EXPLORING EXCELLENCE AND EQUITY WITHIN CANADIAN MATHEMATICS CLASSROOMS

George Frempong

York University, Toronto, Canada

In an attempt to understand the processes that allow all students to successfully learn mathematics this paper conceptualizes a successful mathematics classroom in terms of excellence in mathematics and how equitably achievement is distributed. The study employs multilevel models and the Canadian data from the Third International Mathematics and Science Study to identify the characteristics of successful classrooms. The analysis indicates that the most successful classrooms are those in which students from disadvantaged socioeconomic backgrounds excel in mathematics. Disadvantaged students excel in mathematics classrooms in which instructional practices involve less groupings, the mathematics teachers are specialized, and in schools with lower student-teacher ratio.

INTRODUCTION

One of the major objectives of mathematics education systems around the world is to understand the processes in mathematics education that provide opportunities for all students to successfully learn mathematics. The successful mathematics learning for all requires that schools and school systems function in a way that students' success in learning mathematics is not determined by their background characteristics. That is, in an effective mathematics education system, we would expect the mathematics achievement levels of successful schools/classrooms to be related to their capabilities in helping their disadvantaged students to successfully learn mathematics. In this respect, the relationship between the mathematics achievement level (excellence) of a school and the equitable distribution (equity) of mathematics outcomes within a school is an important indicator of the effectiveness of a school and a mathematics education system.

In a school system where the resources within a school are used in a manner that ensures the successful learning of all students, we would expect equity in high achieving schools. While the relations between excellence and equity are important in defining successful education systems, research on school effect has emphasized exclusively on either achievement levels or equity. As a result, we do not have studies that provide an understanding of how the best schools in an education system function to ensure the successful learning of all students. This understanding is necessary to inform policies on the use of resources and the processes for improving schooling outcomes. The main objective of this paper is to understand how the Canadian mathematics education systems function to include or exclude their disadvantaged students from successfully learning mathematics.

2005. In Chick, H. L. & Vincent, J. L. (Eds.). *Proceedings of the 29th Conference of the International Group for the Psychology of Mathematics Education*, Vol. 2, pp. 337-344. Melbourne: PME.

The emphasis will be on school effect on the mathematics outcomes of students from disadvantaged socioeconomic home backgrounds. A substantial body of research points to a consistently strong influence of family background factors, especially their socioeconomic background on mathematics achievement (see Secada, 1992). Unfortunately, many researchers hold the view that these factors are the least amenable to change within an educational policy framework and should therefore be discussed in the context of social policy initiatives rather than from the perspective of school effectiveness. Mathematics education for all makes an understanding of how students from disadvantaged socioeconomic backgrounds come to successfully learn mathematics fundamental to understanding school effect on mathematics outcomes. This paper explores three main issues: the extent to which students' background characteristics affect their mathematics achievement, the extent to which differences in classrooms affect students' mathematics achievement, and the characteristics of mathematics classrooms where students irrespective of their backgrounds succeed in learning mathematics.

Theoretical Perspective (Successful Learning Environment)

When we consider learning as situated in a social and cultural context, the sociocultural perspective provides a useful lens for understanding how schools might function to provide opportunities for all children, especially those from disadvantaged backgrounds, to learn mathematics. The theoretical position of this perspective is motivated largely through the work of Vygotsky, who argues that, in general, learning occurs when an individual internalizes a social experience through interacting with a peer or adult (Vygotsky, 1988). The process of learning occurs through cognitive processes that originate and form through social interaction. Leantev (1981) supports Vygotskys view but stresses the importance of engagement in activity. He maintains that learning occurs through interaction and participation in activity. Other researchers emphasize the importance of locating learning in the co-participation in cultural practices (Lave & Wenger, 1991; Rogoff, 1990). In this model, the student's social engagements through interaction with more experienced others, and through participation in cultural activities are the driving forces for learning.

Bourdieu (1986) argues that often schools operate such that the social and cultural upbringing of students from working class families is not consistent with school norms making it more difficult for these students to engage and participate in learning activities. When school norms and the cultural traditions of children conflict, a school can address the problem from two perspectives. One perspective is to leave students to adapt to the school culture. From this perspective, the success of a disadvantaged student depends on the ability and willingness of the student to function within the two cultures. In cases where a student is unable to function within multiple cultures, success in school leads to losing their cultural traditions. Another perspective is to accommodate all cultural traditions to create a micro-culture that allow all students to participate. In this approach, the success of disadvantaged students depends on school processes and facilities in the school to enhance learning of students from diverse backgrounds. Both perspectives seem to suggest that, for students to succeed in school, they need to acquire certain practices related to understanding a particular subject content.

