SHORT ORAL COMMUNICATIONS

MATHEMATICAL KNOWLEDGE CONSTRUCTION: RECOGNIZING STUDENTS' STRUGGLE

Roselainy Abdul Rahman, Yudariah Mohd. Yusof

Universiti Teknologi Malaysia (UTM)

J. H. Mason

Open University, UK

We would like to share some findings of an on-going research on improving students' mathematical learning through reforming classroom practice. The sample of the study is a group of engineering undergraduates at UTM studying Calculus of multivariable functions. The mathematics curriculum is mainly taught as service subjects. It is our belief that the most important attribute that students can carry over to their core study area is the awareness of mathematical processes and problem solving skills. Our presentation will describe the struggle students displayed in trying to make sense and understand the mathematics taught. We could see that part of the struggle is due to existing difficulties the students had such as in working with multiple representations, coordinating procedures, and difficulties in recalling prior mathematical knowledge (Tall & Razali, 1993; Mohd Yusof, Y. & Tall, 1994; Khyasuddeen et al., 1995). We will discuss how we supported our students in enhancing their mathematical understanding through making the mathematical processes and thinking explicit. We had adapted and extended existing mathematical activities and tasks to invoke students' use of their own mathematical powers, assist them in developing these powers further as well acknowledge and address their struggle and difficulties. The pedagogical strategies that we used were devised based on the work of Mason, Burton and Stacey (1985) and Watson and Mason (1998). Excerpts of some students' experience and work will be shown.

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FIGURAL INTERPRETATION OF STRAIGHT LINES THROUGH IDENTIFICATION, CONSTRUCTION AND DESCRIPTION FOCUSSED ON SLOPE AND y-INTERCEPT FEATURES

Dra. Claudia Acuña, DME Cinvestav-IPN

To investigate how high school students use the figural elements, we proposed some tasks of construction of straight lines, to identify of equal or different straight lines in a coordinate system, and to explain conditions under which two different straight lines would be equal. To solve the tasks student need the ability to recognise figural landmarks on the graph, that is, the figural shape of slope is visually associated to the position between two straight lines, this is, how the straight line goes down or up from left to right related with x-axis; meanwhile the y-intercept is a point shared by the given straight line and y-axis. These figural shapes are enough to decide at first glance if two straight lines are the same, but is not enough to recognize slope and y-intercept to accept them as a visual criteria, tasks such as construction, identification or explanation of figural conditions. Our practices show that there are some additional ideas that imposed a different way to see the equality among straight lines. Some additional problems are:

- The Euclidean idea about free straight lines without restrictions can be an obstacle in the interpretation of the figural aspects of the analytical straight line.
- Other kind of obstacles are related to the treatment of the graph as a drawing or as a figure. In this case, students add irrelevant properties taken from the present representation that they even treat them like physical objects.
- Although Gestalt relation is an important aspect of the shape on the plane, students frequently omit it, that is, they joint both figures (grounded and form) in only one; for example, a straight line on the plane looks like a triangle when we join the straight line and axes.
- Most students prefer a good gestalt to graphical composition: although many students used slope and y-intercept as criteria to decide if two straight lines are equal, they base their explanations and constructions on prototypes.

Three different tasks give us three points of observation about figural activity of the straight line. The results of our observation were: About construction the students can achieve a good figural criterion at a global level that allows them to easily obtain an adequate representation. The identification requires focusing on the figural criteria in order to do the adequate election; in this case, the slope and the y-intercept are useful as validation tools for the figural aspect of graphics. Finally, in the task related to the explanation of changes on a straight line that match another one, the figural criterion should be exhibited as a validation tool, but in many cases students avoid this figural criterion in the explanation of changes, and get involved in almost empirical processes based on a natural language, thus, this task has more problems for obtaining right solutions.

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THINKING MATHEMATICALLY: PERSONAL JOURNEY IN THE MODELING OF A CLINICAL WASTE INCINERATION PROCESS

Sabariah Baharun, Yudariah Mohd Yusof, Tahir Ahmad & Khairil Annuar Arshad

Department of Mathematics, Faculty of Science

Universiti Teknologi Malaysia

This paper will share a part of a mathematical journey of the first author in an on-going mathematical research. The main objectives of the research were to model the clinical waste incineration process mathematically and to find suitable solutions to this mathematical model. In this presentation, we will discuss an episode of the experience that we thought to be the most challenging and crucial phase in the problem solving process.

The research work was motivated by the results of the Königsberg bridge problem solved by Euler, solutions of linear equations by Kirchhoff and changes in differential calculus considered by Cayley (Harary, 1969). Comparing and contrasting these applications among others has initiated us to hypothesize that the relationship could be presented as a graph. Thus, the research work begins with the construction of a graphical model to represent the flow of the variables in the incinerator plant (Sabariah et al. 2002a; 2002b). However, the graphical model was found to be an inadequate representation of the phenomenon due to the dynamical nature of the process. It was here that the challenge begins. We adopted Mason, Burton & Stacey's (1982) thinking strategies in our mathematical journey. Reflecting and extending the problem together with long periods of mulling for new insights has led us to continue with the problem solving. Appropriate rubrics, questions and prompts that were used to trigger and to reveal the thinking processes that had helped us to proceed and to get out of the 'STUCK!' situation will be highlighted. Some of the mathematical outcome will also be illustrated.

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THE FRUITFUL SYNERGY OF PAPER & PENCIL AND CABRI GÉOMÈTRE: A CASE STUDY

L.Bazzini* L.Bertazzoli** F.Morselli*

*Mathematics Dept., Univ. of Torino, Italy ** Junior Sec. School, Brescia, Italy

It is widely reported in literature that the introduction of new technologies may change the way of teaching and learning mathematics. Along this stream, several studies deal with the potentialities of new technologies and show how the use of computational environments in teaching can improve students' understanding of mathematics.

This paper focuses on the interaction between two different learning environments (paper & pencil and Cabri) as emerged by the analysis of a teaching experiment carried out in junior secondary school. In the experiment we planned the alternation and integration of the two environments, as classroom culture and conditions allow. A fruitful synergy of the environments emerges along the whole research study, particularly in an episode concerning the production and validation of a conjecture. The activity of two students is presented and discussed.

It is interesting to reflect on the dynamics that take place in the environments and on the role played by drawings and measures. We focus on drawings and measures since they are the source of the production and validation of the conjecture. Furthermore, they represent two "actors" with different characteristics in the two environments: in paper & pencil, the drawing is static, and in Cabri the figure is dynamic; as regards measure, in paper & pencil the process may encompass mistakes, whereas Cabri gives immediate access to a series of measures.

In the first phase of the teaching experiment (i.e. activity in paper & pencil), students' perception, supported by the drawing in paper & pencil, causes an ascending process (formulation of a conjecture); numbers (and calculation) support the descending process, showing that the conjecture cannot be validated. In the second phase (i.e. activity with Cabri), number at first guide the dragging, after they support the production of the right conjecture (ascending process). During this phase, the measures in Cabri allow the students to grasp the relationship existing between the areas of two squares, as required by the problem. Measures also give a first validation of the conjecture (descending process).

We observe that figures and numbers have different value and status according to the environment where action is set; they also have a different role in producing and supporting a conjecture. The study of this role is an interesting issue, worth of development in further research.

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DEVELOPMENT OF A RATIONALE IN A US TEXT AND IN SINGAPORE'S SCHOOL MATHEMATICS TEXTS

Sybilla Beckmann

University of Georgia

The presentation of several standard procedures and formulas in a US 6th grade mathematics textbook and in the texts used in schools in Singapore were examined to determine whether foundational assumptions and definitions were clear and consistent, whether a rationale for the procedure or formula was developed in the text, and how many and what types of problems or activities were provided to help students develop understanding of a rationale for the procedure or formula. Major differences were found between the US text and the texts used in Singapore.

Current reform efforts in mathematics education focus on sense-making, reasoning, and proof. Since curriculum materials are an important factor in classroom instruction, recent research aims to investigate the opportunities that curriculum materials provide for children to engage in reasoning and proving (Stylianides & Silver, 2004). One function of reasoning and proof is to provide a rationale for the procedures and formulas used in solving mathematics problems. Results from the TIMSS 1999 Video Study (Hiebert et al., 2003) indicate that the development of a rationale may be weaker in mathematics lessons in the US than in countries in which children scored higher on the TIMSS assessment.

This study investigated the treatment of several standard procedures and formulas in a traditional US 6th grade text and in the texts used in Singapore. In the US 6th grade textbook, foundational assumptions and definitions were sometimes not clearly specified or were not consistent, unlike in the Singaporean texts. For example, three different definitions were implicitly used in discussing the meaning of fraction multiplication in the US text, but none were tied to each other in any way. Rationales for procedures and formulas were sometimes not given in the US text but were always found in the Singaporean texts. When they were given, rationales were not as fully developed in the US text as in the Singaporean texts.

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HOW CHILDREN SOLVE DIVISION PROBLEMS AND DEAL WITH REMAINDERS

Rute Elizabete de Souza Rosa Borba

Universidade Federal de Pernambuco – Recife - Brasil

This study is part of a greater project that investigated factors that affect division problem solving. The project involved 128 children (aged from 8 to 14 years old) who were asked to solve 16 division problems that varied in meaning for division (partition and quota), in symbolic representation used (tokens, pencil and paper, oral representation or use of calculator), and in size of the remainder (small or large).

The research is based on the same theoretical references and similar methodological approach of previous studies (Borba & Nunes, 2002; Selva, 1995). It is an experimental study based on the theory of conceptual fields (Vergnaud, 1982).

It was analysed how children attending two Brazilian state schools (mean age: 11 years and 11 months) solved division problems with remainder by using pencil and paper. It was observed the representations used, the success in the usage of these symbolic representations and in dealing with the remainder of the division problems.

Most of the children (88%) were able to solve correctly the problems posed and more than half (69%) used the conventional division algorithm. The remaining students used heuristics or drawings (pictorial or similar in form to the objects mentioned in the problems) and most of these were also successful in finding the quotient and the remainder. However, only in 18% of the problems were the remainders treated correctly, splitting remainders in partition problems and increasing the quotient in quota problems, in order to exhaust the total quantities mentioned in the problems.

Division problems need to be discussed thoroughly with children for them to present meaningful answers. Division must be taught related to the study of rational numbers so children can understand what must be done to the remainder of division problems.

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STUDENT MATHEMATICAL TALK: A CASE STUDY IN ALGEBRA AND PHYSICS

Michelle L. W. Bower

Mount St. Mary's University

The aim of the study is to investigate the ways of communicating used by students in mathematics and physics. Language is a critical and potentially overlooked component of classroom culture and the teaching and learning of mathematics. By attending to the ways we talk, we can come to understand what we think and believe. Therefore, discourse, observable dynamic acts of communication in social settings, reflects one's thinking and beliefs about the content of that discourse (Sfard, 2001).

The study design is ethnographic and qualitative. A case study methodology was employed. Data collected include daily observations of algebra and physics classes, observation of groups of students working mathematics and physics, and group student interviews. The length of the study was one semester. Participants were students in one algebra class with an enrollment of 31 students and an introductory physics class with an enrollment of 27 students. The same teacher taught both class sections. The main characteristics of students' talk in the two classes were identified by an iterative analysis process.

Language genres, "talking science" and "talking mathematics" have been identified by several researchers (Chapman, 1997; Lemke, 1982). Student talk, therefore, was categorized in to one of two main types: algebra talk and physics talk. Comparisons were then made of these student talk characteristics.

In algebra talk, student utterances were of two types: tutoring and group problem solving. In physics the students were more likely to be working together to solve the problem and their questions were focused on the underlying concepts and the mathematical calculations. In both student physics talk and student algebra talk, the students' utterances were short, often incomplete, and co-constructed among the group.

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ETHICAL CONSIDERATIONS IN A MATHEMATICS TEACHER EDUCATION CLASSROOM.

Chris Breen

University of Cape Town, South Africa

This short oral will be based on research conducted with a group of students taking a one-year postgraduate course to qualify them to teach mathematics in secondary schools. The two key theoretical underpinnings of the work focus on the issues of listening and the setting up of a community of practice. The issue of listening draws on the work of Davis (1996), who focuses his main attention on developing a different form of listening from the common types of evaluative and interpretive listening – that of hermeneutic listening. Cotton (2002) draws on Lave and Wenger (1991) to examine schools and classrooms as communities of practice. In this way, students in mathematics classrooms engage with each other in practice and develop a sense of self in relation to that community of practice. For some students there is a greater synergy and sense of belonging as they fit in with the group and the teacher's expectations of the class, whereas for others, there is a sense of rejection and little sense of identity within the communities of practice. For those students for whom there is little sense of belonging and a lack of sense of identity, there is greater danger of exclusion from that community of practice.

This report focuses on one particular activity where the class was split randomly into five groups, each containing three members. After introducing the three different levels of listening (evaluative, interpretive and hermeneutic as outlined above) in order to emphasise the importance of listening as a tool for working with others, the student teachers were given the task to work on the Painted Cubes problem as a learning community. They were told that the group's focus should be on the process they developed as they tackled the problem rather than the solution obtained. As an assignment they were asked 'to describe the way in which their community came together and the contributions that the various members made to the experience'. The paper discusses some of the important results and raises the question as to the extent to which mathematics teacher educators need to raise issues of ethical know-how as they emerge in teaching sessions with pre-service mathematics teaching students.

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CECI N'EST PAS UN "CIRCLE"

Tony Brown and Krista Bradford

Manchester Metropolitan University, UK, & Rainbow International School, Uganda

There is a famous painting by Magritte. It depicts a smoker's pipe with the caption "Ceci n'est pas une pipe". The joke was that it was not a pipe; it was a painting of a pipe. The painting has fuelled many discussions about the attachment of signifiers to signifieds: how exactly do symbols and words represent an object? As soon as we enter the domain of language we inevitably move to some sort of ideological frame that, in turn, brings with it a host of filters that condition our understanding of the material we are examining. This paper is concerned with the perception of mathematical concepts and seeks to explore some of the linguistic filters and socio-cultural factors which influence human understanding of such concepts.

The study (presented in full elsewhere, Bradford and Brown, 2005; Atkinson, Brown and England, in press) reports on a teacher's practitioner research which took place at different times within successive modes of immersion in linguistic domains, that she sought to observe, understand, participate within (or resist) and transform through her participation. She recorded successive perspectives on successive actions in her work in a Ugandan school with a focus on how the term "circle" was seen from alternative cultural perspectives. Yet in the research process it was the writing generated by her that provided anchorage to her thoughts, but only in the limited sort of way in which the word "circle" served as an anchor for more mathematically oriented discourse. The word itself was more stable than the way it held meaning. Similarly, the writings simultaneously sought to explain the past and shape the future, but in the meantime provided orientation and a conceptual space for examining how the mathematical terms were being used. Yet each component of this writing was constantly in the process of having its status amongst its neighbours unsettled. The teacher was involved in the production of stories that had a limited shelf life as "stories in their own right". The reflective writings and the mathematical words they contained were historically and ideologically defined entities. The passage of time, however, provided the distance necessary to see the previous frame as being outside of oneself. And of how it had reflexively encapsulated the teacher, the learner and the mathematical objects that they had sought to share.

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DEVELOPMENT AND EVALUATION OF A CONCEPT FOR PROBLEM-SOLVING AND SELF-GUIDED LEARNING IN MATHS LESSONS

Regina Bruder, Evelyn Komorek & Bernhard Schmitz

Darmstadt University of Technology

Knowing how to solve problems is what should be 'kept' from maths lessons, it is part of what constitutes the general educational value of math lessons. Problem-solving contributes to an adequate image of mathematics and covers both a specialist and a cross-curricular component.

From 2002 to 2004 we worked on a material-based teaching concept which is intended for teachers as a guideline on how to learn problem-solving in conjunction with self-guidance. Central part of the concept to integrate problem-solving is to get the pupils used to structured proceeding in the mathematical problem-solving process and to help them find individual problem-solving strategies. Problem-solving elements and strategies of self-regulation were then punctually integrated into successive learning phases of all subjects treated in the maths lessons.

For the development of a teachers training programme a multi-step approach over three project phases, each with another main emphasis, was selected:

At first a comprehensive and practicable teaching concept for the systematic integration of the training contents and intention into the regular maths lessons had to be found, including the development of evaluation tools for the use in subsequent project phases. The Repertory Grid technique was adapted for the registration of subjective ideas on maths problems of the teachers.

