THE "A4-PROJECT" - STATISTICAL WORLD VIEWS EXPRESSED THROUGH PICTURES

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This paper arises from an interest in how students at university level view statistics. We asked 394 students to express their views on statistics by designing an A4 sheet of paper. After providing the theoretical framework, we present the analysis of a random sample of 15 pictures. Based on these works we suggest three categories to describe the expressed statistical world views.

INTRODUCTION

In a world where the handling and interpretation of data is becoming increasingly important, basic statistical skills and statistical literacy build the foundation for many of our decisions (Niederman & Boyum, 2003; Wallmann, 1993; Watson & Callingham, 2003). Indeed, the NCTM standards state that "a knowledge of statistics is necessary if students are to become intelligent consumers who can make critical and informed decisions" (NCTM, 1989, p. 105). Recent dialogues on quantitative literacy or numeracy point in a similar direction (Steen, 1997; 2004).

At the same time, more and more researchers are taking into consideration the role of world views or beliefs as a hidden variable in mathematics education (Leder, Pehkonen & Törner, 2002). The term *statistical world views* is chosen in accordance with the term *mathematical world views* by which we understand subjective beliefs and personal theories related to mathematics (Schoenfeld, 1985; 1998). As many researchers point out, the learning and success in mathematics is influenced by student beliefs about mathematics and about themselves as mathematics learners (Schoenfeld, 1992; Hannula, Maijala & Pehkonen, 2003). While there is substantial research on global beliefs about the nature of mathematics, teaching, and learning (Cooney & Shealy, 1997; Lloyd, 1999), in this study we direct our attention to views about statistics.

This focus on statistical world views seems to us to be fruitful since views on statistics might remarkably differ, for example, from views on algebra. Following Törner (2002), Aguirre (to appear) employs the term *domain-specific beliefs* to describe this phenomenon. Domain-specific beliefs are characterized as beliefs that are associated with a special field or domain of mathematics such as calculus, geometry, or statistics.

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MATHEMATICAL WORLD VIEWS

Dionne (1984) suggests that world views or beliefs are composed of three basic components called the traditional perspective, the formalist perspective and the constructivist perspective. Similarly, Ernest (1991) describes three views on mathematics called instrumentalist, platonist and problem solving, while Törner and Grigutsch (1994) name the three components as toolbox aspect, system aspect and process aspect. All these different notions correspond more or less with each other. According to Dionne (1984), beliefs constitute a mixture of the three components. It is possible that more than one view is expressed by a person, so no clear classification can be made.

In this work, we employ the notions of Törner and Grigutsch (1994) and use this section to briefly explain what is understood by them. In the "toolbox aspect", mathematics is seen as a set of rules, formulae, skills and procedures. In the "system aspect", mathematics is characterized by logic, rigorous proofs, exact definitions and a precise mathematical language. In the "process aspect", mathematics is considered as a constructive process where relations between different notions and sentences, as well as the invention or re-invention of mathematics, play an important role. Besides these standard perspectives, another important component is the usefulness, or utility, of mathematics (Grigutsch, Raatz & Törner, 1997). This is particularly relevant for the students in this study who were undertaking an applied course in statistics.

PICTURES AS A MEANS FOR INVESTIGATING WORLD VIEWS

Traditionally, mathematical world views are investigated with the aid of questionnaires or interviews. In our work we have used pictures for investigating the world views, much less common in the literature. As one research example, restricted to beliefs on learning, Berry and Sahlberg (1996) used four pictures, each showing a real life situation. They asked 13-year old students to choose the picture that – in their opinion – best describes a good learning situation and to give reasons for their choice.

In contrast, we have asked students to express their views on statistics by designing an A4 sheet of paper themselves (see Methodology below). With regard to producing pictures that reveal views on mathematics, there also exist some experiences in Germany. On the occasion of the World Mathematical Year 2000, launched by the International Mathematical Union and supported by UNESCO, students in a competition were invited to draw a picture representing their ideas of what mathematics is. Some of those pictures were awarded with prizes and about 30 pictures were part of a special exhibition (Exhibition, 2000), but as far as we know they were unfortunately never the topic of research in mathematics education.

