THREE CASE STUDIES ON THE ROLE OF MEMORISING IN LEARNING AND TEACHING MATHEMATICS

Sue Gordon and Jackie Nicholas

The University of Sydney

We extend an earlier study on the role of memorising in learning science with three case studies on mathematicians' experiences as learners of mathematics and their views on memorising in students' learning. Each report reflects a different view of memorising — from a minimal role in learning mathematics, to a 'stepping stone' role to memorising as a key learning strategy. The mathematicians' views about memorising in their students' learning appears consistent with their reports on memorising in their own learning. Our data leads to the conjecture that personal experiences of memorising underpin attitudes to memorising in teaching mathematics and that further investigation is warranted on this little researched aspect of "coming to know" mathematics.

INTRODUCTION

How did academic mathematicians learn mathematics when they were students and how do they perceive their students learn mathematics? We investigate this question with a focus on perceptions of memorising in learning mathematics. Our discussion draws on and extends our study on the role of memorising in learning physics, physiology, mathematics and statistics (Cooper, Frommer, Gordon & Nicholas, 2002). In that study we conducted interviews with 16 academics from the respective discipline areas. A phenomenographic analysis of the interviews revealed three categories of conceptions of the role of memorising in learning science: memorising plays a minimal role, memorising serves as a stepping stone to learning and memorising is a key strategy in learning.

A set of phenomenographic categories represents a collective awareness rather than an individual awareness expressed by Marton and Booth as "a description on the collective level, and in that sense individual voices are not heard" (Marton and Booth, 1997, p.114). Thus, in phenomenography, individuals are seen as the "bearers of fragments" (Marton and Booth, 1997, p.114) of differing ways of experiencing a phenomenon. In contrast to the "stripped" categories of description of memorising in learning science presented in our earlier paper we now take a deeper look at the views held by three mathematicians, whose conceptions were included in and contributed to the construction of the three major categories outlined above.

University teachers' earlier experiences of teaching and their perceptions of the teaching context have been shown to be significant dimensions of their current teaching approaches (Prosser & Trigwell, 1999). Moreover, there are relationships between the way teachers approach their teaching and students approach their

2005. In Chick, H. L. & Vincent, J. L. (Eds.). *Proceedings of the 29th Conference of the International Group for the Psychology of Mathematics Education*, Vol. 3, pp. 57-64. Melbourne: PME.

learning (Trigwell, Prosser & Waterhouse, 1999). Our framework brings to the foreground mathematicians' experiences both as learners and teachers of mathematics. We concur with Akerland (2004) that "a focus on academics' experience of teaching separated from their larger experience of being a teacher may encourage over simplification of the phenomenon of university teaching, in particular in terms of academics' underlying intentions when teaching".

Our theoretical framework draws on and amplifies Burton's theories on "coming to know mathematics" (Burton, 2001, 2003). Burton (2003, p. 13) proposes that "coming to know mathematics is a product of people and societies" and is heterogeneous, connected with experiences and with aesthetic and other feelings, and interdependent on networks of practice. We conjecture that mathematicians' experiences of coming to know mathematics at university are important to how they view the process of learning mathematics and that their beliefs and ideas impact on teaching. Hence mathematicians' experiences of memorising in their own learning of mathematics and their perceptions of the role of memorising in their students' learning are important to pedagogy. These are areas in which there is a dearth of data. Further, Burton (2001) proposes that mathematicians at university have considerable impact on the continuing development of the discipline and the enculturation of the next generation of mathematicians and mathematics educators.

> Mathematicians in universities have great power to influence what is learned and how it is learned in the discipline. They do this, in part, through the respect which society accords their discipline, and consequently those within it. ... But these mathematicians are also responsible for teaching the next generation of teachers and, consequently, for contributing to the definition and history of the practices which are seen to be appropriate to the communities that they touch (Burton, 2001, pp. 589-590).

We provide insights on the perceptions and experiences of three mathematicians as indicators of the connectedness of learners, learning and teaching mathematics at university and to highlight a need for further research in memorising to help develop theories on coming to know mathematics.

METHOD

We propose and answer the following research questions.

- How did the mathematicians learn mathematics and view memorising in their own learning?
- What were the mathematicians' perceptions of student learning and memorising?