The next step was to establish a training programme for teachers in the first training phase. This project was run in a university course at the Technical University Darmstadt and evaluated. The training programmes were then tested in the second phase of the teacher training.

In order to integrate the developed teaching concept, which proved to be acceptable and practicable, in regular maths lessons, concepts for continual teacher training with different support systems have been tested in the present project phase since June 2004.

Evaluation measures were used in each of the three project phases to analyse the variables "acceptance/identification with the concept" and "skills in the application of the concept" as adopted by the teachers.

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MATHEMATICAL PROBLEM-SOLVING IN A SPREADSHEET ENVIRONMENT: IN WHAT WAYS MIGHT STUDENT DISCOURSE INFLUENCE UNDERSTANDING.

Nigel Calder

University of Waikato, NZ

This study is concerned with how investigating mathematical activities in a spreadsheet environment might filter understanding, with particular consideration of variance in discourse. What is the nature of the learning process in this environment, and how might this particular pedagogical medium shape children's approach to the activities?

A group of twenty ten-year-old children developed individual and collaborative approaches to problem solving. Part of the facilitation of this process was their engagement in investigating mathematical situations with spreadsheets. While there was an initial instructional process within a mathematical problem-solving context, one aim of the study was the use of the spreadsheet as an investigative tool, and the implications for understanding this evoked.

Central to the study is the place of discourse. This also involves theoretical perspectives such as phenomenology and its relationship with mathematics education (e.g. Brown, 2001), and the social-constructivist viewpoint (e.g. Cobb, 1994). Hence an ethnographic, interpretive methodology underpinned the research. The children were observed, their conversations recorded, and they were interviewed, both in groups and individually. Comparisons with pencil and paper methods to investigating were also analysed, and attitudinal surveys undertaken. This data provided some triangulation, and enabled a more fulsome picture to emerge.

Preliminary analysis showed interesting insights into the way the participants familiarised themselves with the investigations, and how investigating with a spreadsheet led to particular discourse, and approaches to investigation. Typically, participants proceeded by entering formulas to generate organised tables of data. These structured tables often led, through discussion, to the resetting of investigational sub-goals, and further exploration. Participants also commented that the seemingly unlimited space and the speed of response to inputted data were other aspects that affected their approach. How this might shape the children's understanding, and if distinctive, what might get lost, are areas of on-going examination.

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THE VALUE OF PLAY IN MATHEMATICS LEARNING IN THE MIDDLE YEARS

Rosemaree Caswell
Griffith University

Steven Nisbet
Griffith University

The value of play has been well established in the early years of schooling, however in the years that follow, a transmission approach where the learner is 'drilled' in mathematical concepts and processes often dominates the curriculum. Mathematical play provides an alternative to the transmission model by recognizing the need for pedagogy where sensory-motor experiences, metalanguage and metacognition are employed to support learners in the transition from concrete to abstract. This requires an epistemology grounded in the constructivist approach with open-ended inquiry.

The study focuses on two main goals: to describe activities that constitute 'playful learning' in the middle years and to analyse and explain the elements of play that enhance student engagement in learning and contribute to deep conceptual development. The focus of this research is to understand from a student's perspective the value of play activities in enhancing mathematical understanding.

Within the literature on mathematical play, a clear definition of 'play' is difficult to find. Explanations and exemplars of mathematical play focus on the objects of the learning context, encompass an awareness of interactive cognitive engagement and address the links between affect and learning. Mathematical play is interactive and involves social discourse and domain specific communication. The perception of play activities as pre-abstract is in fact a misrepresentation of the application of sensory-motor stimuli and cuing using visual and kinaesthetic representation.

As the intent of the research was to document and analyse students' reflections on the value of play, a retroductive approach was adopted. The research was a case study of a single primary level class, where students had already been engaged in 'play-based learning'. The 27 students in the class ranged from 9 years to 12 years. Students were observed over a ten week period. At the end of each weekly cycle of activities, students were engaged in class conferences, where they described 'play' and made comments or written reflections on how well the activities supported their learning.

The results of the study indicate that the students believe play activities in mathematics engage all students at their level of understanding. Play activities can be uni-conceptual or multi-conceptual requiring students to make links to other mathematical concepts. They are challenging and diverse, not repetitive. Both 'cognitive conflict' and 'cognitive challenge' were identified through the study as features of play activities that enhance mathematical understanding and application. Play activities allow a continuum from concrete to abstract that engages all students. The study noted that interactive play created a supportive environment in which there was no failure. The responses from the students in the study were overwhelmingly in favour of play activities as an effective learning context.

THE POTENTIAL OF CAS TO PROMOTE CHANGES IN TEACHERS' CONCEPTIONS AND PRACTICES

Tenoch Cedillo

National Pedagogical University, Mexico

This presentation reports a four year study on the potential of computer algebra systems as a vehicle to promote changes in lower secondary school mathematics teachers' conceptions and practices. In 1999, the Mexican Ministry of Education equipped one hundred lower secondary schools spread out in the country with TI92 calculators, such that each student in the school had individual access to the machine in the mathematics classroom at least twice a week.

Method: The categories proposed by Franke et al (1997) were used. These categories allow us to distinguish four levels of teachers' performance and provided a referent to follow up the evolution of the teachers throughout the field work. At the beginning of the project the teachers answered an initial questionnaire inquiring about their previous teaching experience, their teaching method(s) and their professional background. In order to complement these data, an individual interview was administered each year of the project to 30 teachers chosen out from 800 taking part in the study. The teachers were accompanied during three years by professional instructors who worked with them in their respective schools four hours on Friday and Saturday every six weeks. The training program was outlined by teaching materials especially designed for this project and focused on using the calculator as a cognitive tool and discussing ways to define the teacher's and students' role in the classroom (http://sec21.ilce.edu.mx/matematicas/calculadoras/). All work sessions with the teachers were videotaped and used as a data source for the present study.

Results: The chart below summarizes the changes that were registered in the 30 teachers throughout the three years of the study. The evidence provided by the field work strongly indicates that these changes occurred due to the use of CAS.

	Initial profile	1st year end	2nd year end	3rd year end
Level 1	25	22	4	2
Level 2	5	7	21	8
Level 3	0	1	5	10
Level 4	0	0	0	10

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PRE-SERVICE TEACHERS' SELF-EFFICACY TOWARD ELEMENTARY MATHEMATICS AND SCIENCE

Chang, Y. L., Wu, S. C.

MingDao University, Taichung Healthcare & Management University

Beginning with research in the 1970s, teacher efficacy was first conceptualized as teachers' general capacity to influence student performance. Since then, the concept of teacher's sense of efficacy has developed continuously and currently is discussed relevant to Albert Bandura's (1977) theory of self-efficacy, indicating the significance of teachers' beliefs in their own capabilities in relation to the effects of student learning and achievement. Thus, this study was to compare the difference of their sense of efficacy, including two cognitive dimensions, personal teaching efficacy and teaching outcome expectancy, after receiving various long-term programs of teacher training. Two teachers' self-efficacy belief instruments were used for data collection from 340 senior students in ten departments of National Taichung Teachers College, Taiwan.

Both pre-service teachers' sense of efficacy toward mathematics and science were significantly different among these ten programs. Further, both groups of students from Department of Mathematics Education and Department of Science Education had more confidence in their own teaching abilities than other students who did not specialize in either mathematics or science, as well as in providing efficient teaching in the classroom. Moreover, statistically significant relationships were found between efficacy ratings toward mathematics and science as well as all subscales.

In summary, the traditional teacher preparation program designs, oriented in cultivating elementary generalists, are inadequate for accomplishing the requirement of having qualified teachers in every classroom and for every subject area. As students have diverse needs and distinct characteristics, it is truly essential that specialized teachers exist for every subject area in every school. To enhance prospective teachers' sense of efficacy toward mathematics and science, all faculty members of teacher preparation programs should rethink the program design, the curriculum structure, and the content provided and the pedagogy used in preparing them to teach mathematics and science. Further, even though more preparations in these two subject areas are no guarantee of higher quality of pre-service teachers and their better understandings of their subjects, insufficient preparation will definitely result in inadequacy of content and pedagogical knowledge and teaching skills in mathematics and science. This inadequacy will surely have a great influence on the quality of future teachers and the performance of their students and should be the core of ongoing educational reforms.

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STUDENTS AND SOFTWARE: TALES OF ANXIETY, SONGS OF SUPPORT

Patricia Cretchley

University of Southern Queensland

This report offers insights into the views of students facing their first experience of using professional scientific software (MATLAB) for doing and learning mathematics. The data was captured from samples of a population of 508 undergraduate students at different stages of their technology-integrated learning experience in two early undergraduate Algebra & Calculus courses. The findings illuminate and quantify the range of reactions. Examples are offered of the voices of the majority who chorused songs of support for the use of software for learning and doing mathematics in advance of the initiative (more than three quarters of those entering the courses). In contrast, equally important tales of anxiety expressed by the vulnerable minority who felt negative about the prospect of using computer software for learning are reported.

BACKGROUND AND SUMMARY OF FINDINGS

This study forms part of an investigation of learning and attitudes in a technology-enriched early undergraduate learning environment over the years 2001 to 2004. The majority of students (76% of those surveyed early in 2003 and 2004) expressed positive beliefs and attitudes about using software for learning mathematics in their responses to open questions on entry to the course. Typical examples of these "voices" are presented. Affective responses hinged on perceptions of the use of computers as enjoyable, novel and fun, and some students were clearly excited about the technology intervention. Cognitive responses hinged on the time-saving benefits of computer power and efficiency, and opportunities for deeper learning and investigation. The belief that learning to use professional software would be of later value in their studies and careers was clearly a strong and important motivation.

On the other hand, students' espoused fears and concerns focused on personal feelings of inadequacy when using computers, and beliefs that computers do not aid learning. Some were very anxious about their lack of experience and confidence with computers: two reported negative prior experiences. Students' attitudes to the intervention were generally not closely related to their liking for mathematics: in fact, the most negative technology attitudes came from students who said they like mathematics. Conversely, the positive computer software attitudes of a potentially vulnerable group of students who were not enthusiastic about learning mathematics, made it clear that computer interventions of this kind have the potential to motivate levels of engagement in learning tasks. These early base-line beliefs are an important and reassuring finding for any technology learning intervention that seeks to harness powerful computer software for the learning of mathematics.

COMPOSITION AND DECOMPOSITION OF 2-DIMENSIONAL FIGURES DEMONSTRATED BY PRESERVICE TEACHERS

Robert F. Cunningham   Suriza van der Sandt

Mathematics and Statistics Department, The College of New Jersey, Ewing, 08628

International and national studies highlight the vital role of teachers' content knowledge in mathematics learning. These studies concur that many practicing teachers in USA elementary and middle schools have impoverished conceptual understanding of many of the (Algebraic) mathematical concepts and processes they are required to teach. This study investigates prospective teachers' Geometry content knowledge with specific focus on the process of composing and decomposing two-dimensional figures. For this study, this process will be referred to as composition.

Composition appears infrequently in many elementary and middle school curricula yet teachers of geometry are expected to possess conceptual understanding of composition in order to provide students with meaningful opportunities to transform, combine and subdivide geometric figures. Despite the curricula, the understanding of composition is a vital component of content knowledge for teachers of geometry at all levels as it contributes to such vital skills as perceptual constancy, position in space-perception, visual discrimination, perception of spatial relationships and figure-ground perception. To site just a few examples of its importance in the school curriculum, the knowledge of composition facilitates working with area, with congruence, and with angle computations of polygons where the ability to recognize that a polygon can be decomposed into triangles is critical.

This study examines the conceptual understanding of composition by 125 preservice teachers who were enrolled in a college on the East coast of the USA in the Fall of 2004. While results from surveys and interviews showed that many preservice teacher could successfully compose and decompose figures, over half had severely limited ability to recognize alternative methods of composition. Since these students are enrolled in a geometry course designed for preservice teachers, they will again be surveyed in the Spring 2005 semester. Final and comparative results with suggestions for improved pedagogy will be reported.

The research was funded in part by The Council of Deans at The College of New Jersey

WHAT'S IN A NAME? ANONYMITY OF INPUT IN NEXT-GENERATION CLASSROOM NETWORKS

Sarah M. Davis

University of Texas at Austin

This study looks at student use of anonymity of input with next-generation classroom networks in a pre-calculus class. Results show that the more activity input can be considered right/wrong, the more students want to submit anonymously.

Next generation classroom networks are poised to become a significant presence in schools. In contrast with current networks which connect students to the internet and outside information, these networks harness and share the knowledge within classrooms, sharing and aggregating data among all the members. Common to all of the next-generation classroom networks is the feature of anonymity of input to the public display. Yet, no research has been done to show if anonymity is an important design element in network-supported learning and, if so, what about anonymity is significant. This project looks at anonymity of input across a series of activities in a pre-calculus classroom seeking to answer the question: Does activity type influence students' use of anonymity?

Next-generation classroom networks allow for a positive view of anonymity. Anonymity opens the information that has been displayed to the whole class for interpretation. With a range of mathematical responses collected from the class displayed, students can talk about any one of the answers as if it was theirs. Or, as if it was someone else's. Once information has been submitted to the public display, a student can assume any of the identities and advocate it as if it were there own. In this way, anonymity opens up the classroom allowing students to try on new roles.

Early analysis of project data shows that students attune most closely to the ability of their answer or participation to be considered incorrect when deciding whether to show or hide their identity in the display space. Activities ranged across submitting responses to homework questions, controlling a point in a scatter plot, networked Sim-Calc lessons, and HubNet (networked NetLogo) simulations. Students were most likely to hide their names if their input could be seen as "wrong". In this way the activity design had a strong impact on students' use of anonymity. Additionally, female students were more likely to need to be confident of the correctness of an answer, before choosing to display their name. Finally, there were two negative impacts to classroom interactions from tying names to responses in the display space. First, the teacher's use of the class' responses became directive rather than inclusive. Instead of opening up the discussion of responses to all students, the teacher called on the student who submitted the response. Second, students no longer felt free to critique the responses in the display space. With names associated, discussing responses become "personal" and no longer just about the ideas.

STUDENTS' USE OF DIFFERENT REPRESENTATIONS IN PROBLEM SOLVING AT HIGH SCHOOL LEVEL

Jaguthsing Dindyal

National Institute of Education, Singapore

This study reports how high school students used different forms of representations during problem solving and some of their related conceptual difficulties associated with the use of these representations. The focus students solved nine problems which involved the use of algebraic thinking in geometry. One important aspect of algebraic thinking is the use of different forms of representations which include: verbal, numerical, graphical, and symbolic representations. The chosen problems involved one or more of these different forms of representations.

Theoretical perspectives from Goldin (2002) and Dreyfus (1991) were used in the study. Goldin (2002) has claimed that individual representations belong to a *representational system*. He has postulated two types of representations: internal and external. Dreyfus (1991) proposed a theory about representation of concepts which complements Goldin's theory described above. Dreyfus claimed that to represent a concept means to generate an instance, specimen, example, or image of it. A symbolic representation is externally written or spoken, usually with the aim of making communication about the concept easier, whereas a mental representation refers to the internal schemata or frames of reference which a person uses to interact with the external world. For Dreyfus, learning processes consist of four stages: (a) using a single representation, (b) using more than one representation, (c) making links between parallel representations, and (d) integrating representations and flexibly switching between them.

The results demonstrate that students could work separately with each of these four forms of representations. However, they had difficulties switching from one form of representation to another and making links between parallel representations of the same concept. The algebraic form of representation took precedence over the verbal form in some cases, such as the description of the Pythagorean Theorem. Sometimes not understanding a particular term caused some representation problems for the students. In other situations, the students overlooked the use of a diagrammatic representation in finding a solution and this led them to incorrect solutions

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AUTOMATISED ERRORS: A HAZARD FOR STUDENTS WITH MATHEMATICAL LEARNING DIFFICULTIES

Maureen Finnane

University of Queensland

While researchers have demonstrated the benefits of automatising number facts for the carrying out of problem solving and complex algorithms (e.g., Cumming & Elkins, 1999), there has been relatively little research on the problems caused by automatised fact errors. This may be a critical pedagogical issue for students with learning difficulties in mathematics, who are characterised by pronounced difficulties in mastering basic facts (Ginsburg, 1997), by a high level of errors on retrieved facts, and by a distinctive pattern of counting errors (Geary, 2004).

Recently, however, after confirming Barouillet's discovery that students may substitute counting string associations to one of the addends in a basic fact combination (Barouillet et al., 1997), Geary (2004) has proposed that difficulties in inhibiting the retrieval of irrelevant associations may be an underlying cause of difficulties in mastering arithmetic facts. Furthermore, Hopkins and Lawson (2004) demonstrated that the variable response times noted for retrieval of facts by students with mathematical learning difficulties may in part be caused by increased response times for trials which immediately follow trials where students have made errors.