METHODOLOGY

Sample

The subjects in this study were students undertaking the course "Analysis of Biological Data and Experiments", run at an Australian University in Semester 2, 2004 (from the end of July to the end of October). This course was taken by 433 students who were studying biological sciences, including genuine science students, but also students aiming to study medicine, dentistry, and other professions. A total of 394 students (91%) submitted pictures as part of their assessment for the course.

Study

In the first two weeks of the course, students were asked to use an A4 sheet of paper to describe their views on statistics. Little had been covered in the lectures by this stage, with the pictures aiming to capture the initial understanding that students brought to the course. The wording on the instructions directed students to "take a blank sheet of A4 paper and draw, write, paint, doodle, or whatever" suits them best to express their views on statistics. Since the paper was A4, we referred to this as the "A4-Project".

Analysis

The methodological framework to analyse data about students' statistical world views is informed by grounded theory (Strauss, 1987). The guiding principle of grounded theory is that theorizing grows from the data rather than from a pre-existing theory. We therefore randomly selected 15 out of the 394 pictures, here referred to as pictures (001) to (015). We independently analysed this random sample of 15 pictures and identified some significant themes in the pictures. After discussion between the authors, these significant themes were refined to three key aspects that students appeared to be expressing in their work.

RESULTS

In this section, we first describe the categories by giving a rough definition and an overview of the typical features of each category that were presented in the pictures, providing as foundation some concrete examples that support the category in question. We then summarize these categories and suggest that the statistical world views might be seen as a hierarchy.

Due to the format of the paper, it is unfortunately not possible to include all of the 15 pictures here. Figures 1 and 2 show two examples: pictures (002) and (011), respectively, reproduced with the permission of the students.

Course Aspect

Several pictures do not describe statistics as a discipline. In these the focus is on "statistics" as the course that the students are undertaking. Picture (012) is set at one of the lecture rooms for the course, but goes beyond the course aspect by showing statistical symbols (see Toolbox Aspect) entering the brain of a student. Moreover,

the students often express their attitudes towards this course by drawing a person having a certain facial expression, such as concerned (002 - Figure 1), first anxious, later happy and optimistic (008) or smiling (004, 005). In addition to a smiling person, picture (004) consists of a house, a flower, the shining sun; no views on statistics are expressed, but there is a pleasant atmosphere. Only a sheet of paper in the hand of the person with the name of the course on it suggests that the picture has something to do with statistics.

