PRIMARY STUDENTS' KNOWLEDGE OF THE PROPERTIES OF SPATIALLY-ORIENTED DIAGRAMS

Carmel M Diezmann

Queensland University of Technology, AUSTRALIA

The ability to select an appropriate diagram to represent the structure of problem information is a critical step in reasoning. This paper reports on an investigation of Grade 3 and Grade 5 students' knowledge of the properties of spatially-oriented diagrams. The task required the students to select the diagram that corresponded to the structure of a particular problem and to justify their selection. The results revealed that primary students have difficulty in selecting an appropriate diagram and adequately justifying their selections. Although Grade 5 students outperformed Grade 3 students in some aspects, the similarities between Grade 3 and Grade 5 performances on other aspects suggests that it is fallacious to assume that students' knowledge of the properties of diagrams will increase substantially with age.

Diagrams are an important visual-spatial representation in mathematics because they facilitate the representation of problem information (e.g., Diezmann, 2000; Novick, 2001). Diagrams have three key cognitive advantages in problem solving. First, diagrams facilitate the conceptualisation of the problem structure, which is a critical step towards a successful solution (van Essen & Hamaker, 1990). Second, diagrams are an inference-making knowledge representation system (Lindsay, 1995) that has the capacity for knowledge generation (Karmiloff-Smith, 1990). Third, diagrams support visual reasoning, which is complementary to, but differs from, linguistic reasoning (Barwise & Etchemendy, 1991). However, students of all ages are reluctant to employ diagrams, experience difficulty using diagrams or lack the expertise to use diagrams effectively (e.g., Veloo & Lopez-Real, 1994). Thus, students' use of diagrams can inhibit rather than facilitate their mathematical performance.

DIAGRAMMATIC KNOWLEDGE

Three useful diagrams that have broad applicability in mathematics and unique spatial structures are the matrix, network, and hierarchy (e.g., Novick, Hurley, & Francis, 1999) (see Figure 1). For example, the row and column structure of a matrix makes it useful for depicting a combinatorial relationship between two distinct sets.

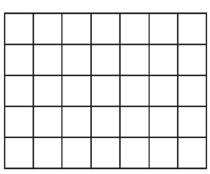
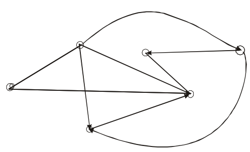
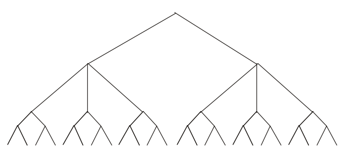
Matrix	Network	Hierarchy
		

Figure 1: Three general purpose diagrams.

While diagrams have been studied intermittently over the past three decades (e.g., Diezmann, 2000), it is only recently that a cohesive framework of ten distinguishing properties of spatially-oriented diagrams has emerged from research with college students (Novick, 2001; Novick & Hurley, 2001). This is a major advance in diagrammatic research because these properties constitute the "building blocks" of diagrammatic knowledge, and are applicable to all spatially-oriented diagrams. Novick and Hurley (2001) confirmed the existence of these properties but found that only six of the ten properties were sufficiently discrete to be readily investigated. These six properties are shown in the first vertical column on Table 1. Each of these properties differs according to the particular spatially-oriented diagram, as shown in the overview on columns two to four on the table.

Properties of Diagrams	Matrix	Network	Hierarchy
1. *Global structure*: the general form	a factorial structure	lacks formal structure	an organisational structure
2. *Number of sets*	ideally 2 sets of information	1 set of information	no limit on sets of information
3. *Item/link constraints:* how items link together	factorial structural constraints	no constraints	organizational structural constraints
4. *Link type:* links between items are best conveyed by a particular diagram	associative non-directional links	flexible links	directional links
5. *Linking relations:* one-to-many links, many-to-one links or both	not salient, but can have both linking relations	both linking relations	either linking relation but not both
6. *Transversal:* the possible paths	paths are not relevant	multiple paths connect item "A" and "B"	only 1 path connects items "A" and "B"

Table 1: Discrete properties of spatially-oriented diagrams.

The ability to identify the properties of diagrams is fundamental to the selection of an appropriate diagram for problem solving (Novick, 2001). This ability involves the recognition of particular representations and knowledge of "where to look and what to look for or look at" (Rogers, 1995, p. 482). Hence, if diagrams are to be useful cognitive tools for problem solving, students of all ages need to know their properties. The focus of this paper is on primary students' knowledge of the properties of diagrams.

METHODOLOGY

This investigation is part of a larger study that aims to increase our understanding of primary students' knowledge about the properties of diagrams and to identify influences on the development of that knowledge. The larger study has an accelerated longitudinal design, in which two differently-aged cohorts are being studied for three years. This paper reports on two aspects of this study which were:

1. To document Grade 3 and Grade 5 students' knowledge about the properties of a matrix, and
2. To determine whether the ability to identify the properties of a matrix increases with age.

The Participants

There are a total of 137 participants in the larger study. The results of eleven students were excluded from this investigation for various reasons (e.g., inconclusive coding). Hence, the results are reported for a total of 126 students comprising 62 Grade 3 students (8- or 9-year-olds) and 64 Grade 5 students (10- or 11-year-olds).