This presentation will demonstrate the hazard of recurring errors for students with learning difficulties by presenting data from sessions focussed on teaching the ten facts to a 9 year old student with a significant mathematical learning difficulty. Conversely, by paying active attention to his errors, the student was able to successfully teach himself the nine times tables to the point of mastery.

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TEACHING GEOMETRY IN TWO SECONDARY CLASSROOMS IN IRAN, USING ETHNOMATHEMATICS APPROACH

Zahra Gooya: Shahid Beheshti University

Tehran, Iran

Azar Karamian: Mathematics Educator, Ghom, Iran

In a study that was conducted in the 2003-2004 school year, a mathematics instruction was designed for teaching geometry in grades 10 & 11 (2^{nd} & 3^{rd} year high school) using the ethnomathematics approach. The purpose of this study was two folded; the first was to investigate the ways in which, this notion could be used in a mathematics classroom in Iran- taking into account that the system of education is extremely centralized, and teachers are responsible for every page of every textbook they are teaching. The second was to study the effect of ethnomathematics teaching on students' perception about mathematics, as well as their understanding of the geometric concepts of high school.

The data for the study were collected through two geometry classes in a girl's high school, in Ghom in which, the second author was teaching geometry in both classes (all schools in Iran are segregated). The data constituted of teacher's observations and her field notes, students' reflective comments about the instruction that they received, and students' responses to two questionnaires about their perceptions about mathematics. The data were analyzed using Bunks (1994) framework of ethnomathematics approach.

The results of the study showed that, even in a highly centralized system, ethnomathematics approach could be taken to teach geometry. This was especially important for Iranian students, since, they expressed their great appreciation for their cultural and scientific heritage, and the contribution that they made to the development of mathematics at the local and global level. Furthermore, students became interested to do more inquiry about the role of mathematics in traditional artwork and handicrafts, including tiling, painting, and woodworks in Iran. The ethnomathematics approach helped students to change their perception about mathematics, as well as a better understanding of geometric concepts.

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TEACHING STRATEGIES TO SUPPORT YOUNG CHILDREN'S MATHEMATICAL EXPLANATIONS

Susie Groves & Brian Doig

Deakin University

The research reported here forms part of a small-scale international collaborative study, *Talking Across Cultures*[1], investigating children's mathematical explanations during mathematics lessons in Australia, Hungary and Japan, during the first year of school.

In this report, we identify strategies used by an expert Year 1 Japanese teacher to support young children's mathematical explanations in a lesson based on identifying children's solutions for the subtraction problem 14 – 8. These strategies, which included interweaving the concrete with the abstract, public and permanent recording of explanations, giving children ownership of ideas, and promoting high level written explanations, are examined briefly from the perspective of their cultural, pedagogical and traditional bases, to establish their pertinence to other educational settings.

As Clarke (2002) argues in his discussion of the problematic nature of international comparative research, the purpose of studying international classroom practices is not merely to mimic them, but rather to support reflection on our own practice. Thus it is important to distinguish between those classroom practices that are specifically cultural, those that are based on deliberate pedagogical decisions, and those that are the unintended consequence of other actions and decisions.

A high quality Australian lesson, also based on subtraction, was video-recorded as part of this study. The contrast between the teachers' strategies used in the Japanese and Australian subtraction lessons, suggests that there are aspects of each that could be profitably explored in the other country. However our analysis also suggests that there may be significant barriers to adopting practices from different cultures. For example, while Australian teachers want to give students ownership of ideas, this is very difficult to do when there is no tradition of identifying, recording and attributing these ideas to individual children for later use.

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[1] *Talking Across Cultures* was funded by the Deakin University *Quality Learning Research Priority Area*. The project team is Susie Groves, Brian Doig (Deakin University), Toshiakira Fujii, Yoshinori Shimizu (Tokyo Gakugei University) and Julianna Szendrei (Eötvös Loránd University, Budapest.

HOW DO WE PROVIDE TASKS FOR CHILDREN TO EXPLORE THE DYNAMIC RELATIONSHIPS BETWEEN SHAPES?

Alice Hansen and Dave Pratt

St. Martin's College, England and University of Warwick, England

We consider to what extent van Hiele's levels of geometrical understanding can be used at the classroom level and raise the issue of appropriate tasks for children to engage with in order to challenge and stimulate their understanding of geometric definitions.

Within a design research methodology (Cobb *et al.,* 2003) we considered how task design can begin to meet the needs of children who are struggling with geometric definitions. van Hiele (1986) offers an overview of children's geometrical understanding. However we think of van Hiele's levels as belonging to a family of macrolevel theories and our focus is much more humble. We focus on the classroom, where we see a child whose knowledge is in a state of flux and under constant pressure from outside influences. Of those many structuring resources in the classroom setting on which we are now focussed, we ask, "What is the contribution of the task to this complex and excitingly unsmooth dynamic?"

It was through the classroom-based use of a product (which in this case was a design for a task for nine- and ten-year olds) within an iterative process that children's definition of quadrilaterals was explored with the aim that we would first be able to abstract principles related to the design of a task about geometric definitions and subsequently propose more generic principles for task design.

It became clear from analysis of the data that there were several sources of confusion about the nature of geometric definitions. These included the identification of instance versus class, the attributes of the shapes, the inclusive nature of definitions, and the definitions themselves.

In light of our findings, we will present the principles that will underpin our next task and explain how they are being operationalised through the *Constructionist* (Harel & Papert, 1991) tenet that technology facilitates the construction of knowledge through use of that knowledge (see the *Power Principle* in Papert, 1996).

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PARTICIPATION, PERFORMANCE & STAGE FRIGHT: KEYS TO CONFIDENT LEARNING AND TEACHING IN MATHEMATICS?

Tansy Hardy, Sheffield Hallam University

This session will be an exploration of both theorisations of what is often named 'identity' and of what it means to be confident in learning and teaching maths. Some models of self-image and individuality can produce restricted understandings of the experience of many learners and teachers of mathematics (Henriques et al. 1984). I will discuss the notion of 'subjectivity' and consider ways in which it offers a better analytical frame. This research was started in the PME 27 discussion group on the interface between psychological and sociological paradigms for mathematics education research (Gates et al. 2003).

I present these explorations in a mix of textual commentary and a patchwork of vignettes from my research experience. These are brought into juxtaposition, using 'what is to hand' to create something new; in a form of bricolage (e.g., Levi-Strauss 1966). This is intended to evoke connections and parallels that are concealed by more traditional modes and to offer an account of how a re-examination of practices operates and new meanings are formed in education research. This bricolage uses data and reflections from particular research projects that I have undertaken. These were 'to hand'. The first of these is an analysis of teacher guidance video material to explore the discursive practices of teachers and children in exemplar mathematics lessons (Hardy, 2004). In a second project, exploring the effects of whole-class interactive teaching, I worked with practicing teachers and pre-service student teachers. I have also borrowed incidents related in interviews with teachers and made connections with other mathematics education research. I will outline how, through this analytical tactic, constructs of a 'good learner' or a 'good teacher' of mathematics can be shaken up and discuss new understandings that are generated. The key theme of being willing and able to participate in mathematics classrooms in ways that are seen as valid was highlighted through this research and will be offered for discussion.

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AN EMERGENT MODEL FOR RATE OF CHANGE

Sandra Herbert and Robyn Pierce

University of Ballarat

It has long been a concern that students develop only procedural competence with differentiation and seem to lack a deeper understanding.(Orton, 1984; Stump, 2001). The aim of this study was to explore the possibility of using the simulation software JavaMathWorlds to develop a 'model of' (Gravemeijer, 1999) a rate of change as motion leading to an emergent model for any context.

JavaMathWorlds, a product of the successful SimCalc project (HREF1), simulates the motion, of a lift or characters walking, and provides links to the numeric, graphical and symbolic representations of motion. The lift animation was used, as a starting point to investigating rate of change in a motion context, because it draws a strong connection between floors in the building and scale on the vertical axis. This encouraged the forging of stronger links between an experientially real situation and its graphical model thus supporting the development of a mathematical 'model of' the motion. Of particular interest was the notion that experience with problem solving, in a motion context only, is sufficient for the development of a transferable 'model for' rate of change regardless of the context.

The study involved year 9 (15 year old) students from two classes at an Australian secondary school. Both teachers used material consisting of four lessons introducing the software and posing problems for students to solve. It was hoped that the cognitive residue of the instructional sequence would be a more complete concept image for rate of change than is usual for students of this age and stage.

Data collected include students' pre and post test scripts, notes based on conversations with teachers and transcribed student interviews. Pre and post tests used consist of both motion and non-motion questions probing students' understanding of the concept of rate of change across multiple representations of the context.

Findings indicate that, for many students, this technology enriched learning environment, based on motion alone, does facilitate the development of an emergent model for rate of change.

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THE ROLE OF ACTIVITIES IN TEACHING EARLY ALGEBRA

Derek Holton — University of Otago
Chris Linsell — Dunedin College of Education

The mathematics curriculum in New Zealand, like many similar documents around the world, emphasises the use of meaningful contexts and practical activities (Ministry of Education, 1992). Some justifications for this are that the mathematics encountered in real life is always in context and the mathematics taught in schools should prepare students for this. Also the use of contexts and activities is likely to motivate students and promote mathematical understanding (de Lange, 1996). However little is known about how students in New Zealand high schools make use of activities when learning algebra.

This work, which forms part of a doctoral thesis, is a qualitative study of learning in a Year 9 class, which monitored four students during twenty-seven consecutive lessons. The data set consisted of videotapes of the lessons, students' written work, stimulated recall interviews and field notes. The teaching programme made extensive use of cooperative group activities.

All four students were engaged in the activities, and enjoyed them. For the two less numerate students, the activities gave them only a vague idea of the purposes of algebra, but for the more numerate students, the activities allowed them to write equations for situations and have a purpose in solving them. However the activities did not directly facilitate the students to develop an understanding of formal solution processes. A possible reason for this is that the students did not usually make use of the contexts when solving equations, working at the symbolic level instead. The students' use of activities when learning to solve equations was very different to when they were learning to operate on integers. During the work on addition and subtraction of integers the physical activity provided a metaphor for the intended mathematical activity, allowing meanings to be constructed through mental and verbal juxtapositions. However none of the activities used in the study provided a metaphor for the formal method of solving equations. The few examples of keeping the context in mind when solving equations could be regarded as metaphors for solving equations by the strategy of *guess and check*. This study reinforces de Lange's (1996) claim that only some contexts are useful for the development of concepts.

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COORDINATED ANALYSES OF TEACHER AND STUDENT KNOWLEDGE ENGAGED DURING FRACTION INSTRUCTION

Andrew Izsák
The University of Georgia

Adults and children often understand joint activity in different ways, but little research has investigated consequences of such differences for classroom teaching and learning. In fact, teachers' knowledge and teaching has been a separate sub-field of educational research from students' cognition and learning. The present study reports a coordinated analysis of one U.S. sixth-grade teacher's and her students' understandings of lessons that used linear and area models to develop fraction multiplication. The main research questions were (1) what conceptual structures did the teacher and her students have available for interpreting the tasks contained in the instructional materials and (2) how were students' opportunities to learn shaped by the ways in which they and their teacher engaged those structures. The theoretical perspective on classroom instruction was informed by Cohen and Ball (2001), who argued that instruction is shaped fundamentally by interactions among teachers, students, and content as mediated by instructional materials. The theoretical perspective on cognitive structures in the domain of fractions was informed by Steffe's (e.g., 2003) recent work examining how students can construct understandings of fractions using their understandings of whole numbers as they work with linear and area models. Data for the present study came from videotapes of (1) Ms. Archer's instruction every day in one class over a period of 6 weeks; (2) concurrent, weekly interviews with four pairs of students from the same class during which students worked tasks like those in the lessons and interpreted video excerpts from Ms. Archer's related instruction; and (3) concurrent, weekly interviews during which Ms. Archer interpreted the instructional materials, explained her pedagogical decisions, and interpreted the same video excerpts from lessons and further video excerpts from her students' interviews. The method for inferring conceptual structures involved fine-grained analysis of talk, hand gesture, and drawing as captured in the videotapes. Results indicate that Ms. Archer and her students evidenced a range of conceptual structures relevant for using linear and area models to understand fraction multiplication but did not consistently engage those structures during the lessons. As a result of under utilizing their available cognitive resources, Ms. Archer and her students often misunderstood one another and, as a result, constrained the opportunities to learn.

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DIFFERENCES IN LEARNING GEOMETRY AMONG HIGH AND LOW SPATIAL ABILITY PRESERVICE MATHEMATICS TEACHERS

Elizabeth Jakubowski
Florida State University

For many years now the International Association for the Evaluation of Educational Achievement (IEA) has conducted several international comparative studies of the mathematics and science performance of students around the world. One of the most recent, TIMSS-R, was conducted in 1999, and continued to show that United States (US) students' mathematical achievement lagged behind that of several other countries. More specifically, in the geometry content area, United States student achievement was in the bottom third of all countries tested. When geometry scores are examined, Japanese students performed at the top with a score of 575. The international average score was 487 and the US scored 473 in the geometry content area. Among the 38 countries, 26 countries outperformed the US in the geometry content area (Mullis et al., 2000). One question to consider is why is US student performance in geometry so low when compared with their peers in other countries?

While investigating the learning process, one has to consider the role a teacher plays during instruction. A primary consideration has to be the content-knowledge a teacher brings to teaching. The objective of this study was to investigate and characterize the geometric thinking of four preservice middle mathematics teachers while considering spatial ability levels. Specifically the study was guided by the following research question: *what differences, if any, exist between preservice middle and secondary mathematics teachers with different spatial ability levels and their understanding of geometry?* This report is a part of larger study, focusing on four contrasting cases in terms of their spatial ability levels. The study used the van Hiele model to provide a description of geometric thought. Participants were chosen using the Purdue Visualization of Rotations test (Bodner & Guay, 1997) from among a pool of preservice middle and secondary mathematics teachers (n=26) at a major research university. Using the Mayberry (1981) protocol four participants' van Hiele levels were identified at the beginning and end of an informal geometry course for mathematics education majors. Field notes were kept for all class sessions and student work was reviewed. Results indicated that students with a high spatial score had larger gains in van Hiele levels than preservice teachers with low spatial ability scores.

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MATHEMATICAL KNOWLEDGE FOR TEACHING PROBABILITY IN SECONDARY SCHOOLS

Mercy Kazima

University of Malawi/University of the Witwatersrand

The paper will report on an ongoing study that is attempting to identify and describe the mathematical knowledge teachers need to know and know how to use in order to teach probability well to secondary school students. Research questions are (i) what mathematical knowledge is evident in curriculum documents? (ii) what mathematical knowledge(s) do teachers draw on while teaching probability?

Shulman (1986) argues that teaching entails more than knowing the subject matter, it requires “pedagogical content knowledge” which “goes beyond knowledge of the subject matter *per se* to the dimension subject matter knowledge for teaching” (1986, p9). Adler (2004), Ball *et. al.* (2001), Brodie (2004), among others, emphasise that mathematics teachers need ‘mathematical knowledge for teaching’ (MKFT) which includes knowing how to do mathematics as well as how to use the mathematics in practice (teaching). Therefore, MKFT can only be identified and described by studying practice.

The theoretical framework that underpins the study is that MKFT is situated in the practice of teaching (Adler, 2004). Therefore, and also from literature, a study of MKFT entails an analysis of the curriculum in both (i) documentation and (ii) practice. The paper focuses on MKFT in practice and reports on findings from observations of Grade 8 probability lessons in one township school in Johannesburg, South Africa. The main source of data is videotapes of the lessons. Each lesson will be broken down into episodes of ‘what the teacher was doing’. Within each episode, attempts will be made to describe the knowledge sources the teacher draws on.

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STUDENT TEACHERS' VIEWS ON LEARNING THROUGH INTERACTIVE AND REFLECTIVE METHODS

Sesutho Koketso Kesianye

University of Botswana

INTRODUCTION

First Year mathematics student teachers at Tonota College of Education in Botswana participated in a study on the preparation of preservice teachers on the Integration of Assessment and Instruction (Kesianye, 2002). The study employed teaching and learning methods designed to create an environment where student teachers interacted with others and reflected on their learning. Data was collected through "free writing reports", lesson diaries and structured questionnaires. Qualitative types of analysis were employed to interpret data. Student teachers' views about the learning environment created in the study are the substance of this presentation.