This suggestion is consistent with an emerging perspective in mathematics education that highlights both the social and mathematical norms in a mathematics classroom. Yackel and Cobb (1996) distinguish between social and sociomathematical norms. Social norms refer to classroom practices that teachers and students engage and that develop gradually over time. They include practices such as learning to participate in group work. Sociomathematical norms are lenses through which teachers and students assess their choices of mathematics teaching and learning activities. To the extent that these norms play important role in learning, we would expect that equitable access to these practices is likely to ensure that students from disadvantaged backgrounds successfully learn mathematics. The question is, whether these practices are the norm in high achieving schools, and to what extent do these practices account for the successful learning of students from disadvantaged backgrounds?

Multilevel models

The concept of "successful schools" defined in terms of excellence and equity poses a considerable methodological challenge as it requires estimates of school achievement levels and inequalities in school outcomes, and most importantly, the processes that account for the variation in these estimates. In the past, researchers assessed school effectiveness through production function models (e.g., Bridge, Judd, & Moock, 1979) from multiple regression statistical techniques, where schooling outcomes are regressed on variables describing students and their schools. However, during the 1980s, there was intense debate as to whether the student or the school was the correct unit of analysis for estimates of school effects (Burstein, 1980). This debate culminated in the development of multilevel statistical models that allow researchers to examine the separate effects stemming from processes at the student, classroom, and school levels (see Goldstein, 1995). These multilevel techniques are now used fairly routinely in analyses of educational data, but rarely by researchers in mathematics education.

This paper employs multilevel statistical analysis techniques that we can describe as regression analyses within and between groups, in this case, schools/classrooms. The analyses provide estimates of regression intercepts (levels of school outcomes) and regression coefficients within schools (measures of, for example, SES achievement gaps). These intercepts and regression coefficients can be regressed on school and classroom characteristics so that the characteristics of schools with high achievement levels and narrow SES achievement gap can be easily identified.

TIMSS

The multilevel models are estimated using the 1995 Third International Mathematics and Science Study (TIMSS) data for Canada. TIMSS is a study of classrooms across

Canada and around the world involving about 41 countries, which makes it the largest and most comprehensive comparative project to assess students= school outcomes in mathematics. TIMSS targeted three populations: population 1 B students in adjacent grades containing a majority of 9-year-olds (grades 3 and 4 in most countries), population 2 B students in adjacent grades containing a majority of 13-year-olds (grades 7 and 8 in most countries), population 3 B students in their final year of secondary schooling (grade 12 in most countries). This research study utilized the Canadian population 2 data describing the mathematics achievement levels of 13-year-old students in Canada. In Canada, these students are in grades 7 and 8 (Secondaire I and II in Quebec). Both grades are part of the secondary school system in all provinces except British Columbia, where grade 7 is part of the elementary program.

The TIMSS Canada population 2 data were collected from a random sample of Canadian schools and classrooms. The random sampling and selection were carried out by Statistics Canada and data were collected in the spring of 1995. Over 16 000 students and their teachers and principals participated in the population 2 component of the study in Canada. Students wrote achievement tests that included both multiple-choice and constructed-response items which covered a broad range of concepts in mathematics. The students also responded to questionnaires about their backgrounds, their attitudes towards mathematics, and instructional practices within their classrooms. Principals completed a school questionnaire describing school inputs and processes, and teachers responded to questionnaires about classroom processes and curriculum coverage.

Instructional Practices and other School Processes

Students responded to a wide range of questions in the questionnaire about instructional activities within their mathematics classroom. In this paper, the classroom instructional practices are classified as grouping, problem solving, traditional, technology, and assessment. Grouping is the extent to which students work in pairs or small groups during mathematics lessons or on projects. Problem solving is a composite score of variables describing the extent and nature of problem-solving activities students are exposed to in a mathematics classroom. The problem-solving activities included giving students problems involving practical and everyday life experiences. Traditional is a composite score of three instructional techniques where students usually copy notes from the board, often work from worksheets, and rely extensively on textbooks. Technology is a description of the extent to which calculators and computers are used in mathematics classrooms. Assessment includes quizzes and homework.