The Tasks

Students' knowledge of the properties of diagrams was investigated in the larger study through a series of 15 scenario-based tasks, which were designed to focus on a range of properties of the matrix, network and hierarchy. Appendix A presents the Matrix task which was the focus of this investigation. The 15 tasks were designed in accordance with the principles used by Novick and Hurley (2001) in the design of scenario-based tasks for college students. The first sentence or two of the scenario tasks sets up a cover story. The same broad scenario of "The Amusement Park" was used for all tasks with primary students to avoid them selecting their responses on the basis of the cover stories rather than the structural information. The next sentence or two focuses on a particular property of a diagram (e.g., the number of sets). The final sentence indicates that someone wants a diagram for a purpose relevant to the cover story. Only two (correct/incorrect) spatially-oriented diagrams were presented for each task. In one of these diagrams, the property was correctly represented, and in the other diagram, the property was not represented (see Appendix A). The scenario-based tasks required students to (1) select a diagram that best suits the given information and to (2) justify their selection and (3) non-selection of particular diagrams. These 15 tasks were presented to students in two individual interviews to avoid undue fatigue. During the first interview, students engaged in a task that emphasised that the diagrams presented were representative of a specific class of diagram rather than the particular problem. This paper reports on one of these tasks.

Students' knowledge of the properties of diagrams was determined by their correct/incorrect selection of a diagram. Categories were developed from the reasons that students gave for selecting and not selecting particular diagrams and frequencies of students' responses calculated.

Results and Discussion

The results reported here are necessarily limited. They focus on the number of students at each grade level who selected correct/incorrect diagrams on the *Sandwich Bar* task (see Appendix A) and the reasons why students correctly selected the matrix to represent the problem information. The reasons why students incorrectly selected the hierarchy and the reasons why students did not select either the matrix or the hierarchy are not discussed here.

The results of the *Sandwich Bar* task for Grades 3 and 5 of 66.1% and 71.9% respectively indicate that many students had difficulty identifying which of the two diagrams (matrix, hierarchy) would best show the information given (chance accuracy = 50%) (see Table 2). The mere 6% difference between the Grade 3 and Grade 5 results suggests that additional two years of schooling have limited impact on students' ability to select the correct diagram.

	Grade 3 (n = 62)		Grade 5 (n = 64)	
Diagram Selection	Number Correct	Percentage Correct	Number Correct	Percentage Correct
Correct	41	66.1%	46	71.9%
Incorrect	21	33.9%	18	28.1%

Table 2: Number and percentage of students selecting a correct/incorrect diagram.

The explanations for students' correct responses are presented on Table 3 together with the frequency of these responses. As shown on Table 3, there was great variation in the type of explanations given by the 87 students who correctly selected the matrix as the best diagram to represent the given information. Of the 16 types of explanations given by students, 11 types of response (indicated by *) were specific to tasks in which the matrix was the correct diagram and five types of response (indicated by $^{\#}$) were more generic and could have referred to any of the spatially-oriented diagrams (see Table 3).

Only three of the total 87 students (3.45%) provided an exemplary or ideal response for their selection of a matrix with a reference to the representation of combinations (CO). However, a further 16 students' explanations showed they had some understanding of the matrix as having a row and/or column structure (LR, LC, RC). Hence, a total of 19 students provided an explanation that was either fully (CO) or partially correct (LR, LC, RC). There was only a 4% difference between the performance of Grade 3 (19.5%, n = 8) and Grade 5 (23.9%, n = 11) students who made fully and partially correct responses.

A total of 32 students (36.8%) based their explanations for selecting the matrix on another visual representation that is used in mathematics, such as a picture graph (PG), (non picture) graph (GF), co-ordinates (UC) or a tally reference (grid) (TR). Fewer Grade 3 students made this type of response (21.9%, n = 9) than Grade 5 students (50%, n = 23). A possible explanation for students' references to other visual representations in mathematics is their attempts to capitalise on prior knowledge of mathematics to make sense of a novel representation. Although some diagrams and other visual representations can be informationally equivalent and content transfer between these is desirable (Baker, Corbett, & Koedinger, 2001), the informational equivalence of representations cannot be assumed. For example, the content of a co-ordinate representation is unlikely to be informationally equivalent to that of a matrix. Of the 28.1% difference between Grade 3 and Grade 5 students who referred to other visual representations used in mathematics, 21% of the variance can be accounted for by the differences between Grade 3 and Grade 5 students' references to a graph format (GF). This response was made by 7.3% (n = 3) Grade 3 students and 28.3% (n = 13) of Grade 5 students. A possible explanation for the greater percentage of Grade 5 than Grade 3 students making reference to a graph could relate to recent instruction about graphs or the use of graphs in the Grades 4 and 5 curricula.