DISCUSSION

Research indicates that student-teachers arrive into teacher education programmes with certain conceptions of teaching, some of which may be vague and difficult to articulate, and which appear resistant to substantial change, as observed by Haggarty (1995). However, Amit et al. (1999) suggest that teachers should be provided with experiences where these conceptions are challenged and that they be given opportunities to reflect on and rethink their conceptions. The findings reflected that student teachers made critical and constructive observations about their learning experiences. They articulated their learning progress and made suggestions for improvement. Their reactions indicated a deeper understanding of the purposes and practicalities of employing these methods, which changed previously held conceptions of teaching.

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LOW-ACHIEVEMENT STUDENTS' RATIONALE ABOUT MATHEMATICS LEARNING

Kim In Kyung

Korea National University of Education

Adopting the perspective of activity theory, Mellin-Olsen argues the case for the significance of a student's rationale for engaging in classroom activity. He identifies two rationales for learning. These are the S-rationale (Socially significant) and the I-rationale (instrumental). And then, Simon adds the P-rationale (practice) and the N-rationale (no rationale). This study investigates which of these four rationales low-achievement students have. This study also intends to find out what teaching method is best for low-achievement students.

The subjects are from the lower 5% first year students of each class of an urban high school. First, I had an interview with their teacher. And then, I had interviews with students in December. The interview was conducted once per student, and when more information was needed, I conducted additional interviews (about 3 students). It took an hour or an hour and a half hour at half-structured interview.

The result showed one of those students as having N-rationale while the others were either of I-rationale or S-rationale. In contrast to my suspicion, though they are low-achievement students, they know that they need mathematics and want to study mathematics. But compared with primary and middle school students, high school students need to be provided with more complementary measures. They have not ever talked and consulted with a mathematics teacher, classroom teacher, parents, nor another person about mathematics learning. So we have to provide a more careful concern for low-achievement students.

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CHARACTERISTICS OF EARLY ELEMENTARY STUDENTS' MATHEMATICAL SYMBOLIZING PROCESSES

Kim, NamGyun

CheongJu National University of Education

Mathematical symbolizing is an important part of mathematics learning. But many students have difficulties in symbolizing mathematical ideas formally. If students had had experiences inventing their own mathematical symbols and developing them to conventional ones natural way, i.e., learning mathematical symbols via expressive approaches (Gravemeijer et al., 2000), they could understand and use formal mathematical symbols meaningfully. These experiences are especially valuable for students who meet mathematical symbols for the first time.

Hence, there are needs to investigate how early elementary school students can and should experience meaningful mathematical symbolizing. The purpose of this study was to analyze students' mathematical symbolizing processes and characteristics of theses.

We carried out teaching experiments that promoted meaningful mathematical symbolizing among eight first graders. And then we analyzed students' symbolizing processes and characteristics of expressive approaches to mathematical symbols in early elementary students.

As a result, we could places mathematical symbolizing processes developed in the teaching experiments under five categories. And we extracted and discussed several characteristics of early elementary students' meaningful mathematical symbolizing processes.

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TEMPORAL ORDER IN THE FORMAL LIMIT DEFINITION

Jessica Knapp — Arizona State University
Michael Oehrtman — Arizona State University

Mathematically a definition determines how one may structure a valid mathematical argument and proof. This study describes a difficulty advanced calculus students have with the formal definition of proof and how this might affect their proof writing. Moore (1994) suggests that students' lack of understanding of both the role of definitions in proving and the meaning of a formal definition are an integral part of their struggles with proving. Davis and Vinner (1986) found calculus students have a naïve conceptualization of the limit concept which impedes their understanding of the formal limit definition. Inherent in this definition is an underlying process. Davis and Vinner call this the *temporal order;* i.e. first given an ε, then a δ must be found which makes the implication which follows true, hence δ is a function of ε. We offer examples of students' lack of understanding the role of the temporal order process in the limit and we comment on the implications of this difficulty on their proof writing.

This research report compares data between semester long workshops for freshmen calculus students and juniors in an advanced calculus course. Students were asked to determine if the limit of a particular function exists and then to produce a proof to justify their answer. Student discourse was coded for the students' use of language associated with understanding of the process in the epsilon-delta definition.

This report focuses on the students' evaluating the limit as $x \to 0$ of $s(x)$, where the function $s(x) = x + 1$ for $x \in \boldsymbol{Q}$ and $s(x) = 1$ for $x \notin \boldsymbol{Q}$. The students were focused on how to prove the limit exists. However, they began looking for an ε, rather than looking for a δ determined by the given ε. In fact, Molly stated, "If we choose $\varepsilon = \delta$, we've got it." No one in the group objected to Molly's goal. Kelly clarified the suggestion, "If we could pick a δ such that $[|x - a| < \delta]$ is true then we can pick the same value for ε." Notice the group did not see δ as a function of ε. Molly explained, "First we resolve δ, then we go on to resolve ε."

These students knew some aspects of the formal definition of limit. In fact, the group eventually produced a correct proof including a given statement for ε and defining δ as a function of ε. This suggests that at some level they have learned an appropriate structure for a proof that a limit exists, but do not grasp the underlying temporal order of the formal definition. This implies they likely do not see the role that the structure of the definition plays in determining the structure of an appropriate proof.

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RESEARCH ON THE PROCESS OF UNDERSTANDING MATHEMATICS: WAYS OF FINDING THE SUM OF THE MEASURE OF INTERIOR ANGLES IN A CONVEX POLYGON

Masataka Koyama

Graduate School of Education, Hiroshima University, Japan

This is a part of the series of research on the process of understanding mathematics based on the “two-axes process model” that consists of two axes, i.e. the vertical axis implying levels of understanding such as mathematical entities, relations of them, and general relations, and the horizontal axis implying three learning stages of intuitive, reflective, and analytic at each level (Koyama, 2000, 2003, 2004). The purpose of this research is to closely examine the 40 fifth-graders’ process of understanding the sum of the measure of interior angles in a convex polygon in a classroom at the national elementary school attached to Hiroshima University.

In order to improve their understanding of the sum, with a classroom teacher, we planned the teaching unit of “The Sum of the Measure of Interior Angles” and in total of 8 forty-five minutes’ classes were allocated for the unit in the light of “two-axes process model”. Throughout the classes we encouraged students to think the sum in a various and logical/mathematical way. The data collected in the observation and videotape-record during the classes were analysed qualitatively to see the change of students’ thinking and the dialectic process of individual and social constructions through discussion among them with their teacher in the classroom.

As a result, the teaching unit starting from the tessellating congruent triangles to the finding/explaining ways for the sum of the measure of interior angles in quadrilaterals, pentagons, and hexagons could improve the students’ mathematical understanding and logical thinking. Especially, their whole classroom discussion on the various way of finding the sum in a hexagon was effective for the students to share with and reflect on their ideas leading to the general formula for the sum in a convex polygon with n sides.

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MATHEMATICS: COPING WITH LEARNER SUCCESS OR FAILURE

Daniel Krupanandan Mathematics Subject Advisor

KZN Department of Education, Durban, South Africa

Despite the valiant efforts of community organizations, dedicated teachers and others, success in mathematics has eluded the majority of learners in South Africa. This is a growing concern for mathematicians and parents.

The National Department of Education has embarked on a national science and technology strategy instituted to promote and popularize mathematics. This is done at a time when fewer learners are choosing to study mathematics at high school level. Also there has been a marked decrease in the number of learners studying mathematics at Higher Grade, the “grade” of study needed for further studies in scientific fields of study.

South African is on the brink of curriculum transformation in the high school years. This will be implemented in 2006. The question on many minds is whether this will “turn the tide” in the mathematics classrooms.

THIS RESEARCH

This research looks at the challenges facing South African learners and teachers in the South African Mathematics classroom. This study is undertaken through interviews with learners who have experienced repeated failure in doing mathematics despite their honest attempts to do well. These learners are taken from both rural and urban schools in our city, Durban in South Africa. This testimony does not do much in my attempts as subject advisor to promote the subject amongst learners who do not want to hear us speak of mathematics.

In addition interviews are undertaken with mathematics teachers who often have given up hope in some instances to reach out to learners who often time have lost interest in the subject. My engagement with teachers suggests that they are often frustrated with poor results. This research will attempt to itemize some reasons why children perform poorly in mathematics.

Improving mathematics results or popularizing the subject is a challenge worldwide. It is hoped that the research will give us a new perspective to learner difficulties, and teacher initiatives to improve mathematics results.

AN ANALYSIS OF TEACHER-STUDENTS INTERACTION IN KOREAN ELEMENTARY MATHEMATICS CLASSROOMS

MinSung Kwon   JeongSuk Pang & Kyung Hwa Lee

Daegu Chilsung Elementary School   Korea National University of Education

Recently mathematics educators have made an effort to change teacher-centered instruction to student-centered. However, many teachers still have difficulties with their instructional changes. These difficulties come not merely from the complexity of instruction itself but also from the lack of understanding what is constituted of student-centered and what kind of classroom culture needs to be established.

The purpose of this study was to provide useful information for instructional improvement by analyzing various teacher-students interaction in reform-oriented mathematics classrooms. As an exploratory, qualitative, comparative case study, 4 classrooms were selected in which teachers attempted to implement student-centered instruction. As a total of 34 mathematics lessons of fraction were videotaped and individual interviews were conducted with the teachers. A theoretical framework of data analysis resulted mainly from Hufferd-Ackles, Fuson, & Sherin (2004), in conjunction with Pang (2000) and Wood (2003). Specifically, teacher-students interaction in each classroom was identified by 4 levels in questioning, explaining, and the source of mathematical ideas, respectively.

Detailed analyses of classroom episodes showed what kinds of interaction were fostered. As for questioning, teachers asked for reasons rather than simply answers, but the degrees of students' questions were different. As for explaining, the quality of students' justification varied depending on the teacher's role of listening. As for the source of mathematical ideas, students came up with multiple ideas but they tended to focus on the diversity of representations rather than solution methods. Whether students discussed similarities and differences of their various ideas was different across classrooms. The differences in three components set forth implications of what aspects of teaching and learning need to be focused for a real instructional change.

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STUDENTS' GRAPHICAL UNDERSTANDING IN AN INQUIRY-ORIENTED DIFFERENTIAL EQUATION COURSE: IMPLICATION FOR PRE-SERVICE MATHEMATICS TEACHER EDUCATION

Oh Nam Kwon Mi-Kyung Ju So Youn Kim

Seoul National University Silla University Seoul National University

Although graphical understanding has been emphasized in teaching and learning of mathematics, research shows that school mathematics curriculum is not enough to facilitate the development of students' graphical understanding (Dyke & White, 2004; Knuth, 2000). From this perspective, the inquiry-oriented differential equations course (IODE) was designed to emphasize the integration of multiple mathematical methods such as graphical, numerical, and qualitative methods as well as analytic method. The research measures the impact of the IODE by investigating students' graphical understanding and sought for implications for teacher education. For comparative analysis, pre- and post-tests, made up of problems requiring graphical understanding, were given to two groups of students of a university in Seoul, Korea. Since IODE was a course offered in a university pre-service program, the experimental group consisted of 36 pre-service students in the department of mathematics education. The control group consisted of 30 students who enrolled in a traditional differential equations course based on lecture (TRADE) in the mathematics department of the university. Following are the results of the analysis:

The IODE group got statistically higher mean score than the TRADE group did.

The IODE group tended to use the graphical/qualitative method, while the TRADE tended to use the analytical method only.

The IODE group was significantly better than the TRADE group at the problems requiring the connection between an equation and a graph. In particular, while the rate of NO ANSWER to this type of problems was very high in the TRADE group, it was very low in the IODE group.

These results show that the IODE contributed positively to develop the students' graphical understanding, specifically, their abilities to use graphs for problem solving, to interpret the meaning of a graph, and to grasp the connection between a graphical method and other methods. Moreover, this research implies an alternative model to the current pre-service mathematics teacher education programs which instruct mathematical knowledge and pedagogical contents separately. By combining subject matter knowledge and pedagogical content, the IODE not only transformed the quality of the students' graphical understanding but also provide opportunity for the pre-service students to reflect on how to teach to develop graphical understanding in their future teaching career.

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TYPES OF VISUAL MISPERCEPTION IN MATHEMATICS

Martin Lamb & John Malone

Curtin University of Technology

A *misperception* is the act of perceiving via a single sensory modality (e.g. seeing in mathematics or hearing in music) something that is different from reality or an imagined reality (e.g. visualising a rotated shape in maths).

Visual misperceptions seriously inhibit students' efforts to learn. The authors have found misperceptions occurring not only with school students but also with pre-service and practicing teachers (Lamb, Leong & Malone 2002). Teachers who perceive in their mind's eye something that is different from reality are likely to mis-teach, and students who misperceive will experience learning problems also.

This ongoing study of 720 Year 8 Australian school students has revealed three different types of misperception occurring in very simple mathematical tasks. During three separate tests, over 40% of the participants misperceived at least once.

The topic selected for the study was linear transformations, a topic that lends itself to displaying the misperception phenomenon (Kuchemann, 1982; Sherris, 2003; Edwards, 2003). The students in our study had been taught about reflections and rotations approximately two years beforehand.

The negative effects of misperceptions on learning have been largely unappreciated, and are usually misdiagnosed to be the result of student errors or misconceptions (Shaw, Durden & Baker, 1998). The study has revealed how misperceiving students can be identified, and how using manipulatives and specially designed software, some misperceptions can be corrected.

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DEVELOPING STUDENTS' HIGH-ORDER THINKING SKILLS THROUGH INCREASING STUDENT-STUDENT INTERACTION IN THE PRIMARY CLASSROOM

King-man Leung

University of East Anglia, Norwich, UK

Mathematics curriculum reform at the primary level in Hong Kong has been a concern of primary mathematics teachers since the implementation of the curriculum guide (CDC, 2000) in 2002. This study investigates mathematics learning and teaching in the primary classroom with a particular focus on the changing role of the teacher as a facilitator who helps students develop high-order thinking skills through using mathematical tasks and classroom discussion (as specified in the 2002 policy document "Learning to Learn Key Learning Area Mathematics Education"). Recent research suggests that the majority of teachers in HK largely still use the textbook in a routine, "chalk & talk" mode as the main material for the introduction and consolidation of mathematical concepts by students (Wong, N.Y., Lam, C.C., Leung, F.K.S., Mok, I.A.C. & Wong, P.K.M., 1999). These teachers' classrooms are dominated by traditional teaching practices. Furthermore the rare teacher training provided towards the implementation of the new curriculum seems to have had minimal influence on teachers' philosophies and beliefs about the learning and teaching of mathematics (CDC, 2000). The study reported here was set up with the intention to encourage more student-student interaction in the classroom, to enhance students' thinking and communication skills and to use diversified learning activities and tools (including mathematical tasks & information technology) for improving learning and teaching. In collaboration with two teachers in two elementary schools, learning materials were developed for selected topics. Trial lessons were conducted in these teachers' schools over the last three years. Data was collected in the form of audio-taped interviews with teachers and groups of students, video-taped classroom observations, field notes, documents and students' annotated work. Preliminary analysis suggests that the new learning environment contributed significantly in the development of students' high-order thinking skills; that by using the mathematical tasks, it created a new platform for students to learn in a collaborative mode; and that increased opportunity for communication meant that students, even those usually reticent in the classroom, could freely put forward their ideas and suggestions.

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SUPPORTING TEACHERS ON TEACHING FRACTION EQUIVALENCE BY USING RESEARCH-BASED DATA

Pi-Jen Lin National Hsinchu Teachers College, Taiwan

The emphasis on anticipating students' learning process and integrated it into teaching is developed on current reform in mathematics teaching. The success of CGI is a case (Fennema et al., 1996). Simon's model indicates that the teacher's knowledge evolves simultaneously with the growth in the students' knowledge (Simon, 1995). The continual change of hypothetical learning trajectory as a result of the increment of understanding students' thinking can be as an indictor of the process of a teacher constructing knowledge. This study offered the teachers with the key ideas suggested in the literatures of research on children learning fraction equivalence [FE] (Post, 1992). How a teacher learned the results of the research on fraction equivalence and integrated them into classroom teaching is the focus of the study.

Six teachers participated in the study, while only one teacher, Jing-Jing, teaching in fifth grade, was report here. The data collected in the study included: videotapes of five classes, audiotapes of three weekly meetings, teacher's reflective journals, students' pre- and post-test of FE, and students' responses to the assessment tasks.