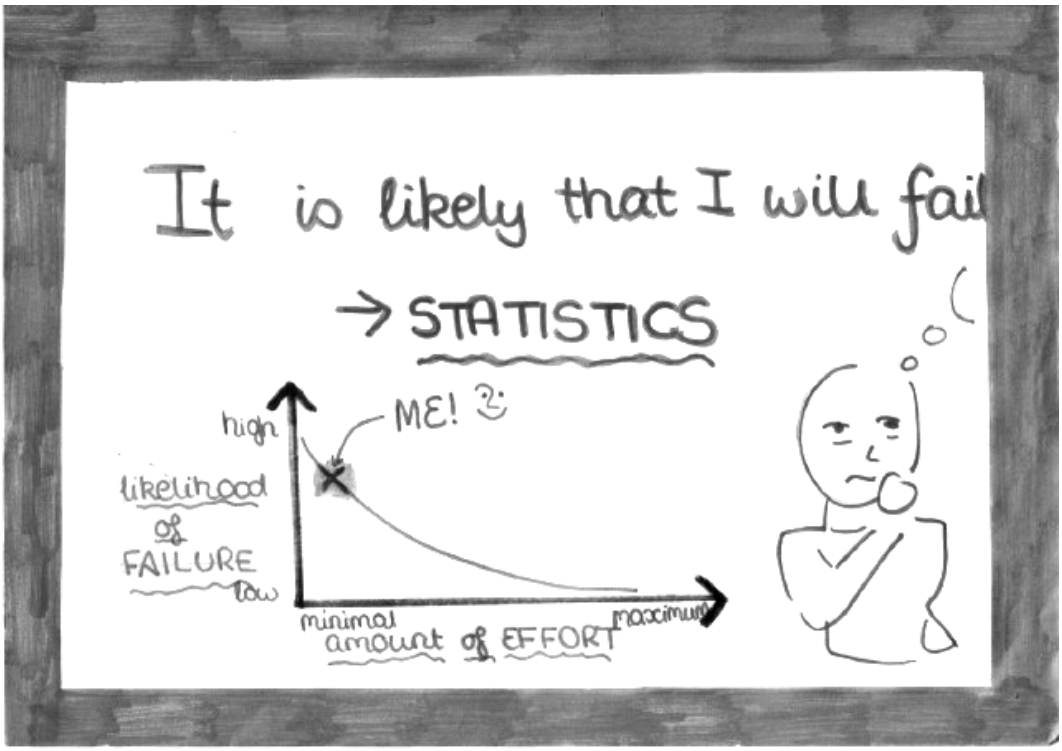

Figure 1. Example picture (002)

Toolbox Aspect

A very common theme is to show statistics as a collection of tools. Typical elements that indicate this view are diagrams, plots and histograms (001, 006, 007, 009, 010, 011 – Figure 2, 013). Another indicator is the inclusion of statistical key words. These are just given, like vocabulary, without explaining the context, such as the mean, null hypothesis, and variation (006), or the mode, median, and mean (011 - Figure 2, 013). Symbols are also used (001, 012), again without explanation of what they mean, as are statistical formulae (009).

Related to statistics as a collection of tools is the view of statistics as numbers and data. This is perhaps expressing lower knowledge of technical statistics, but in the pictures these usually accompany the general toolbox aspect. For example, in two cases (001, 006) numbers and symbols form the frame of the picture, while another picture (015) has a background of numbers.

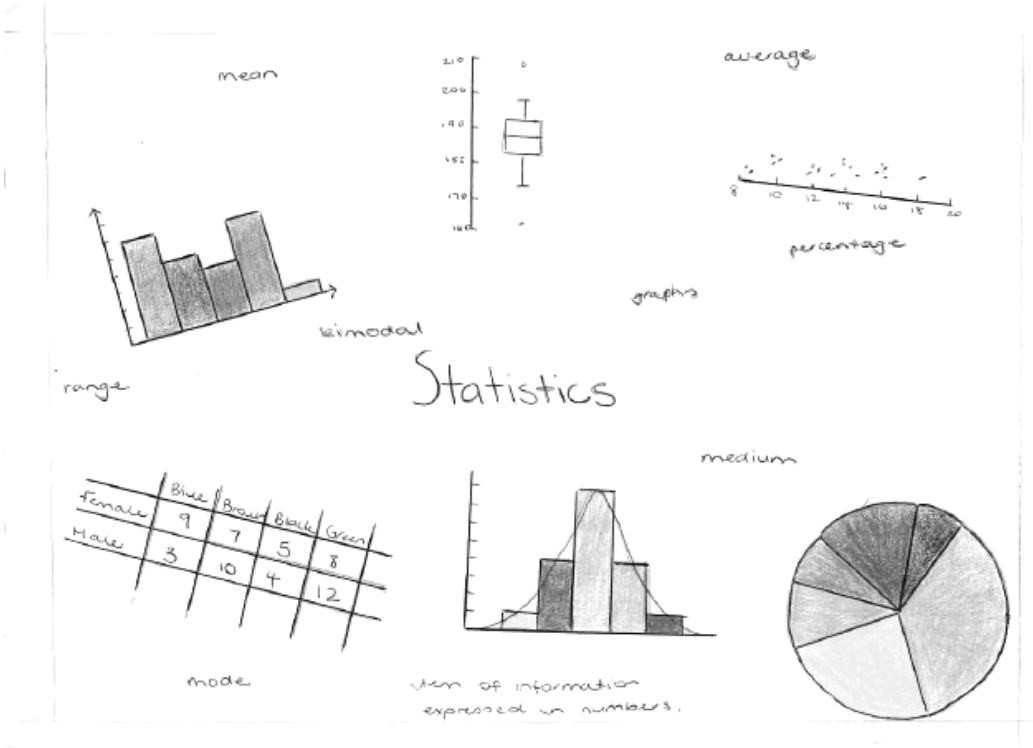

Figure 2. Example picture (011)

Utility Aspect

The final aspect we consider is the expression of the utility of statistics. Picture (001) mentions the role of statistics in "forming the foundations for many reports". Picture (003) includes a crystal ball with text stating "statistics – a crystal ball to model the future", while in another corner it hints that statistics helps solving everyday problems.