The other school process variables are teacher-specialize, student-teacher-ratio (STR), remedial-tracking, and school disciplinary problems. Teacher-specialize was constructed by dividing the total number of periods a teacher is scheduled to teach mathematics by the total number of periods allocated to that same teacher. This variable served as a proxy for a teacher's specialization in mathematics teaching. Given the challenge mathematics teaching poses to a number of teachers, one will expect that teachers who spend relatively more time teaching mathematics are likely to specialize in this field. There may however, be cases where teachers are assigned to teach mathematics because there are no qualified mathematics teachers. STR is the total number of students per teacher in a school. The STR variable was constructed by dividing the total school enrolment by the full-time teacher equivalent (FTE) of a school. Remedial-Tracking is a dummy variable denoting whether in a particular school, students in remedial classes are removed from regular classes. School disciplinary problems measured the extent of disciplinary problems, such as stealing, in a school.

The dependent variable, students' mathematics scores, is scaled such that the mean score for grade 7 is 7 and the mean score for grade 8 is 8. The scale represents "years of schooling", seven years for an average grade 7 student and 8 years for an average grade 8 student, and is intended to re-express the magnitude of the differences in mathematics scores in a metric based on the mathematics test scores for grade 7 and 8 students in Canada.

Analysis and findings from the multilevel models

	Models			
	1	2	3	4
Within-School Effects				
Socioeconomic status		.52	.50	.50
Female		-.15	-.15	-.16
Immigrant		.36	.36	.34
Variance Components				
%Var. among classrooms	19.2	18.2	17.9	13.6
Corr. Intercept/Gradient		-.14		
Within classrooms (S.D)	2.20	2.13	2.13	2.13
Amongst classrooms (S.D.)	1.07	1.00	.99	.84
Effect on mean achievement				
School mean SES			.47	.38
Students grouped				-.10
Traditional approach				-.12
Use computers				-.15
Use calculators				.26

Regular homework				.15
Disciplinary problems				-.36
Teachers Specialize				.61
Student-Teacher Ratio				-.02
Topic Covered This Year				.31
Topic Covered Last Year				.18
Effects on SES Gradients				
School Mean SES			.14	.09
Students Grouped				.08
Traditional Approach				-.08
Regular Homework				.08
Teachers Specialize				-.14
Student-Teacher Ratio				.02
Note: Bold indicates statistical significance at p<.05				

Table 1: Estimates from the multilevel models

Variation among Schools and Classes in Mathematics Scores

The first model tests the hypotheses that schools and classrooms differ in their unadjusted scores. The results indicate that they do indeed differ: 19.2% of the variance is among classrooms (and therefore 80.8% is within schools). The model also yields estimates of the magnitude of the within- and between-school components. Within classrooms, the standard deviation is about 2.20 years. This suggests that in a typical middle school, at each grade level, about two-thirds of all children would have scores within about 2.2 years of the average for their age. But about 16% of all children would fall above or below that range. What this means for most middle school teachers is that in a class of 25 pupils they can expect to have 4 students with scores that are at least *two* year behind those of their peers, and 4 pupils with scores that are at least *two* year above those of their peers. Similarly, the range of classroom means scores vary considerably. The results indicate that about two-thirds of all classrooms have average scores that fall within a year of the national average.

The second model in Table 1 asks whether there is variation among classrooms at these levels after taking account of students= characteristics and family background. The covariates accounted for only about 5.2% of the variance among classes (reducing it from 19.2% to 18.2%). Thus, one cannot claim that schools vary in their mathematics scores mainly because of the types of students they enroll. This model also indicates that the socioeconomic gradients vary significantly among classes in the TIMSS. The intercepts were correlated negatively with the SES gradients providing evidence of converging gradients (magnitude of the correlation, -.14 is small). That is, variation among classes in their mathematics achievement levels tends to narrow at the higher SES level. This seems to suggest that low SES students tend to do well in schools with high mathematics achievement levels.