The processed of constructing pedagogical knowledge of FE were characterized as: suspecting the instructional sequence scheduled in textbook, conjecturing and justifying hypothetical learning trajectory, and reflecting on FE teaching. Two suspects relevant to instructional sequence of FE Jing-Jing addressed included suspecting the priority of continuous model in FE teaching and suspecting the activities of generating FE multiplying denominator and numerator by a nonzero number. The hypothetical learning trajectory was supported by the following arguments: 1/2 as a reference point strategy of ordering fractions, FE first developed in discrete model and followed by continuous model, and naming a fraction in more than one way by various ways of packing. The study found that the teacher elaborated and refined her pedagogical knowledge of FE on the basis of the objectives scheduled in the textbook was resulted from research-based data of students' learning fraction provided in the teacher professional development program.

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PUPILS' TOOLS FOR COMMUNICATING META KNOWLEDGE

Nora Lindén

Bergen University College

How do young pupils with learning difficulties and their teachers understand the special education provision? Is it possible that differences in understanding creates a less positive learning context for pupils with special educational needs, and contributes to, or reinforces, their difficulties?

These questions emerge from a study of how teachers, pupils and parents understand the classroom context. The phenomenon of understanding belongs to the micro context of a person. The understanding constitutes in term an individual rationale for action (Lindén 2002). The participants understanding will differ from each other because of the different part they play in the special educational arena. They have different obligations and expectations regarding the provision. In the classroom situation the pupils are supposed to acquire new knowledge. At the same time the pupils acquire knowledge about knowledge, about learning itself. This is what Bate son (1972) calls meta-knowledge. The meta-knowledge represents a person's understanding of the situation. If the pupils do not understand what learning is about, if the subject taught in school does not belong to the pupils' field of interest, the goal for learning is not part of the pupils understanding. The reason for learning is not present. In this perspective it is interesting to search for an explanation to some of the learning problems developed in school.

The presentation will focus on and discuss the pupil's tools for communicating their understanding as they appear in the study.

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MATHEMATICS EDUCATION, CULTURE AND NEW TECHNOLOGIES

Dr. Abigail F. Lins (Bibi Lins) Dr. Carlos F. de Araújo Jr. Carlos H. de J. Costa
Dr. Iara R. B. Guazzelli Rosângela M. C. Bonici
UNICSUL - Brazil

This talk discusses some theoretical issues related to culture and meaning production for technology that come from previous research done by Guazzelli (2002, 2004) and Lins (2002, 2004) during their doctoral studies. For this reason, methodological issues will be narrowed discussed here. The three-year research study is about a collaborative work with 13 Brazilian secondary state school teachers. Two are Master students of the Graduate Program in Science and Mathematics Education at University of Cruzeiro do Sul (UNICSUL). Mathematics Education can play the role with respect to the anthropological dimension showed in the culture. In terms of theoretical perspective, it is our intention within the research study to exploit the four poles proposed by Morin (1977, 1981, 2000): the day-by-day experience, knowledge, code and patterns in a way of elucidating the meaning production to technologies, which expresses the way of being of that community. D'Ambrosio (2003) calls our attention to a new conception of mathematical education in favour to the development of complex thinking, one of the most important change nowadays; it relates to getting rid of linear thinking and integrating qualitative and quantitative dimensions in a richer and more complex synthesis. Another change concerns the use of technologies. Their uses were mostly linked to the sense of power, control and economical growth. Education, in particular Mathematics Education, can take the challenge of joining complex thinking to the use of technologies and appropriate them as mediation to a new cultural perspective. We believe that by carrying out this research study will give room for discussing new theoretical, methodological and epistemological perspectives to the use of technologies in Mathematics Education.

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INVESTIGATION OF STUDENTS' VIEWS OF MATHEMATICS IN A HISTORICAL APPROACH CALCULUS COURSE

Po-Hung Liu

National Chin-Yi Institute of Technology, Taiwan

The role of college students' views of mathematics in the leaning of advanced mathematics has been cited in several research literatures (e.g., Carlson, 1999; Kloosterman & Stage, 1991), endorsing Schoenfeld's claim that developing a mathematical point of view is a potential indicator of strategies or approaches students adopt while engaging in mathematical tasks. On the other hand, several documents also call for awareness to enhance students' understanding of the nature of mathematics (AAAS, 1990; NCTM, 2000). To this end, one of the general goals for all students is learning to value mathematics, to focus attention on the need for student awareness of the interaction between mathematics and the historical situations from which it has developed (NCTM, 2000).

The purpose of this study was to investigate how Taiwanese college students develop their mathematical point of views in a historical approach calculus course. At the beginning of the semester, by means of administering all students an open-ended questionnaire, conducting follow-up interviews to a random sample, the study attempted to examine students' initial conceptions of mathematics. During the subsequent academic semester, the students experienced a calculus in which the sequence was structured in historical order and historical problems plays a central role serving to lead students to search for solutions and compare diverse thinking mode of mathematicians in history. Near the end of the semester, all participants answered the identical questionnaire and the same students were interviewed to pinpoint what shift their views on mathematics had undergone. It was found that participants initially tended to hold an instrumentalist view of mathematics, yet were more likely to value logical processes in doing mathematics afterward and leaning toward a conservative attitude toward certainty of mathematical knowledge. Their focus seemingly shifted from mathematics as a product to mathematics as a process.

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CONJECTURE ACTIVITIES FOR COMPREHENDING STATISTICS TERMS THROUGH SPECULATIONS ON THE FUNCTIONS OF FICTITIOUS SPECTROMETERS

Shiang-tung Liu[1] Feng-chu Ho[2]

[1] National Chiayi University, Taiwan

[2] Yan-Shuei Elementary School at Tainan County, Taiwan

Mathematics teachers are supposed to design meaningful tasks to motivate students' interest and to enhance students' communication and reasoning. Under various contexts, if meaningful tasks are designed by mathematics teachers for students to work out, then students would benefit more from those contexts of problem solving. For example, in the unit for learning three statistics terms, i.e. *median*, *mode*, and *range*, the authors provided opportunities for students to formulate, verify, and modify their presumptive rules rather than directly told them the rules to find those three terms. Such a learning process might result in better student performance.

The purpose of this study was to describe students' problem solving performance when they were initiated to make conjectures for comprehending three statistics terms. To give students the opportunities to formulate, verify, and modify their conjectures, three terms first were temporarily replaced by three spectrometers in the instructional activities. Students then tried to conjecture the functions of the three spectrometers and thus to give names for the three spectrometers according their intuitive perceptions.

During the conjecture activities, as developing computational ability was not the focus of the study, the authors encouraged students to utilize calculators to find their answers, so as to reduce their mistakes. There were three stages in the conjecture activities. First, the data and fictitious spectrometers were introduced to students. Second, based on the analyses the relationships between data and answer, students identified the functions of the three spectrometers. Finally, students were asked to name the three spectrometers according to their functions. After the three stages, students were encouraged to discuss the applications of the median, mode and range.

The findings of this study were as follows: 1) Students could comprehend statistics rules from inductive speculation. 2) Students could find the functions of three spectrometers from the process of conjecturing, verifying and modifying. 3) Students could provide intuitive terms for the functions of the three spectrometers 4) Students could make sense of three terms from the process of giving names. 5) Students modify those names successfully through peer discussion.

TURNING MATHEMATICAL PROCESSES INTO OBJECTS

Mollie MacGregor

University of Melbourne

Conceptualising mathematical processes as abstract objects (commonly referred to as 'reification' in the literature) is considered to be an essential mathematical activity. According to several authors, it is an ability that is difficult to acquire. In everyday life and language people have no difficulty thinking about processes and actions as things (e.g., "a rise in interest rates; "consumption of junk food"; "global warming"). In the field of mathematics education, thinking about processes as things is considered to be a major difficulty at all levels of algebra learning. It is well known that beginners in algebra are surprised to find that an expression (e.g., $x + 5$) can denote an operation to be carried out (i.e., a process) and also denote the result of that process - a mathematical object. However this initial obstacle is overcome with appropriate teaching. In contrast, the cognitive adjustment required to conceptualise a process itself - not its result - as an object is considered to be difficult for many students at all levels of mathematics and perhaps impossible for some. I have found no reports of empirical studies that explain how evidence for this difficulty has been obtained, how widespread it is, and how it may be overcome. Nevertheless there are various theories about the cognitive actions and structures that may be involved.

It is interesting to note that turning processes into objects is a major focus of literacy teaching in the middle grades of schooling. Mastering grammatical features of language enables actions to be described as things and new concepts to be developed. Many students who are able to write about "what I did" and "what happened" in the primary grades need much instruction and practice in the middle grades as they learn to describe these events as things. When processes become things (expressed by noun phrases instead of verbs) they can be reflected on, generalised, placed in causal relationships with other things, and discussed; they can take part in other processes. For example, "the temperature of planet Earth is increasing" becomes "global warming", which can now be placed in relationships with other things such as "the melting of Antarctic ice sheets" and "extinction of species". Similarly, mathematical processes can be conceptualised as things that can be used in chains of deductive reasoning, placed in relation to other things, and take part in other processes.

In ordinary language, people learn how to turn processes into things. Why should they not learn, with similar success, how to turn mathematical processes into things? With this question in mind, I discuss speculations, opinions and theories in the literature on how reification of mathematical processes takes place.

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THE BENEFICIAL AND PITFALL ROLE OF THE SPOKEN LANGUAGE IN THE INFORMAL DEFINITION OF STATISTICAL CONCEPTS

Michal Mashiach-Eizenberg, Ilana Lavy

Emek Yezreel Academic College

The formal definitions of mathematical concepts constitute of a symbolic language which is independent of any spoken language. Yet, since these definitions in their symbolic format are difficult to teach and understand, we use the spoken language to define them non-formally. In case of the Hebrew language, we use various words to describe mathematical concepts. Some of these words have the same meaning in the everyday use as in the mathematics such as 'average (mean)'; some of them have a different meaning such as 'mode', and some have an opposite meaning such as 'significance level'.

Researchers explored various aspects regarding the understanding of statistical concepts (Falk, 1986) such as how technology can help students understand, integrate, and apply fundamental statistical concepts (Chance et al., 2000). In the current study we examine students' difficulties while defining statistical concepts informally and how they are influenced by the everyday meaning of the same words.

A questionnaire which included several statistical concepts and everyday expressions bearing the same meaning as the statistical concepts was given to second year college students that had already studied probability and statistics. The students were asked to write down a definition to each concept in their own words (informally) and to add an example of its use.

Categorization of the students' definitions revealed the following: using the meaning of each word separately; confusing between related concepts; using the concept within its definition; using the word's stem or tone; bringing the mathematical notation of the concept and others.

In the presentation, we will provide our full categorization and examples for each one of the revealed category and also bring possible explanations regarding the influence of the spoken language and the everyday use of words on the informal formalization of statistical concepts.

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A DIDACTIC PROPOSAL FOR SUPPORTING PUPILS' PROPORTIONAL REASONING

Christina Misailidou

The University of Manchester

This communication aims to propose ideas concerning the teaching of the topic of 'ratio' and proportion' in primary school. Results from teaching sessions focused on 'ratio' tasks are the focus of the presentation. These sessions were part of a case study concerning the teaching of 'ratio' in a primary school classroom consisting of 29 pupils (aged 10-11).

A diagnostic test for ratio and proportion was used, prior to the teaching sessions, for exposing the pupils' strategies and errors in varying 'ratio' items (Misailidou and Williams, 2003). Selected problems from that test (Misailidou and Williams, 2004) were used as central tasks for the subsequent teaching sessions. The role of discussion and of the generation of arguments was considered crucial in aiding the pupils' proportional reasoning. Thus, 'tools' for facilitating the pupils' arguments in discussing the 'ratio tasks' were designed and used.

During this communication, some representative episodes from the teaching sessions will be outlined; the tools that have been used for facilitating discussion and the resulting pupils' argumentation will be presented and discussed.

It is argued that the main characteristic of an effective didactic proposal for supporting pupils' proportional reasoning is the exchange of arguments in discussion. Such an exchange can only be productive when supported by tools specifically designed for the task under discussion.

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AN INVESTIGATION ON PROOFS EDUCATION IN KOREA

GwiSoo Na

Cheongju National University of Education

In this article, we investigate the various attempts in didactical transposition by teachers and the difficulties which students have in learning proofs. Finally, we suggest implications for improving proofs education.

THEORETICAL FRAMEWORK

The results and discussion that follow in this article arise from the "didactic transposition theory" originally described in the works of Chevallard (1988; Kang, 1990), and "quasi-empiricism" originally described in the work of Lakatos (1976).

METHODOLOGY

The proofs-classes analysed and discussed in this article are 30 classes of 8-grade. I used the participating observation method to analyze the features of proofs-classes. In addition, we informally interviewed two teachers and some students by using descriptive questions.

RESULTS AND DISCUSSIONS

- The Various Attempts in Didactical Transposition by Teachers
- The Weak Points in Teaching Proofs
- The Difficulties in Learning Proofs

THE IMPLICATIONS FOR PROOFS-EDUCATION

The first implications for teaching proofs on the basis of the results of analysis in this study in that we should teach proofs as a dynamic reasoning activity that unifies the analytical thought and the synthetical thought. Second, we should make students guess the conclusion by themselves by giving the assumption alone instead of giving both assumption and conclusion, then make students perform the proofs in order to justify the truth of their own conclusion.

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IMPROVING SPATIAL REPRESENTATIONS IN EARLY CHILDHOOD

Andreas Oikonomou, Marianna Tzekaki

Aristotle University of Thessaloniki

In this paper, a teaching intervention programme aiming at improving spatial reasoning in early childhood is presented. An initial study (Oikonomou, 2005), which investigated the development of spatial representations in pre-school children (4, 5 – 6 years old) showed that, despite important individual differences, the ways in which the subjects represented and handled spatial situations evolved: from the exploitation of either a holistic or partial approach in the beginning to the control of elaborated one and two dimensions situations. These results contributed to a better understanding of the development of children's spatial representations, as detected and discussed by Case and Okamoto (1996) and Newcombe and Huttenlocher (2000) and more in accordance to Siegler's than to Piaget's approaches (Siegler, 1998).

Based on these findings, a teaching intervention programme was designed aiming at the investigation of the possibility to help children to improve the ways they represent and handle spatial situations and hence the knowledge and the abilities required.

The 52 pre-schoolers who participated in the experiment were pre-tested and classified in different groups according to their performances. The teaching intervention included group activities, where the task was related to the reproduction of material or to graphical configurations. The spatial relations involved were relative positions or locations in space, locations according to a reference system, colinearity, horizontality, perpendicularity. The children were post-tested a month after the end of the intervention and their performances was compared to that of a control group.

The comparison of the children's performance in the diagnostic and the evaluative tests showed that the improvement of the experimental group was very significant (54%, with an effect size=1,24), whereas that of the control group was modest (28%). The improvement concerned all the dimensions of the test, thus showing that, working with appropriate tasks, the children of the sample could ameliorate their spatial reasoning.

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THE DEVELOPMENT OF PATTERNING IN EARLY CHILDHOOD

Marina Papic

Macquarie University, Sydney

It is well recognised that patterning is fundamental to the abstraction of mathematical ideas and relationships and the development of mathematical reasoning. (English, 2004; Mulligan, Prescott & Mitchelmore, 2004). Educators in the early years can promote the development of patterns and relations, by helping children think beyond the 'play' situation, building on everyday mathematics but incorporating traditional strands of the mathematics curriculum (Ginsburg, 2002). This study raises two key research questions: Is there a link between a child's ability to pattern and their development of pre-algebraic and reasoning skills? Can an intervention program focused on identification and application of patterns, show long term benefits for children's overall mathematical development?

This project tracks the development of 53 young children's pre-algebra (patterning) skills from preschool to the second year of formal schooling. Case-studies of two matched preschools ('intervention' and 'non-intervention') examined the influence of a mathematics intervention promoting children's patterning over a 6 month period. Individual task based interviews were conducted at three intervals over an 18 month period. Tasks comprised construction of towers using blocks, subitising dot patterns, arrays, grids, patterns in the formation of borders and hopscotch, as well as numerical sequences.

Children who performed poorly on patterning tasks at all interview points were identified as low achievers on other numeracy assessments. Children from the intervention program consistently showed a greater level of improvement in patterning tasks than the non-intervention sample at the end of the pre-school year. This was sustained at follow up interviews one year later. Survey and interview data from participating teachers highlighted their lack of confidence and awareness of the importance of patterning in mathematical reasoning and understanding.

