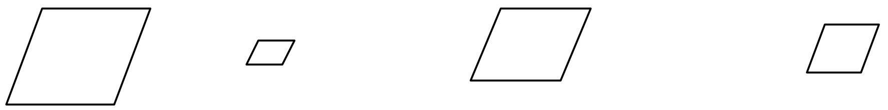

Fig. 5: The identification of figures of different sizes

Type 6: identification of extremely obtuse figures

The example of Type 6 questions is shown in Fig. 6. The passing rates of the triangle concept were 65.85% and 43.85% for quadrilateral. It showed that students could easily identify the figures of extremely obtuse figures in triangular concept and have difficulties on quadrilateral.

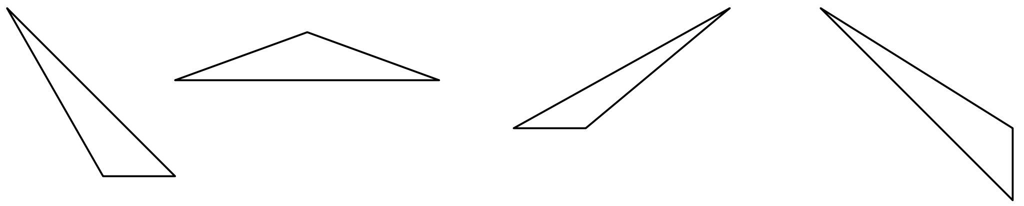

Fig. 6: The identification of extremely obtuse figures

Type 7 (identification of wide and narrow figures)

The example of Type 7 questions is shown in Fig. 7. The passing rates of the triangle concept were 57.91% and 59.22% for quadrilateral. It showed that students could easily identify the figures of wide and narrow figures in quadrilateral concept and have difficulties on triangular.

Fig. 7: The identification of wide and narrow figures

Type 8 (identification on width of the contour line)

The example of Type 8 questions is shown in Fig. 8. The passing rates of the triangle concept were 88.51%, 84.32% for quadrilateral, and 79.59% for circle. It showed that students could easily identify the width of the contour line in triangular concept and have difficulties on circle.

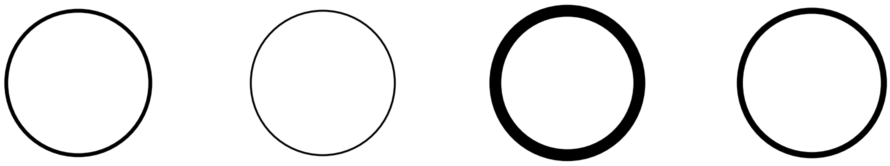

Fig. 8: The identification of width of the contour line

Type 9 (identification on filled and hollow figures)

The example of Type 9 questions is shown in Fig. 9. The passing rates of the triangle concept were 81.88%, 78.30% for quadrilateral, and 75.04% for circle. It showed that students could easily identify the filled and hollow figures in triangular concept and have difficulties on circle.

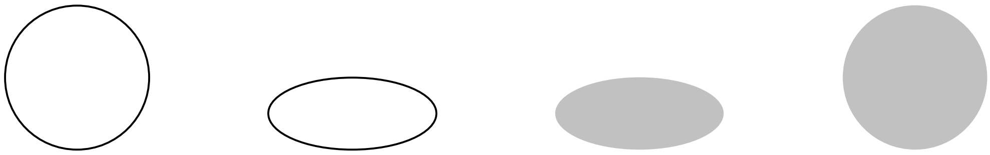

Fig. 9: The identification of filled and hollow figures

CONCLUSION:

At the first van Hiele level (visual), students judged the figures by their appearance. Among these nine different types of figures in this study, Type 3 (identification of straight line and curve line) is the easiest for students and Type 6 (extremely obtuse figures) is the most difficult one. The circular concept is the easiest for students; on the other hand, the concept of quadrilateral is the most difficult to students.

The results of this study identified the easiest and the most difficult concepts for students, it is important to investigate the reason(s) behind this result. The authors of this study are interested to investigate why elementary students have difficulties in identifying extremely obtuse figures. One reason might be that extremely obtuse figures are rarely shown in the textbook, and in their daily lives. Researchers might consider this as their research interests.

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