In the third model, I added the mean socioeconomic status of a class to the second model. The estimate of the effect for mean socioeconomic status is .47, which is comparable to the effect associated with the socioeconomic status of the child. In practical terms, this means that if a child has a socioeconomic status which is one standard deviation below the national average, he or she is likely to have a mathematics score that is the equivalent of about six months below that of his or her peers. But if this child also attends a classroom that has a low average socioeconomic status, say one where the average for the classroom is also one standard deviation below the national average, the child is likely to be a full year (*i.e*., .50 + .47) below national norms. The classroom mean SES was also positively related to the SES gradient, indicating that the SES gradient is shallower in low mean SES classrooms, and steeper in high mean SES classrooms.

The last model in Table 1 includes several variables describing school and classroom variables. These were modeled on both the intercepts and the gradients, although the model for the gradients was reduced as most of the processes did not have a significant effect. The results indicate that the most successful classrooms are those where: (a) less grouping is practiced, (b) calculators are used but computers are not, (c) there is regular homework, (d) there are few discipline problems, (e) teachers specialize, and (f) there is low student-teacher ratio. Results for the model describing socioeconomic gradients indicate that classrooms have more equitable results (i.e., shallower slopes) when: (a) less grouping is practiced, (b) there is less homework, where teachers are specialized, and (c) there is low student-teacher ratio.

CONCLUSION

This paper conceptualized successful schools in terms of achieving the twin goals of excellence and equity. The analysis indicates that there are schools in Canada that are successful in achieving both excellence and equity. Successful schools and classrooms tend to be those which have relatively high achievement levels for students from lower socioeconomic backgrounds. These schools have low student-teacher ratio, specialized mathematics teachers who rarely employ grouping in their instructional practices. The finding pertaining to small grouping is not consistent with the theory that holds that interaction among students within small groups through discussion, debating, and expressing ideas creates the opportunity for multiple acceptable solutions to mathematics problems. The belief is that, through these interactions, students would experience cognitive conflicts, evaluate their reasoning, and enrich their understanding about mathematical concepts. However, as Springer, Stanne, and Donovan (1999) have noted, without the appropriate structures to make each member of a small group accountable for learning, the expected benefits of small groupings may not be realized, since the interaction would be in most instances merely sharing answers instead of ideas. Effective interactions characterized by high-level deliberations about issues that enhance conceptual understanding occur when teachers clearly define issues, give specific guidelines, and define roles for members in a group (see Johnson and Johnson, 1994). The data from TIMSS do not provide details about small grouping practices in school to allow for further analysis. The finding however calls for a better understanding of how current reform practices should work to provide opportunities for all students to learn mathematics. Current reform in mathematics education in Canada advocates for a more interactive mathematics classroom.

References

Burstein, L. (1980). The analysis of multilevel data in educational research and evaluation.In D. C. Berliner (Ed.), *Review of Research in Education*. (pp.158-231), Washington, DC American Educational Research Association.

Cobb, P., Yackel, E., & Wood, T. (1992). A constructivist alternative to the representational view of mind in mathematics education. *Journal for Research in Mathematics Education, 23,* 2–33.

Goldstein, H. (1995). *Multilevel statistical models*. London: Arnold.

Johnson, D. W., & Johnson, R. T. (1994). *Learning together and alone: Cooperative, competitive, and individualistic learning*. Boston: Allyn & Bacon.

Lau, C. J. (1979). Educational production function. In D. M. Windham. (Ed.), *Economic dimensions of education*. Washington, D.C: National Academy of Education.

Lave, J. & Wenger, E. (1991). *Situated learning: Legitimate peripheral participation*. Cambridge, Cambridge University Press

Leot'ev, A. N. (1981). The problem of activity in psychology. In J. V. Wertsch (Ed.), *The concept of activity in society psychology*. Armonk, NY: Sharp

Springer, L., Stanne, M. E., & Donovan, S. S (1999). Effects of small group learning on undergraduates in science, mathematics, engineering, and technology: A meta-analysis. *Review of Educational Research* 69(1): 21-25.

Secada, W. G. (1992). Race, ethnicity, social class, language, and achievement in mathematics. In D. A. Grouws (Ed.), *Handbook of research on mathematics teaching and learning:a project of the National Council of Teachers of Mathematics* (pp. 623-660). New York, Macmillan Publishing Company.

Vygotsky, L. S. (1978). *Mind and society: The development of higher psychological processes*. Cambridge, Harvard Education Press.