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THE DEVELOPMENT AND PILOTING OF A SIX-MONTH PRE-ITE MATHEMATICS ENHANCEMENT COURSE

Adrian J Pinel, Maria Dawes, Carol Plater

University College Chichester, University of Portsmouth, University of Brighton, UK

In order to expand the pool of potential secondary mathematics teachers, a Mathematics Enhancement Course [MEC] was designed and run. This has already enabled graduates with a broad range of first degree subjects to access initial teacher education courses. Our pilot MEC led to some insights into [a] what priorities are needed in such courses, and [b] what fresh perspectives such graduates can bring to the subject and to their subsequent teacher preparation course. Our key research question was whether a course based upon consistent attempts to develop profound mathematical understanding could succeed over an intensive six-months.

THE DESIGN OF MEC

The fundamental design concept was to follow the lead given by Ma [1999] encapsulated by the maxim *'Know how, but also know why'*. Profound understanding of fundamental mathematics was built into our approach to course design and in-built network of connections between its taught units; it also formed the basis for our assessment strategies.

Responses and persistence of MEC students

The piloting was very closely monitored by the funding government agency and by their appointed evaluator. Apart from the natural 'goldfish bowl' effect, this added to the range of data collected on student responses. Some students found the whole idea of pursuing profound understanding quite counter to their prior experience and cultural assumptions. These students were the most 'at risk' and despite a very strong group support ethic fostered among the students, four of the original 25 did not complete the six-month course. Responses of those who completed the course were very positive, and 20 moved directly on to a one-year teacher preparation route.

Initial findings

Half way through the teacher preparation route, 19 are still continuing, one having intermitted but not withdrawn. The indications at this stage from all observers are that the MEC has been successful. This short presentation will draw out some key issues.

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AUTHENTIC ASSESSMENT IN MATHEMATICS CLASSROOM: A PARTICIPATORY ACTION RESEARCH

Yachai Pongboriboon, Khonkaen University, Thailand
Sompong Punturat, Khonkaen University, Thailand
Anchalee Chaiyo, Nakon KhonKaen School, Thailand

Authenticity is an important element of new model of assessment. Some defined authentic assessment as a synonym for performance assessment, while others argue that authentic assessment put a special emphasis on the realistic value of the task and context. The definition of authentic assessment used in this study is an assessment requiring students to use the same competencies, or combinations of knowledge, skills, and attitudes that need to apply in the criterion situation in professional life.

The purpose of this study was to select and develop procedure for multiple authentic assessment tools and techniques (such as: task, work project, open-approach question, observation, interview, and self-evaluation) involved in real life or authentic tasks and contexts of mathematics in geometric content area for grade seven students. The study was also described in which university instructors served as the participatory researcher to provide collaborative research with school mathematics teachers. Participatory action research process and characteristics was derived including a series of six phases. They were phase 1: forming collaboration; phase 2: problem identification for action research; phase 3: data collection and analysis; phase 4: data synthesis and generation of recommendation; phase 5: design of data-driven action/intervention; and phase 6: evaluation of intervention.

Among the salient finding identified were (a) to ensure the effectiveness of authentic assessment should be linked to authentic instruction and learning activities, (b) regarding impact on student's learning, varying multiple assessments and criterion situations should be related to meaningful real-life situations, and (c) scoring rubrics of content knowledge, skills, and attitudes preferred venues of communication assessment criteria and results to both students and teachers. Methods for developing and refinement of authentic assessment procedure were described.

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THE CANDY TASK GOES TO SOUTH AFRICA: REASONING ABOUT VARIATION IN A PRACTICAL CONTEXT

Chris Reading
SiMERR National Centre, University of New England

Helena Wessels & Dirk Wessels
University of South Africa

Lack of recognition of the role that variability plays in sampling prompted research into developing the 'candy' task (Reading & Shaughnessy, 2004), various forms of which have been given to students (Grades 4 to 12) in a number of countries. Responses were categorized according to *Centre* and *Spread* (Shaughnessy et al., 1999) exhibited when students predicted samples. Of interest was how the reasoning about variation in a sampling situation by students from the Republic of South Africa (RSA) compared with that of students in the 'other' countries where the task had already been implemented. A sample of 100 students, from primary (Grade 6) and secondary (Grades 8 and 10) schools, were given a detailed *Demonstrated Questionnaire* (a compromise between a questionnaire and an interview) with questions in three different formats, *List, Choice* and *Range*. Students were offered the opportunity to alter responses after viewing demonstrations of the sampling.

Differences between performances RSA students and those from 'other' countries were significant, with more *correct* (in relation to both *Centre* and *Spread*) responses for RSA students in all three question types. For *Centre*, RSA students exhibited a similar number of incorrect (poorly centred) responses but showed a distinct trend to be too low, compared to too high for the 'others'. For *Spread*, the RSA students exhibited more incorrect responses but these were similar to the 'other' responses in terms of incorrect variation expectations as too narrow or wide. While Grade 6 students performed much better than the 'others', there was a lack of comparable improvement across Grades 8 and 10. Most RSA students with correct responses chose not to alter them when given the chance; of those who did change, Grade 6 and 8 students mostly produced better responses while many Grade 10 students reduced the quality of their response ('other' data for changed responses was not available for comparison). Teachers already acknowledge the importance of personal experiences that students bring to learning situations. Further investigation is needed to determine what learning environment factors in the RSA could contribute to performance differences on such sampling tasks, influencing students' reasoning about variation.

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SOME CHARACTERISTICS OF MENTAL REPRESENTATIONS OF THE INTEGRAL CONCEPT – AN EMPIRICAL STUDY TO REVEAL IMAGES AND DEFINITIONS

Bettina Rösken, Günter Törner

University Duisburg-Essen, Germany

This work deals with students' mental representations of the integral concept. By means of empirical data, students' conceptual images and corresponding solving competence for this specific mathematical concept were analysed on the theoretical basis of the concept image and concept definition model. This dual confrontation is the core of Tall's and Vinner's theory (1981) which emphasizes the interaction of intuitively and heuristically influenced images of a mathematical concept and its formal definition. Our empirical study (Rösken, 2004) shall demonstrate the fruitfulness of this approach and is based on the following questions: Which concept images and which concept definitions exist in the conceptual field of the definite integral? Which incoherence is there between concept image and concept definition? Could this be the reason for misconceptions? On the background of the theory described, students' concept images and concept definitions of the integral concept are established by combined survey methods. First, following Tall and Rasslan (2002), we assume that the concept image will become clear by working with the corresponding questions. Furthermore, for the concept image the students have to create a structured mind map to develop a suitable description of the cognitive structures via a graphical representation. The representations of the concept definition are examined by query of the definition line.

The results of the empirical study show that the students developed varied concept images of the definite integral. The mind maps revealed that the majority of students knew all aspects relevant to the concept. As expected, the definition line was represented rather weakly. Part of the students had major problems with the concept image tasks. The answers concerning the concept definition already showed that the geometric interpretation of the integral as an area was reflected as only one limited aspect.

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PRE-SERVICE TEACHERS' MATHEMATICS SUBJECT KNOWLEDGE: ADMISSIONS TESTING AND LEARNING PROFILES

Julie Ryan Barry McCrae

Australian Council for Educational Research

We will report the development of an assessment instrument that provides a profile of the mathematical ability of pre-service students for each strand of the curriculum: Number, Measurement, Space and Shape, Chance and Data, Algebra, and Reasoning and Proof. We will describe the test developmental cycle, our research analyses and test validation involving a sample of 430 pre-service students in the first year of their training. We will report our evidence for the predictive power of the test for course achievement.

Our test was not only summative but also provided opportunities for diagnostic assessment. Errors and misconceptions were collected and analysed for all items. We will report the patterns of errors of these adult learners. We will also outline how course teachers can use such errors to support their students' learning.

Students seeking admission to primary teacher education courses (both undergraduate and post-graduate) come with a variety of mathematics backgrounds. Since they will be required to teach mathematics, their mathematical attainment level is of importance in admission decisions. In Australia the range of mathematical credentials of students seeking admission to teacher education courses makes informed selection difficult. Additionally, a single achievement grade provides no detail of student areas of strength or weakness. Evidence of mathematical attainment thus is weak.

We sought to strengthen that evidence. We suggest that better 'tools' can be used to find potentially strong teachers who may not have taken traditional routes in the school curriculum. Our research makes a contribution to knowledge by providing fine-grained detail about pre-service teacher subject knowledge in mathematics, including current attainment, patterns of errors and misconceptions and predictors of course success.

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MATHEMATICS TEACHERS' PEDAGOGICAL IDENTITIES UNDER CONSTRUCTION: A STUDY

H. Sakonidis

Democritus University of Thrace, Greece

It is well argued today that for any educational reform to succeed, it should aim at constructing new pedagogical identities, that is, new 'forms of consciousness' in teachers and learners. These identities emerge as reflections of differing discursive acts (Bernstein, 2000).

Most of today's mathematics education reforms tend to see mathematics as a fallibilistic discipline, its learning as meaningful in its own right and also to life and its teaching in socio-constructivist terms. In this context, a challenging way of pursuing the formation of corresponding pedagogical identities by teachers would be through discourse and reflection within the context of a community of practice (Lave & Wenger, 1991). In such a perspective, teachers would come to see themselves as being joined with colleagues to discuss teaching practice, develop consensus on alternative ways to promote students' mathematical thinking and support each other through difficult points in the change process.

The work presented here reports on an attempt towards this direction. Its purpose was to introduce alternative mathematics teaching approaches to secondary schools with culturally mixed students' classes in the north-eastern part of Greece. The 15 rather traditionally educated and teaching teachers involved in the two years study were asked to exploit a package of mathematical activities in their classrooms. The constitution of this package was based on offering opportunities for autonomous learning to students, on utilising their everyday experiences and on interacting in small groups. In parallel, the teachers participated in regular meetings with colleagues and mathematics educators at school and also at prefecture level to share dilemmas, failures, convictions, beliefs, etc. In these meetings, mathematics educators systematically used the accumulative knowledge of the field to feed in the teachers' specific actions and understandings of the learning environment under construction. The results showed that such an approach has the potential to develop a powerful and robust sense of teacher identity by making explicit, deconstructing and problematising his/her personal theories through reflection and discourse.

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MIDDLE SCHOOL MATHEMATICS AND THE DEVELOPMENT OF MULTIPLICATION CONCEPTUAL KNOWLEDGE

Rebecca Seah and George Booker

The need to think mathematically has become essential for students (Booker, 1998) in a new millennium 'awash in numbers' and 'drenched with data' (Steen, 2001). Many middle school students are increasingly disengaged from school in general and mathematics in particular and are not gaining basic mathematical ideas. In particular, a lack of multiplicative thinking/reasoning ability appears to be a major cause of students' difficulties with further mathematics (Thomas & Mulligan, 1999). Multiplication is part of a larger context of what Vergnaud (1994) termed a 'multiplicative conceptual field' – a bulk of situations and concepts that involve multiplication and division. An ability to engage in multiplicative thinking requires a clear conceptual understanding and full knowledge of mathematical processes and the relationships between them. Many high school teachers assume that such basic, fundamental ideas are taught in primary school and tend to focus solely on higher mathematics learning and abstract reasoning irrespective of their students' readiness.

Two Year 8 classes in a socio-economically disadvantaged area were examined to ascertain the level of mathematical understanding and investigate classroom interactions that promote mathematics learning. Class A emphasised collaborative teaching and 'open dialogue' to construct mathematical ideas. Class B employed a traditional 'initiation-reply-evaluation' approach (Richards, 1991) where the teacher taught for the first 10 - 20 minutes then asked students to work on a set task individually. Results indicated that the most students' knowledge of multiplication was restricted to procedural rather than conceptual understanding. Class A students demonstrated a deeper degree of conceptual knowledge than Class B.

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A REVISIT ON PROBLEM SOLVING

Shy Haw-Yaw, Feng Shu-Chen, and Liang Chorng-Huey

National Changhua University of Education

A problem is traditionally considered as a set of criteria and tasks to be accomplished. In the issues of problem solving, the focus has been centered on problem understanding and strategy planning, which also indicate the mark of two different stages. Most models on the problem solving implicitly suggest that the picture of a problem is quite robust and will maintain its appearance throughout the solving procedure. However, the fact is that solvers usually tackle the problem with misunderstanding and hence fail to finish the job.

The objective of this study is to explore, by using qualitative methodology, the key points where a solver is assuming a good understanding or realizes his mistake on a problem. By analyzing the extent of a problem, recognized by a solver, in the course of problem solving, we try to provide a different perspective on describing a problem and problem reading. Twenty samples have been selected out of different grades, abilities and experiences. The interview data is collected right after samples have finished their work on a set of word problems.

The finding shows that the extent of a problem is not robust. Solvers are frequently bringing in not only useful but also irrelevant or false material in reading the statement of the problem or in the process of problem solving. A solver may find his conception on the problem mistaken when he has new findings which do not agree with the previous ones. Sometimes a solver may come up with a wrong answer serving as the best candidate to meet all the requirements asked by the problem. And the solver can find no hindrance when carrying out the checking procedures due to an incorrect realization of the problem. According to the findings of this study we propose that a problem, in a solver's mind, should be considered as an organic object and its extent will grow or mutate along the course of the problem solving. The problem reading is then better considered as a cognitive activity in realizing every single piece of the material sensed by the solver from the very beginning to the end of the problem solving. Surprisingly, the problem solving efficiency can be greatly improved merely by reminding the solver to keep reevaluating his understanding about all the data in his mind.

PROBING INDIGENOUS STUDENTS' UNDERSTANDING OF WESTERN MATHEMATICS

Dianne Siemon, Fran Enilane & Jan McCarthy

RMIT University, NT DEET

The Supporting Indigenous Students' Achievement in Numeracy (SISAN) Project 2003-2004 was aimed at researching the impact of authentic (rich) assessment tasks on the numeracy outcomes of middle years Indigenous students in a targeted group of remote schools in the Northern Territory[1]. The project involved trialling and evaluating a range of tasks aimed at identifying starting points for numeracy teaching. Initial results suggested that while the rich tasks helped identify 'what works' and highlighted important areas of learning need more generally, for example, number sense and mathematical reasoning, much more work was needed to develop these tasks to the point where they could be used more widely to support remote Indigenous student numeracy learning.

As a consequence, a small number of more focused tasks were introduced which provided a broader range of response modes and allowed teachers to identify learning needs more specifically. Originally developed to support pre-service mathematics teacher education at RMIT University, the Probe Tasks, as they were referred to, were chosen because they require relatively low levels of student literacy and focus on key number ideas and strategies, the area broadly identified by student responses to the rich assessment tasks. Participating teachers typically reported that as student responses to the Probe Tasks were more readily observed, interpreted, and matched to expected levels of performance, they felt more confident about identifying and responding to student learning needs in a targeted way, and as a consequence, more likely to have a positive impact on student numeracy learning. This was particularly the case for the Indigenous teacher assistants and secondary-trained teachers with a non-mathematics background. This suggests that the Probe Tasks, and the associated Probe Task Advice developed to support the work of teachers in this instance, offer a useful means of building remote teacher's pedagogical content knowledge for teaching mathematics.

This presentation will illustrate the Probe Tasks and explore the implications of the teachers' responses to the use of the tasks in relation to improving the levels of Indigenous student numeracy in remote communities.

1 Funding for this project was provided by the Australian Government Department of Education, Science and Technology under the *National Literacy and Numeracy Strategies and Projects Programme*. The views expressed here are those of the author and do not necessarily reflect those of the Australian Government Department of Education, Science and Technology or the NT Department of Employment, Education and Training.

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ASSESSING BEGINNING PRE-SERVICE TEACHER KNOWLEDGE: AN EARLY INTERVENTION STRATEGY

Kelli M. Slaten Sarah B. Berenson Maria Droujkova Sue Tombes

North Carolina State University

The purpose of this preliminary study is to investigate a way of assessing beginning preservice teachers' knowledge and philosophies of teaching and how that assessment could potentially be used as an early intervention strategy for knowledge development in teacher education programs.

Much of the research in teacher education has divided teacher knowledge into two separate categories: subject matter knowledge and pedagogical knowledge (Ball & Bass, 2000). Learning to teach requires the development of both subject matter knowledge and pedagogical knowledge. Shulman (1986) described pedagogical content knowledge as the knowledge teachers need to teach a particular subject. Grossman (1990) further addressed four knowledge components of pedagogical content knowledge: knowledge of instructional strategies, curricula, how students learn, and why teaching a particular subject is important. This study addresses relationships between pre-service teachers' conceptions of teaching mathematics and knowledge of instructional strategies. It is conjectured that these relationships can address the development of knowledge of how students learn.