Our three participants were experienced mathematicians and mathematics educators working in metropolitan universities in Australia and South Africa. The interviews were semi-structured and consisted of seed questions such as: *What role did memorising play, in your own learning of mathematics? How do you think students learn mathematics*? *Do you feel there is a relationship between memorising and* *understanding*? Follow up questions depended on the responses and aimed at an in-depth understanding of the participants' experiences and perceptions. Interviews were audio-taped and transcribed.

In our initial phenomenographic analysis (Cooper et al, 2002) we focussed on the construction of broad categories from the data to form an outcome set, ignoring personal and individual perceptions. Our re-analysis of the interviews for the current investigation had a different purpose — to listen to and represent the individual reports of our participants. We examined independently and extensively the mathematicians' responses as they related to the two research questions. We compared, discussed and integrated our individual analyses and reviewed the interviews in ongoing cycles. We present case studies to illustrate views that best fit with each of the three categories describing memorising. However the view of each participant presented is more than an expansion and delineation of a category – it is an individual construction as interpreted by us, the researchers. This methodology can be considered as ethnographic. All three participants are well known to us as fellow mathematicians and mathematics educators, and in one case, also a fellow postgraduate student of mathematics. Our interpretations are cognisant of the environments in which our participants work, the resources available to them and the constraints which organise and shape their teaching.

RESULTS

Case Study 1: Peter

For Peter memorising was not a conscious strategy in his learning and therefore played a minimal role if any. His major strategy for making mathematics meaningful was writing out his lecture notes so he could understand them.

> I wrote out everything as much as I could of what he (the lecturer) said and what he wrote. And then I tried to write it in a way that I could understand it. I know that it was the effort of writing it out that helped me understand it.

Peter described his learning as including doing many exercises, essential to gain proficiency and understanding, and reported that learning mathematics was an effortful business. He said: "I did many exercises, so when I finished each course I knew how to do the problems and I had an idea of what the lecturer was saying I think". He continued: "I was lucky, because I didn't have any tutorials, so I had to grapple with stuff".

When asked for his perceptions of how his students learned mathematics, this mathematician talked for the most part about a large group of students who he thought were memorising without understanding.

> I think many of them memorise. ... I'm looking at one student now who really shouldn't be in second year. She has serious algebraic difficulties, and I know she is hanging on my every word and thinks all she has to do is repeat and she'll be right.

He considered this strategy had little value in learning mathematics as it was the "shallowest thing which was active" and "it is not sufficient just to memorise facts without any understanding of them".

He was of the view that the majority of students arrived from school with "an incorrect image of mathematics in their heads". Their approach to learning mathematics consisted of learning formulae and techniques and passing the examinations was a matter of applying those. However, he considered that these students in first year were "serious about learning" and they were "quite able to grow" even though they struggled to understand what the lectures were about. He believed that by third year this seriousness about learning had disappeared and students expected everything to be given to them easily, using tutorials as the principal venue for their own learning.

> They don't understand what the lectures are about, they think there are hoops, that they have to jump through these hoops, and all they've got to do is do that, and they pass and go on to the next thing.

These students, he felt, were able to pass examinations with a minimal understanding of the mathematics course by learning techniques and memorising rules. He thought that the examinations were "testing a very basic manipulative ability to follow the rules that we have given".

On the other hand, Peter acknowledged that there was another group of students who were serious about learning mathematics, and that "the student who gets a rudimentary understanding of mathematics I think does a great deal of learning".

Case Study 2: Mary

Reflecting on her learning Mary recounted that memorising fulfilled a role as a "stepping stone". She explained that by memorising she could ensure that an idea was captured with the necessary completeness, precision and conciseness. This allowed her to use an idea with confidence and certainty enabling her to progress. Memorising was interlinked with personal understanding and this was a dynamic, incomplete process.

> Memorising would never be like the lecturer wrote it, like 1, 2 and 3. I'll write it as 1, 2, and 3 because it did make sense to her (the lecturer) as 1, 2 and 3 but now it makes sense to me as well as 1, 2, and 3 so I'll learn it as 1, 2 and 3.

Mary reported that as a student had learned mathematics by making a line by line examination of a section of work trying to understand each and every facet. When asked how she knew she understood, she replied as follows.