Code	Explanation	Grade 3 (n = 41)	Grade 5 (n = 46)
*BR	<u>B</u>ox <u>R</u>eference (not a list); used as a storage space	7	3
#CA	<u>C</u>orrect <u>A</u>ppearance; "looks right", "would work"	1	2
*CL	<u>C</u>reate a <u>L</u>ist	4	4
*CH	A <u>Ch</u>ecklist; uses ticks	3	1
*CO	Ideal response that described <u>Co</u>mbinations	1	2
*GF	<u>G</u>raph <u>F</u>ormat - not a picture graph	3	13
*LR	Create a <u>L</u>ist using <u>R</u>ows	4	5
*LC	Create a <u>L</u>ist using <u>C</u>olumns	3	1
#NO	<u>N</u>ot the <u>O</u>ther diagram	3	0
#NS	Response makes <u>N</u>o <u>S</u>ense, illogical, vague response, insufficient information supplied	3	1
*PG	<u>P</u>icture <u>G</u>raph	5	2
*RC	Create a list using <u>R</u>ows and <u>C</u>olumns	0	3
#SI	<u>S</u>ize <u>I</u>ssues, could be the right size	0	1
*TR	<u>T</u>ally <u>R</u>eference (number)	1	7
*UC	<u>U</u>sed for <u>C</u>o-ordinates	0	1
#VD	<u>V</u>isual/pictorial <u>D</u>escription eg., shelves, bread slice	3	0

Table 3: Explanations for why the matrix was selected for the Sandwich Bar task.

A number of students' explanations were inadequate. One type of inadequate explanation focussed on the matrix as a visual representation at a surface level. Students' visually-oriented responses ranged from the broad explanation that the matrix had the correct appearance (CA) to more specific comments about boxes (BR), or a visual/ pictorial description (VD). These responses were made by 10.9% (n = 5) of Grade 5 students and 26.8% (n = 11) of Grade 3 students. It is encouraging that older students made fewer of these types of response than younger students. Another type of inadequate response made by three students was the adoption of the default position that they selected the matrix because the correct response was not the other diagram (NO). All default explanations were made by Grade 3 students (7%).

CONCLUSIONS

Overall, the results suggest that Grade 3 and Grade 5 students have a limited knowledge of the properties of diagrams, in particular the matrix. Although there were some indications of improvement in performance with an increase in age, this was not universally true. There were five key results related to students' performance. First, Grade 3 and Grade 5 students' performed similarly in their ability to correctly select the matrix to represent problem information. Second, students' selections were based on a variety of reasons that may be fully or partially correct, incorrect or inadequate (e.g., default responses). Third, less than 24% of Grade 3 and Grade 5 students made responses that were fully or partially correct. Although Grade 5 students outperformed Grade 3 students, the percentage difference was small. Fourth, over 36% of all students featured another visual representation used in mathematics in their explanations. There was a large difference between Grade 3 and Grade 5 responses with more than double the percentage of Grade 5 students (50%) proposing this type of explanation compared to Grade 3 students (21.9%). Finally, students made a variety of inadequate responses, which included basing their explanations on the surface features of a matrix or providing a default explanation. These visually-oriented responses were made by Grade 3 students (26.8%) substantially more than Grade 5 students (10.9%). Additionally, only Grade 3 students gave default explanations (7%).

If diagrams are to be effective in problem solving, students must be diagram literate (Diezmann & English, 2001). Thus, students need to be able to select the appropriate diagram for a particular problem and adequately justify their selection. The results of this investigation suggest that primary students need considerable teacher support in diagram selection and justification. There are particular concerns with students' performance related to: the scant exemplary responses of students for selecting a matrix; the small differences between Grade 3 and Grade 5 students' performance on correct diagram selection, and the numbers of fully or partially correct responses; and the possible negative effect that an increased familiarity with graphing may have with Grade 5 students making inappropriate transfers between knowledge of graphs and knowledge of the matrix. Limitations of this investigation are that the results are based on the analysis of one task and that task focused on only one of the three spatially-oriented diagrams. However, the generalisability of these results will be informed by other aspects of the larger study, which includes a further 14 tasks, which incorporate the three spatially-oriented diagrams, and the monitoring of students' performance on diagram selection and justification over a 3-year period.

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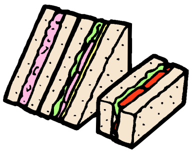
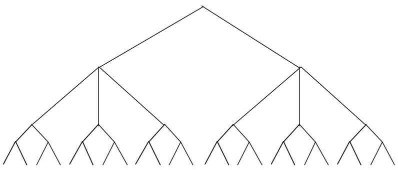
Appendix A. Sandwich Bar Task



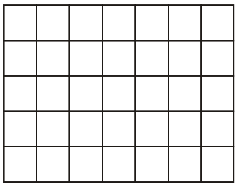
The Sandwich Bar sells sandwiches made with different types of bread and different kinds of meat. The Sandwich Bar Manager wants to know which different combinations of bread and meat are ordered the most, so that she can get her workers to prepare the right types of sandwiches for the busy lunch time rush. The Manager would like a diagram to record how many people buy each different combination of bread and meat during one lunch time.

Which type of diagram would best show the information given?

Hierarchy



Matrix



1. Tick the box

2. Why?

3. Why not?