The researchers acted as co-teachers in a 14-week undergraduate methods course for students preparing to become middle or secondary school mathematics teachers. Each week, students were required to develop four different representations based on introducing concepts related to proportional reasoning. Students were also required to write a personal philosophy of teaching for a teaching portfolio. The representations and philosophies of three students were compared and were found to be conflicting. All three pre-service teachers expressed a desire to engage students and teach them the utility of mathematics. Yet, two relied on formalized mathematics throughout their representations. This information can be used to address pre-service teachers' emerging perspectives of teaching mathematics.

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THE IMPEDIMENTS TO FORMULATING GENERALIZATIONS

Bharath Sriraman

The University of Montana, USA

In this paper, the psychological impediments to the process of generalization are explored within the context of classroom experiments. Extant descriptions of the process of generalization are (1) the abstraction of similarities from disparate problem situations Mitchelmore, 1993); (2) *uniframing*, i.e., casting out cases that don't fit a general concept or definition emerging from numerous cases (Lakatos, 1976; Sriraman, 2004), among many others. These descriptions consist of some similar elements, yet the Gestalt (or the whole) remains elusive. One reason why the Gestalt remains elusive is that the literature describes components of "*what is*" generalization. There is a lack of studies that describes components of "*what is not*" generalization, which would greatly add to the existing literature and fill the missing elements in extant descriptions. To study the impediments to generalization a teaching experiment was conducted with 14 year old students at a rural American high school in which students were asked to solve 5 non-routine combinatorial problems characterized by a common principle (namely the Dirichlet principle) over a 3-month period. Since the research was concerned with exploring factors constituting individual student's psychology of generalization all student work was non-collaborative. Data collected via journal writings and clinical interviews was analyzed using the constant comparative method of Glaser & Strauss (1977) to sieve out factors that impeded students from formulating generalizations. As a result, similar "impeding" focussing factors fell under the categories of "*repeated use of similar examples*", "*focus on superficial (numerical features) of representation*" and "*focus on context*". The study reveals that these focussing phenomenon play an important role in how and what students abstract from a given problem situation and often impede the formulation of generalizations. One implication of these findings is that the theoretical properties of focussing phenomenon need to be further studied by constructing different classes of problems in which the complexity is varied gradually via the use of a complexity load metric. This will allow researchers to document the finer perturbations/variations within focussing phenomena that lead to false generalizations

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STUDENT NUMERACY: A STUDY OF THE NUMERACY DEVELOPMENT OF TEACHING UNDERGRADUATES

Noel Thomas, Wendy Hastings and Bob Dengate

Charles Sturt University, Bathurst, Australia

Teacher education is increasingly coming under pressure to be accountable to government for the quality of the graduates going out into the schools. This is at a time when University funding in Australia has been effectively cut back so that even provision of the essential professional experience component of course is under scrutiny because of costs. The literacy and numeracy competencies of teacher graduate teachers are being analysed and questioned. Teachers were once held responsible for lack of literacy and numeracy of students, now it is the University Teacher Education programmes that are being blamed for 'lack of standards' in schools. So the political stance seems to be that 'teachers are not being taught how to teach reading, writing and arithmetic'! In Australia, Institutes of Teaching (Vic) or Teachers (NSW) have been established in each state and are responsible for endorsing courses and programs of teacher preparation, as well as for managing teacher accreditation against determined teaching standards for beginning teachers. The Victorian Institute of Teaching (VIT) requires beginning teachers to build a portfolio containing evidence that teaching standards for registration have been met during a period of induction. The study described here is significant as it addresses one aspect of this public concern, that is, the numeracy competence of undergraduate primary school teachers.

This study aimed to identify the relationship between teacher education students' prior numeracy skills and knowledge and to track the development of numeracy competence during their undergraduate course. All students who entered the BEd(Primary and EC) and BSocSc(Psychology)/BTeach(Prim) courses in 2001 completed an ACER Mathematics Competency test at the beginning of their studies. Ten students from the Primary course were randomly selected and the progress of each of these students was tracked over the passage of their four year course through case studies.

This paper will report the findings from the data collected from the students who have graduated in 2005 and demonstrate growth in mathematical understanding, classroom competence as mathematics teachers and increasing confidence with mathematics as a learning/teaching area in the primary school.

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INTERACTION BETWEEN TEACHING NORMS AND LEARNING NORMS FROM PROFESSIONAL COMMUNITY AND CLASSROOM COMMUNITIES

Wen-Huan Tsai

National Hsin-Chu Teachers College, Taiwan

This study was designed to support teachers on developing teaching norms based on classroom communities in which students are willing to engage in discourse. A collaborative team consisted of the researcher and four first-grade teachers. The professional community intended to generate norms of acceptable and appropriate teaching based on teachers' observation of their students' learning mathematics in classroom.

The study was based on the theoretical perspectives of cognitive development and sociology in order to examine both teaching and learning in professional community and classroom communities.

The teaching norms being developed in the study was an important notion of analyzing the teachers' pedagogical reasoning in their classroom communities (Tsai, 2004). The teaching norms were not predominated by a criteria set by outside of the professional community. Instead, the teaching norms were generated from each teacher's classroom community and were then continually evolved by the professional community between the interactions of the teachers and the researcher based on some events of classroom communities of the teachers.

As we have shown, the teachers established the teaching norms through their negotiations in the professional community were actively restructured personal beliefs and values and then resulted in their increasing ability of autonomous teaching. The result indicated that teaching norms and learning norms were mutually interactive. It is not only developing teachers' practical teaching, but also improving children's learning. Therefore, one way of improving teacher' pedagogical reasoning and students' learning with understanding simultaneously was to create a professional community for developing teachers' teaching norms in which were based on the development of students' learning norms and contributed to the development of students' learning norms.

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"ERRONEOUS TASKS": PROSPECTIVE TEACHERS' SOLUTIONS AND DIDACTICAL VIEWS

Pessia Tsamir Regina Ovodenko

Tel Aviv University

There is a wide call to have mathematics classes in which students make conjectures, explain their reasoning, validate their assertions, discuss and question their own thinking and the thinking of others, and argue about the mathematical validity of various statements (e.g., NCTM, 2000). One of the issues addressed in the literature is the need to provide the students with rich mathematical environments. However, teachers' readiness to adapt innovative instructional practices is related to their beliefs about mathematics instruction. In our study, we developed materials aimed at encouraging the participants to discuss different solutions to mathematical tasks in light of related rules and definitions. The tasks are designed to stimulate the discussion of questions like: "Why is this so?", "Can we justify this?" "Is this explanation acceptable?"

In this paper we address secondary-school mathematics prospective teachers' solutions to an "erroneous task" (i.e., a task that includes contradicting data) and their disposition towards the presentation of such tasks, in high school classes.

Prospective teachers were presented with an "erroneous task", in which they were asked to determine whether it is possible to draw the graph of a function that satisfies four types of given: (a) $f{:}R \rightarrow R$, (b) 5 points $(x_i; f(x_i))$, (c) the range where f'(x) and f''(x) are positive, zero or negative, (d) the asymptotes. If the participants' answer was "yes", they had to draw the graph, and if it was "no", they had to justify their position. Then, the prospective teachers were asked whether they would use this task in their classes.

Our results show that the participants knew that it was impossible to draw the graph, and in their justifications they correctly pointed to different options of contradicting given. However, while they mentioned that by this task they gained extra insight into the relationship between the various components of the function and the related graph, their position regarding the presentation of such tasks in classrooms, varied. Several participants unconditionally supported the presentation by addressing the mathematical potential or the instructional merits of the task. Some rejected the presentation and others specified conditions to be fulfilled in order to allow the presentations of such tasks. The conditions related to the students that would benefit (e.g., only advanced students) and to the adequate timing for presenting such tasks (e.g., as an introductory activity).

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BLIND STUDENTS' PERSPECTIVE ON LEARNING THE GEOMETRIC CONCEPTS IN TAIWAN

Yea-Ling Tsao

Taipei Municipal Teachers College

This study focused on gaining perspective into the blind student's experience with mathematical concepts addressed included the ideas of parallel, perpendicular, angle and slope. The main purpose of this study collected data on the learning and understanding of aforementioned geometric concepts: parallel and perpendicular. The following questions will be addressed in the study: (1) What are experiences of blind students in learning geometry and mathematical concepts?(2)What experiences do blind students share?(3) What obstacles to learning mathematics are encountered?(4) How does learner come to an understanding of the concept?(5)What types of learning experiences are most beneficial?(6)How are the concepts related to their "every day lives"?(6)What do they think might another learner better understanding the concept? Data collection was conducted primarily through semi-structured interviews. Although there will be a basic outline for the interview. The results of this study provide evidence to support the importance of grounding the learning of mathematical concepts in everyday practical experience, especially for the blind student.

The results of this study provide evidence to support the importance of grounding the learning of mathematical concepts in everyday practical experience, especially for the blind student. The blind student's comprehension of certain geometric concepts is based primarily on her or his application of these concepts to real life situations. The more personal and practical the experience, along with the use of sensorimotor activities and embodied processes, the stronger the comprehension of the mathematical concept. Therefore, accommodations must be made to provide meaningful experiences within the formal educational setting that make use of experiential, kinaesthetic and auditory learning if the results of formal instruction in geometry are to be effective and useful to the blind learner.

MATHEMATICAL MODELLING AS LEARNING ACTIVITIES

Tai-Yih Tso

Department of Mathematics, National Taiwan Normal University

The purpose of this paper is to elaborate upon mathematics modelling as learning activities using dynamic geometric software for designing mathematics learning system. This learning system consists of a computer environment to display the properties of objects with multiple dynamic linked representations. The major benefit in learning how to do mathematical modelling for students is to encourage them developing a particular way of reflecting and acting on mathematics through making connection between mathematics and real world. We will discuss some examples to show how the learning activities be used in mathematical modelling instruction.

Mathematical modelling is a process of using mathematical language to describe, communicate, express, and think about the real world. The mathematical modelling process is important in both research and learning, however, the process has been applied in the research field, while, students seldom have experience when learning. Research in cognitive science and cognitive development has made it possible to progressively move to new levels of thinking about educational environments that promote learning. Students should be given opportunities to practice mathematical modelling for translating, interpreting, organizing and verifying the real problem to conjecture and generalize their finding. In this paper, we present a theoretical framework for designing and implementing a learning system with multiple dynamic linked representations based on reflection on action. It consists of a learning environment in which students control and operate the objects by means of mathematical modelling.

A prototype computer-based environment for students making mathematical modelling has developed with dynamic geometric software. The experiment of mathematical modelling as learning activities has shown following impacts on students' mathematics learning.

The computer can be an effective tool for students thinking their own way to solve the problems. Students increase their metaconceptual awareness that mathematics is not just the product of mathematicians work, but continually evolves in response to challenge both internal and external representations. Students realized that it is not necessary to spend much time on tedious calculations and memorizations in mathematics learning. Instead, they noted that it is useful for explore mathematics by the use of computer simulation.

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INNOVATIVE TEACHING APPROACHES IN DIFFERENT COUNTRIES

Marianna Tzekaki, Aristotle University of Thessaloniki

Graham Littler, University of Derby

This paper presents the results of the second year of the project IIATM "*Implementation of Innovative Approaches to the Teaching of Mathematics*". The project is realized by the collaboration of four European Universities (Charles University of Prague, Aristotle University of Thessaloniki, Cassell University and University of Derby) and aims to bring together teachers and teacher-trainers from different European countries. Teachers, across national boundaries, experience common constructive teaching approaches in their classroom, recording and exchanging experiences. International research shows that the shift from a familiar instructional practice to an innovative approach is not easily accomplished (Fennema & Neslon, 1997). Teachers find it very difficult to change from their established transmissive ways of teaching, even if the curricula and their trainers propose very interesting and creative tasks (Desforges & Cockburn, 1987). Research also shows that providing teachers with experiences where their own practices are challenged and opportunities to reflect on and rethink about them, has the potential to facilitate new insights and understandings of the teaching process (Aichele & Caste, 1994). The IIATM project allows teachers to try common activities in culturally different classrooms and encourages them to exchange ideas and experiences, comparing the use of constructive teaching strategies.

During the first year, the groups of researches and teachers established in each institution, developed tasks concerning various mathematical topics (Geometry and Polygons, Functional Thinking, Patterns leading to Algebra, First Arithmetic Concepts) (Tzekaki & Littler, 2004). During the second year, the groups exchanged the tasks and evaluated their use in different environments.

The collection of illustrations of the constructivist approaches with common tasks, including the analysis of classroom experiences from teachers' outcomes, as well as comments, discussion and an overview of remarks gained from this evaluation in two countries (Greece and Czech Republic) will be developed in this presentation. *(The Project is funded by the European Commission's Socrates/Comenius).*

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EQUITY AND TECHNOLOGY: TEACHERS' VOICES

Colleen Vale

Victoria University

Teachers use technology in mathematics to enhance the classroom ambience, assist tinkering, facilitate routine processes, and to accentuate features of mathematics (Ruthven & Hennessy, 2002). However there is some evidence to suggest that the use of technology may be accentuate cultural inequalities (Vale, Forgasz & Horne, 2004). Furthermore, innovations in the use of ICT in schooling, including those involving mathematics, have not targeted students from socially or culturally disadvantaged backgrounds (Kozma, 2003).

Teachers' understanding of diversity and equity is varied and related to their school setting (Quiroz & Secada, 2003). According to the literature, equity involves equal access, equal treatment, fairness and a commitment to achieving equal outcomes and the characteristics of equitable classrooms include: connectedness, collaboration, support, intellectual quality, and respect for difference. However achieving it is a complex task for teachers working in socially disadvantaged schools. In this presentation I will report on a current study that is exploring teachers' understanding of diversity and equity and how this relates to their practice regarding the use of technology in their junior secondary mathematics classes.

Teachers from socially disadvantaged secondary schools in Melbourne were participants in this study. Twelve teachers who use technology regularly in junior secondary mathematics and who gave priority to success for all students in their classrooms were selected. In this first phase of the study, the teachers have been interviewed about the meaning of equity and how they used technology in mathematics. Preliminary analysis of these data will be presented.

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THE STATE AND IMPACT OF GEOMETRY PRE-SERVICE PREPARATION – POSSIBLE LESSONS FROM SOUTH AFRICA

Suriza van der Sandt

The College of New Jersey, Ewing, NJ, 08628

The responsibility of pre-service preparation programs includes making evident the complexity of teaching and adequately preparing teachers for their roles as mediators of learning, interpreters and designers of learning programs and materials. This paper summarizes a two year study investigating the state of prospective teachers' (PTs') (n=254), teachers' (n=18) and students' knowledge (n=103) of Grade 7 geometry (using the van Hiele theory (1986) and acquisition scales of Gutierrez, Jaime & Fortuny (1991)). Three foci direct this study, the impact of different pre-service preparation time frames (3 years versus 4 years) on PTs' geometry content knowledge being the first. The second investigates the possible relationship between teachers' content knowledge and the students' learning gain (measured by the same van Hiele-based geometry questionnaire). The third focus is the effect of teaching experience on both the teachers' own level and degree of geometry acquisition as well as the resulting (self-reported) classroom practice. Results indicate that both teachers and PTs (irrespective of preparation time frame) fail to reach the level of geometric thinking and degree of acquisition expected (van Hiele Level 3- the level teachers are expected to teach). Only one of the four participating grade 7 classes made a practical significant learning gain on the informal deductive level (van Hiele Level 3). There seems to exist a possible relationship between the learning gain made by students and the teachers' pre-service education and years of teaching experience. Results further show that PTs exit school with higher geometrical acquisition than after three years of mathematics content and methodology training or after four years of methodology training. This shocking revelation could indicate that pre-service preparation programs had no significant impact by either maintaining or positively impacting on the already attained thought levels. One conclusion is that PTs and teachers are not adequately in control of the grade 7 Geometry subject matter they have to teach which has implications for classroom teaching and learning. The results have serious implications for pre- and in-service training and suggestions on the features of an improved program are made.

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EXAMINING TASK-DRIVEN PEDAGOGIES OF MATHEMATICS

Fiona Walls

James Cook University

Task-oriented pedagogies predominate in mathematics classrooms around the world. These pedagogies are premised on the belief that children's mathematical learning develops best through teachers' careful management of task selection, allocation, administration, and assessment. This practice must be examined in light of international declarations advancing children's right to participate in decisions affecting their lives, and recognising the benefits of learner-negotiated pedagogies.