In addition to the utility of modelling the world, the utility of communication is also identified. Picture (014) asks, "Does there lurk a sunken treasure chest of ideas, investigation, experimentation… Nestled in the bedrock of scientific communication?" Picture (015) features two facing silhouette heads over a field of numbers, suggesting communication.

Hierarchy of World Views

Table 1 presents the three world views as a hierarchy. The description for each attempts to justify the hierarchical nature by specifying what is missing in one aspect that puts it below the next aspect.

Aspect	Description
1. Course	The picture shows "statistics" as a course that the student is studying. Such pictures often describe attitudes, including apprehension about failing the course or the expectation that the course will be boring. No mention of statistics as a discipline or the content of the course is given.
2. Toolbox	The picture shows statistics as numbers and as a collection of tools, such as different ways of plotting data or calculating summary statistics. No indication of where these statistics come from or how they can be interpreted is given.
3. Utility	The picture shows the utility of statistics in describing the world. Rather than just being a tool, the picture shows some relevance of statistics, such as in communication or in scientific experimentation.

Table 1. Summary of world views identified in the sample pictures

Of course the hierarchy in Table 1 is not perfect. Many pictures were seen to express more than one aspect, though it was usually possible to identify a dominant aspect for a picture. It is also likely that a student might draw a picture that expresses the utility of statistics without having a strong understanding of statistics as a toolbox. Since this was not in an interview setting it is difficult to clarify some of these points.

It is also worth noting that this hierarchy of statistical world views is focussing on statistics as a discipline. In studying the pictures we were also aware that many expressed an affective or attitudinal aspect that is not captured in our hierarchy (although some of the affective component has been mentioned in the Course Aspect above). However, most of the pictures do not clearly present an affective aspect and so for this work we have focussed on the actual statistical content of the picture.

CONCLUSION

Almost half of the examined pictures use statistical plots, such as line graphs and histograms. It is not surprising to see such an emphasis on the visual tools of statistics since the nature of the task emphasised the visual. Based on these works, it is difficult to conclude, for example, whether students do normally think visually about statistics. This highlights that what we are measuring here is the *expression* of statistical world views, not the world views themselves. However, the picture task, with the opportunity to write or draw or paint, provides a greater range of possibilities for this expression than does a standard questionnaire, for instance. By starting with a blank sheet of paper we suggest that the students are also less constrained by what they might think are "good answers" to the task. To conclude we discuss two possible areas for leveraging this aspect of the task.

In a broad sense, the pictures drawn by students may give an insight into their level of statistical literacy. However, our results are quite different to those from other research. For example, Watson and Callingham (2003) give a "statistical literacy construct" that has six levels, from "idiosyncratic" to "critical mathematical". While their research is based on studying how students complete statistical tasks, we have found it more difficult to get fine classifications from the static expression captured by a picture. It would be worthwhile to measure the same students with the tasks given by Watson and Callingham (2003) and see if there are any relationships to our categories.

The timing of the pictures, drawn by students at the start of their tertiary statistics course, was aimed at providing data on the broad beliefs, attitudes, and knowledge of statistics that students brought with them from their prior experience. In itself this has provided academic staff with a richer understanding of the backgrounds of students undertaking this course. However, the wide variety of pictures drawn by students made it difficult to make any clear conclusions from the data. This present study into the categories of statistical world views will make it easier to work with this type of data and provide the needed clarity. For example, we are currently analysing all of the 394 pictures based on the categories given here. Once a larger number of pictures have been categorized it is then possible to look for associations between a student's initial statistical world views and measures of subsequent performance in this particular course, such as marks on project work or exams. If such associations exist then this will suggest changes to the curriculum in order to address how particular world views need to be incorporated.

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