> A feeling that the statements were validated. Yes a feeling that each statement was valid and followed a sequence, that there were no gaps. If there were gaps I would put in those gaps for myself. And those gaps later you would take them out again as your understanding became more sophisticated so your need to fill those gaps became less profound, you could take them out.

This mathematician perceived memorising as playing an important part in all students' learning. She identified two distinct groups of students characterised by a qualitative difference in their memorising. She considered that the majority were the weaker students who used memorising as a "crutch" for their learning mathematics — for them memorising was employed when there was no underlying understanding.

> Many students when they don't understand will learn by rote, learn by heart, learn to reproduce a piece of work according to how they've been given it. But it is quite clear when you ask a leading question they have no understanding to accompany that learning.

She perceived that for these students the goal was to reproduce what they were given.

Mary distinguished these students from the second group of students who, like her, memorise "only after they have come to some understanding about the nature of the subject". She characterised their memorising as a "stepping stone" or "scaffold" to support their growth in learning mathematics.

> In order to complete a piece of learning they might have memorised the theorem, and feel confident in the theorem, these are the students who get the picture of what mathematics is about. And in order to use it frequently and without judgement and without thought, they've memorised it. They can repeat the definition. But they use the definitions with insight whereas the other group merely have learned the words of the definition but they can't use it.

Thus, Mary perceived that memorising fulfilled the same function in learning for both groups of students but there were qualitatively different forms of memorising with differing longevity. Students who used memorising as a crutch were going to use that crutch forever, while students who used memorising as a scaffold for understanding ultimately removed that scaffold.

Case Study 3: Qian

To Qian memorising was a strategy that played a key role in learning mathematics and pervaded every aspect of his own learning mathematics.

> From my experience, definitely memorising is one of the very basic techniques for me, to go through my study. You just start to memorise from the first page to the last pages and exercises you produce. Memorising is very important.

He reported that memorising patterns of proofs and how to do things was essential for remembering in the long term, for problem solving, and for precision. He added that memorising, for him, was essential for speed in examinations as there was no time to derive the required formulae or prove the theorems. His strategy appears analogous to a master chess player deliberately memorising multiple chess games, forming a permanent memory base from which a play could be drawn and speedily applied to a game of chess in progress but also as a springboard for creating new plans and solving problems.

Doing problems was an important strategy in Qian's learning to see if he could apply what he had learned, to test his understanding of a mathematical concept and to identify gaps in understanding.

When asked how his students learned mathematics, Qian replied that students learned mathematics by pattern recognition and learning pattern types which they tried to imitate or copy. He considered that for the majority of students, learning was accomplished largely through doing examples and using those examples as templates for doing the same kind of examples. He estimated that only the top 20% of his students were attempting to learn how to reason mathematically.

Qian reported that memorising was a strategy used by all students. However, consistent with his view of memorising in his own learning, he emphasised memorising as being of prime importance for the top students in the senior years of their degree. He expected these advanced students to remember a lot of proofs and definitions and to apply these to solve complex problems. He emphasised that memorising a "pattern of proof" was an efficient and time saving examination technique for advanced students even though they had the skills to use derivation and deduction.

He considered that there was a qualitative difference in memorising as a strategy for the top students and for the majority of students.

> One is memorising with understanding and the second is memorising hopefully, I guess. Is this the formula, or is that the formula? They did not have understanding so they could not use it.

He stressed that those students who memorised a formula or technique with understanding would remember "forever", while students who memorised a formula "hopefully" and "for the sake of memorising, without even knowing anything", would quickly forget it.

DISCUSSION AND CONCLUSION

In reflecting about their own mathematical learning, the mathematicians expressed their engagement with mathematics at a high level. They were "mathematically curious" and wanted to see how everything fitted together with a focus on developing a personal understanding. For them learning mathematics was effortful. All agreed that it was vital to have a memory bank of precise definitions and mathematical concepts and techniques that could be recognised and retrieved as required. This enabled progress to be made in learning mathematics. Their learning strategies included rewriting lecture notes according to their understanding, solving problems, a line by line analysis of mathematical statements and, for some, memorising. Memorising was employed to learn patterns and proofs, for precision and to facilitate progress, for speed in examinations, for permanency and to enable application.