Recent mathematics research has focussed on classroom norms as a significant variable in children's learning, and particularly on how mathematical tasks enhance children's thinking, reasoning and working mathematically. Children have seldom been viewed as critical collaborators in this process. The following responses from child interviews gathered over three years of research in 33 primary classrooms in New Zealand (Walls, 2003) typify children's experiences of learning mathematics:

Jared: The teacher says, "Go and get your maths books out", and she writes stuff on the board for maths. (Late Year 3)

Georgina: We get into our [ability] groups and do the worksheet. (Mid Year 4)

These statements speak powerfully about everyday classroom cultures in which mathematics and its learners are shaped by teacher-selected mathematical tasks.

Changes in ethical and legal discourse in support of children's participatory rights as global citizens, oblige us to re-examine current pedagogies of mathematics. Pollard (1997) for example, advocates for *negotiated curriculum:* "…rather than reflect the judgments of the teacher alone, it builds on the interests and enthusiasms of the class…Children rarely fail to rise to the occasion if they are treated seriously" (p. 182). Similarly, in an explication of the articles of the 1989 UN Convention on the Rights of the Child, a recent UNICEF report says, "we see…children actively involved in decision-making at all levels and in planning, implementing, monitoring and evaluating all matters affecting the rights of the child" (UNICEF, 2002, p.11).

If we are to honour children's right to significant agency in their own learning journeys, mathematics educators must now consider partnership with young learners as a necessary evolution from adult centred, task-driven pedagogies of mathematics.

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INCLUSIVE MATHEMATICS: CATERING FOR THE 'LEARNING-DIFFICULTIES' STUDENT

Margaret Walshaw
Massey University, NZ

Elizabeth Siber
Pukekohe High School, NZ

Current mathematics education reform efforts require a commitment from schools to offer inclusive learning opportunities that will maximise student potential. At the forefront of this requirement are questions concerning the optimal student learning arrangements for students with learning difficulties. Policy makers recognise that for reform intent to be operationalised in the classroom there needs to be a school–wide approach to the organisational aspects of inclusive teaching and learning. Given our commitment to the mathematical development of students with learning difficulties, we wanted to examine the approaches taken by schools to cater for the mathematics experience of students with learning difficulties/learning disabilities (LD).

In the study we were particularly interested in the approach to mathematics teaching and learning of LD students undertaken by schools to see if it matched anecdotal evidence. Anecdotal evidence suggests that the LD classroom teacher is not always the person who wants to work with LD students. The classroom experience of LD mathematics classes, the story goes, is about keeping the students occupied and busy, and, above all, out of trouble. Achievement and effort grades assigned to these students are often restricted to C, D or E, regardless of student effort and content knowledge. Furthermore, anecdotal evidence suggests that LD students have poor motivation, their self-esteem is very low, and their post-school options are limited.

We wanted to determine how the mathematics learning needs of LD students were being met. Barash and Mandel (2004) report on the introduction of a programme for seventh grade LD students, developed and taught by pre-service teachers. We wanted to know what types of programmes were established for LD students in New Zealand schools. We were interested in exploring how mathematics classes are organised for LD students, how LD students were selected for mathematics groups and by whom, how their learning was assessed and who teaches them.

In the presentation we will report on the findings from our survey research with 74 schools. The research elicited information on a wide range of school issues concerning LD students.

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CHILDREN'S NOTATION ON EARLY NUMBER COMPUTATION

Linda Warner & Glenda Anthony

Massey University

Mental computational strategies have taken centre stage in a range of numeracy programmes world-wide. However, despite the move away from traditional written algorithms notation remains a critical component of students' mathematical development. Reform practices encourage students to be active participants in inquiry processes—as part of the communication process student need to develop notational systems to describe their own mathematical activity. Students' emerging ways of symbolizing and notating provide a vehicle for communication, representation, reflection and argumentation (McClain & Cobb, 1999). Within New Zealand, numeracy curriculum support material (Ministry of Education, 2004) indicates that informal jottings of students are to be encouraged as a "way to capture their mental process" (p. 8) so that their ideas can be shared with others. Notation models provided include the empty number line and annotated ten frames.

This paper reports on a teaching experiment focused on the Year 5/6 students' development of notational schemes within a unit of addition and subtraction. In particular, the research was interested in determining how expectations for using notation to record mathematical thinking could be more firmly established within the classroom and how notational practices might best support students' sense-making practices by providing a reference point within group and class discussions.

While the study provided evidence that teacher focused use of notational schemes can effectively support the norms of explanation and justification, it also highlighted a range of dilemmas for the teacher. These included timing of the introduction of notational schemes for some children, the tension between individual children's idiosyncratic notational schemes and more formalised teacher notation, and the potential for notational schemes to act as a barrier by over-riding children's informal thinking. The findings illuminated the complexity of the classroom and the challenges that the teacher faces in attempts to place children's thinking at the centre of her decision making.

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TEACHER BEHAVIORS AND THEIR CONTRIBUTION TO THE GROWTH OF MATHEMATICAL UNDERSTANDING

Lisa Warner, Roberta Y. Schorr, May L. Samuels, & Darleen L. Gearhart

Rutgers University Newark Public Schools

This research focuses on teacher behaviors and the impact of those behaviors on students' mathematical activity and understanding over the course of several months. We do this by documenting, coding, and analyzing video-data, reflections and other artifacts collected from an eighth grade inner city mathematics class. The classroom visits were part of a professional development project in which the teacher/researcher (first author), who is a mathematics education researcher at a local university, routinely met with local teachers in their classroom and at a course (that the first and second author taught), where they discussed key ideas relating to classroom implementation, the development of mathematical ideas, and other relevant issues.

Our conceptualization of teacher behaviors extends the literature on teacher questions and behaviors (eg., Schorr, Firestone & Monfils, 2003), based on the first author's experience in a variety of teachers' classrooms and inductive analysis of the data in the present study. Some examples of identified teacher behaviors include the teacher: showing evidence of listening to a student's idea; highlighting or placing a high value on a student's idea; encouraging a student to link representations to each other.

This can have important implications for teacher development. For example, we noticed that as the teacher encouraged students to look at the relationship between and amongst their own representations, students were able to link these representations to each other, which contributed to their move to an outer layer of understanding within the Pirie/Kieren model (Pirie & Kieren, 1994). By looking more closely at the relationship between teacher behaviors, student behaviors, and ultimately linking this to the Pirie/Kieren theory for the growth of understanding, we can gain deeper insight into the chain of events that unfold in classrooms. This, in turn, will allow us to obtain a richer understanding of the complexities of teaching in an urban environment, and has the potential to contribute to the research base relating to teacher development and student learning.

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TEACHERS' BELIEFS OF THE NATURE OF MATHEMATICS: EFFECTS ON PROMOTION OF MATHEMATICAL LITERACY

Lyn Webb and Paul Webb

Nelson Mandela Metropolitan University

Various studies have shown that what teachers consider to be optimal ways of teaching mathematics and mathematical literacy is influenced by their beliefs about the nature of mathematics. It is therefore advantageous to determine teachers' conceptions of the nature of mathematics before developing curriculum interventions. In this study various methods were employed to stimulate teachers to both reflect on their beliefs and to make them explicit. A Likert-scale questionnaire was administered to 339 in-service teachers in urban and rural areas of the Eastern Cape, South Africa. A sample of ninety-five of these teachers completed a questionnaire based on videotapes of lessons recorded during the TIMSS (1995) study that they had viewed. These teachers also ranked their own teaching on a continuum ranging from traditional to constructivist approaches and provided explanations for their ranking. A further sub-sample of thirty-six teachers participated in individual interviews, which explored their perceptions of the nature of mathematics and their own teaching practice. In order to investigate whether these beliefs are mirrored in practice, four teachers were videotaped in their classrooms. The data generated by these videos support the findings of similar studies, i.e. that teachers' beliefs of the nature of mathematics are often not reflected in their practice. This has far-reaching implications for the implementation of compulsory mathematical literacy to grades 10, 11 and 12 in South Africa, as the mode of delivery is envisaged to be through contextual problem solving.

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ACTION RESEARCH ON INTEGRATING BRAIN-BASED EDUCATIONAL THEORY IN MATHEMATICS TEACHER PREPARATION PROGRAM

Wu, S. C., Chang, Y. L.

Taichung Healthcare & Management University, MingDao University

At the end of the 20th century, the United States implemented the educational philosophy and principles based on the research evidences of human brain and learning. Educators emphasized that the more understandings of our brain, the more effective curriculum and instruction can be designed by teachers (Chang, Wu, & Gentry, 2005). Thus, the evidences and reflections from brain research influenced educational theories and provided scientific characters for all levels. In Taiwan, the brain-based educational theory is still a new phrase that needs to be introduced and implemented extensively and intensively. Thus, the purpose of this action research, using a mixed method approach along with non-sequential and concurrent triangulation strategies, is to apply it into the real classrooms within the mathematics teacher preparation program and examine the processing changes of pre-service teachers who do not specialize in mathematics education in order to assist them for the future teaching.

In order to face the various myths and misunderstandings of brain's development and confront teaching and learning problems of elementary mathematics education, the better way is to go back to the teacher education and devote extra efforts to train the pre-service teachers. Accordingly, researchers re-designed the course of "teaching elementary school mathematics" by shifting the curriculum and instruction design associated with the brain-based educational theory and the nature of mathematics. By working with 66 pre-service teachers closely in National Hsin Chu Teachers College, Taiwan and providing more integrated contents, hands-on activities, and opportunities of thinking, data were collected qualitatively and quantitatively with pre- and post-tests, observations, and reflections. Results indicated that their self-efficacy ratings toward mathematics increased significantly, as well as rasing their interests and reconstructing confidences in learning and teaching mathematics in the elementary classrooms. Reflections and recommendations were also valuable for revising the course design.

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USING WRITING TO EXPLORE HOW JUNIOR HIGH SCHOOL GIFTED STUDENTS CONSTRUCT MODEL IN PROBLEM SOLVING

Fu-Ming Yen Ching-Kuch Chang

National Chanaghua University of Education

This study explored modeling of junior high school gifted student by thirty 7th-grade students as they participated in model eliciting activity and writing of learning journal in problem solving. This article describes development of student's ability of description and modeling through team cooperative discussion, verify, presentation and comment in inquiry oriented teaching environment, and using Discourse analysis to team cooperative verifying process, evaluation and reflection that are excerpted from student learning journal.

INTRODUCTION

This study uses writing-to-learn strategy in mathematics classes, engage thirty 7th-grade gifted student to participate in model eliciting activity and inquiry oriented teaching environment. The forms of students' journals writing are used in this study involves logs, journals and expository writing (Strackbein and Tillman, 1987). Theoretically, "Modeling cycles" (Lesh and Doerr, 2003) , "Three modes of inference making employed in sense-making activities" and "A past instance of semiosis can become the object of new semiosis" (Kehle and Lester, 2003) are used in this study as foundation of writing text analysis and model eliciting activities.

METHODOLOGY

Essentially, researcher as teachers adopted inquiry-oriented teaching strategy, and guided students writing journal with inquiry of problem-solving task in this class. In practice, there are nine problem tasks in this study. We audio taped teaching and interview sessions, and adopted Discourse analysis (Gee, 1999) to organize, analyse, classify, and consolidate the data which included writing text, then determine themes.

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AN ALTERNATIVE MODEL ON GIFTED EDUCATION

Yen Ying-Hsiu

Hsing Chuang Elementary School

Shy Haw-Yaw, Chen Chun-Fang,

Liang Chorng-Huey, and Feng Shu-Chen

National Changhua University of Education

Due to the limitation on the availability of the resources, students have to pass certain examinations to be able to enter the gifted programs. The precious gems characterizing giftedness, such as creativity and originality, are mostly destroyed in preparing for the exams. To avoid this disaster, we address an alternative teaching model suitable simultaneously both for gifted identification and development.

In this eighteen-month study, the samples consist of seventy fifth graders in Taiwan, with above-average academic achievement. The design of the instruction is based on the philosophy that learning is an opportunity for finding personal intrinsic ability instead of mere knowledge acquisition. Inspired by this philosophy, the instructor plays the role as a supporter, not an examiner, of students' idea. For identification, the learning behaviour scale (LBS) is chosen according to the longitudinal nature of the implementation and the high validity as an indicator of the academic achievement. The principles of the instruction and the factors of LBS are analyzed, compared, and matched. This remains mostly absent in the previous literatures on the topics related to LBS. To test the domain specific ability, the factor concerning the creativity on mathematics is added, an aspect that has long been ignored in LBS.

The results of this study are:

1. Hatched by the sophisticated knowledge from the instructor, students' raw, but exclusive, idea can provide a different view that is more interesting than the traditional way. Students' eminent creativity, demonstrated in treating a new task by devising novel schemes, is nurtured rather than taught.
2. A teaching model to serve both for identification and development is possible on the group with above-average academic achievement.
3. LBS scores do provide a good correlation with the mathematics achievement.
4. The frequency of presenting student's personal thinking has high correlation with student's creativity. This factor yields a powerful facet on gifted identification.

LEARNING MATHEMATICAL DISCOVERY IN A CLASSROOM: DIFFERENT FORMS, CHARACTERISTICS AND PERSPECTIVES. A CASE STUDY

Oleksiy Yevdokimov

Kharkov State Pedagogical University, Ukraine

Nowadays the phenomenon of mathematical discovery, its mechanism and mental processes remain into the educational research limelight (Burton, 1999; Tall, 1980). Indeed, the concept of mathematical discovery has quite many common features with learning process for being considered together. We understand *learning mathematical discovery in a classroom* as a short-term active learning process aimed at the development of students' abilities to assimilate new knowledge through the use and interpretation of their existing knowledge structures with the help of a teacher or with considerable autonomy and only teacher's control of the direction of inquiry activities within the topic studied. The main question of our research was the following: How could students' inquiry work in a classroom be modified to simulate mathematicians' practice and what were the ways of evaluation of students' work in such activities? We tried to answer this question in the context of using three different forms of students' inquiry work in a classroom. We took the position that Active Fund of Knowledge of a Student (AFKS, Yevdokimov, 2003) was the most relevant structure to introduce new characteristics for studying this process. For quantitative evaluation of student's conscious involvement in the process of learning mathematical discovery in a classroom we considered an index I of using own AFKS by every student, i.e. I served as indicator how much AFKS was involved in doing each task. We studied the character (logical or non-logical) of using AFKS within learning mathematical discovery in a classroom. Analysing the data received we found out that we can construct a set of key problems with indicated in advance quantitative scale of using extra-logical processes for students' inquiry activities in learning mathematics. Thus, we can distinguish and regulate the illumination stage of learning mathematical discovery, we can adapt it to the needs of classroom activities or to the thinking process of a certain student involved in these activities.

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STUDENTS' VIEWS ABOUT COMMUNICATING MATHEMATICALLY WITH THEIR PEERS AND TEACHERS

Jennifer M. Young-Loveridge, Merilyn Taylor & Ngarewa Hawera

School of Education, University of Waikato, Hamilton, NZ

This paper reports on the responses of approximately 180 nine- to eleven-year-olds during individual interviews. The children were asked a range of questions designed to explore their perspectives on mathematics learning, including questions about the importance of working out problems mentally, of getting answers correct, and whether they thought that there was only one or several different ways of working out an answer. They were then asked the following questions and the reasons for their responses:

> Do you think it is important for you to know how other people get their answers? Is it important for you to be able to explain to other people how you worked our your answer? What about your teacher - is it important to be able to explain your thinking to your teacher?

Idea	Yes	No	Not Sure	Total
Knowing others' strategies is important	38.8	47.9	13.3	176
Explaining thinking to others is important	55.4	27.1	17.5	178
Explaining thinking to teachers is important	84.6	7.7	7.7	169

Table 1: Percentage of students who thought particular ideas were important

Almost all students concurred with the idea that explaining one's thinking to one's teacher is important. A wide range of reasons was given for agreement with this idea. Some reasons were related to teachers' actions, such as assessing students' understanding, making decisions about grouping students, helping students with their learning, reporting to parents. Other reasons were more to do with students' concerns, such as "proving" that they had worked out answers for themselves. A few students thought that explaining their thinking to their teacher could help that teacher with his/her own mathematics learning. The verbatim quotes from individual children provide insights into the children's unique perspectives on their mathematics learning, and underline the importance of taking children's views into account (Cook-Sather, 2002; Young-Loveridge & Taylor, in press).

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