In discussing their students' learning, all our participants identified a large group of students whose learning was characterised by imitation. Students in this group were perceived as learning mathematics by rote without an underlying basis of conceptual understanding. For these students the primary goal in learning mathematics was to pass assessments and the mathematicians considered these students to be the weaker students or the students who were struggling.

The mathematicians also identified a smaller group of students, described as the good, advanced or top students, who were motivated by understanding the mathematics and who worked at learning mathematics. Not all our participants perceived that memorising was a strategy used in learning mathematics by these students. The mathematicians who believed that memorising was used, and was an appropriate and valuable strategy for good students, perceived the purpose and quality of their memorising to be quite different to memorising as the empty repetition and imitation of the weaker majority.

The three case studies reveal diverse views on memorising in learning mathematics from memorising as rote learning or parroting to memorising as a form of permanent internalising with understanding. Further, while each individual conveyed strongly his or her own view about memorising in learning mathematics, apparently none considered or contemplated other possibilities for the role of memorising in learning mathematics.

The observations of the mathematicians about memorising in their students' learning appeared to be related to perceptions of memorising in their own learning as students. Moreover, there was little acknowledgement by our participants about the impact of their own learning experiences on their teaching. If personal experiences of learning mathematics are not recognised as being related to teaching mathematics at university we can expect to perpetuate the idea that mathematics is a pure body of knowledge that is learned through logic and rational thought alone, independent of social context, emotion, personal history and human interaction.

The case studies demonstrated little appreciation or nurturing of diversity in learning mathematics and the contributions of students' life experiences did not appear to be acknowledged in the mathematicians' reports. Burton (2001) distinguishes between mathematical 'knowledge' and mathematical 'knowing' and concludes that while the journey of coming to know is seen as important for research in mathematics it is notably absent from views of teaching mathematics at university. All three participants of our study talked about "good" students and "weak" students and their concepts of these appear to be in terms of acquiring knowledge or making an effort to acquire knowledge. A good student is one who works to gain the knowledge set out by the teacher — a poor student does not try or does not succeed in attaining this discipline knowledge.

Our participants reported constraints in the institutional context for teaching mathematics at university, which impacted on their teaching strategies, including pressures to have satisfactory pass rates, pressures to make things easy for students and colleagues, time constraints and large class numbers. Peter even proposed that students' engagement with learning mathematics on entering university was systematically undermined during their undergraduate years. The impact of context is important to understanding memorising in learning mathematics and we are investigating this further (Gordon & Nicholas, In preparation).

Anecdotally mathematicians appear to expect students "to know" and remember the mathematics they are taught, to "have" a body of mathematical knowledge in their memories. Our data provides insights into the relationship between memorising and memory and suggests that it is not a simple connection. More research is needed to explicate the strategy and outcomes of memorising in learning mathematics.

Questions also arise as to the intentions of mathematics teachers at university — is the primary goal that students acquire a body of knowledge, whether by memorising or other learning strategies or do we focus on promoting mathematics enquiry (Burton, 2003) and diverse ways of learning mathematics? The rhetoric of mathematics education emphasises the importance of engaging students to construct mathematics and appreciate mathematical thinking but in our universities the rhetoric and the reality may not match.

Acknowledgement

We gratefully acknowledge the financial assistance for this project through a research grant from the University of Sydney Science Faculty Education Research Group (SCIFER).

References

Akerlind, G. (2004). A new dimension to understanding university teaching. *Teaching in Higher Education*, 9 (3), 363-375.

Burton, L. (2001). Research mathematicians as learners—and what mathematics education can learn from them. *British Educational Research Journal*, 27 (5), 589-599.

Burton, L. (2003). *Mathematicians as enquirers*. London: Kluver Academic Publishers.

Cooper, I., Frommer, M., Gordon, S., Nicholas, J. (2002). University teachers' conceptions of memorising in learning science. *Higher Education Research & Development*, 21 (3), 305-321.

Gordon, S. & Nicholas, J. (In preparation). Memorising, learning and teaching mathematics at university.

Prosser, M. & Trigwell, K. (1999) *Understanding learning and teaching*. Buckingham: Open University Press.

Trigwell, K., Prosser, M. & Waterhouse, F. (1999). Relations between teachers' approaches to teaching and students' approaches to learning. *Higher Education*, 37, 57–70.