INVESTIGATING THE PROBLEM SOLVING COMPETENCY OF PRE SERVICE TEACHERS IN DYNAMIC GEOMETRY ENVIRONMENT

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This study investigated the problem-solving competency of four secondary pre service teachers (PSTs) of University of London as they explored geometry problems in dynamic geometry environment (DGE) in 2004. A constructivist experiment was designed in which dynamic software Cabri-Géomètre II (hereafter Cabri) was used as an interactive medium. Through participating in the study, the PSTs demonstrated their competency exemplified by using their subject knowledge to construct, to make conjectures and verify their conjectures on their way to solve the given geometric problems.

INTRODUCTION

Being a mathematics teacher educator I felt the gap between the technology infused learning experiences in geometry class and the current status of teacher preparation programs that in general pay little attention to this aspect. For a solid prescription, I needed an empirical data involving PSTs and DGE. The first issue that stroke me was: How competent are the PSTs in using subject knowledge to solve problems in DGE? I decided to pursue my investigation to find answer for this question.

Related Studies

There exists very few experiential data to inform us about the competency of PSTs in using DGE. Saads and Davis (1997) investigated the van Hiele levels in three-dimensional geometry and the spatial abilities of a group of secondary PSTs and found that PSTs' language depends on a combination of the PSTs' general geometric level, their spatial ability and their ability to express the properties of the shape using language. Pandiscio (2001) studied how secondary mathematics PSTs perceive the need for and the benefits of formal proof when given geometric tasks in the context of dynamic geometry software. Jiang (2002) observed the actual learning processes of two PSTs as they explored geometry problems with dynamic geometry software and the effects of using the software on developing their mathematical reasoning and proof abilities.

'Knowledge-in –Action' Design

The aim of the study is to find out the PSTs' problem solving competency in DGE. I developed a 'Knowledge–in–Action' Design (Fig 1) based on geometrical constructions in Cabri using dragging and other functions to illustrate the problem solving process of PSTs. The process of converting the conceptual understanding of the given geometric problems into action demands the PSTs to apply their content

knowledge to understand the given problem, construct the dynamic figures, make conjectures, verify the conjectures, and solve similar problems. Accordingly the competencies are defined as follows:

- Competency 1: PSTs construct figures in DGE.
- Competency 2: PSTs make conjectures in DGE.
- Competency 3: PSTs verify their conjectures in DGE.

It is been hypothesized that if the PSTs are able to solve the given problem using their subject knowledge by constructing, conjecturing and verifying the properties in DGE then their competency is testified. The study builds on the assumption that learning involves the active construction of knowledge through engagement and personal experience (Ernest, 1994; von Glasersfeld, 1995). The Van Hiele theory of learning geometry also provided a useful conceptual framework for this study.
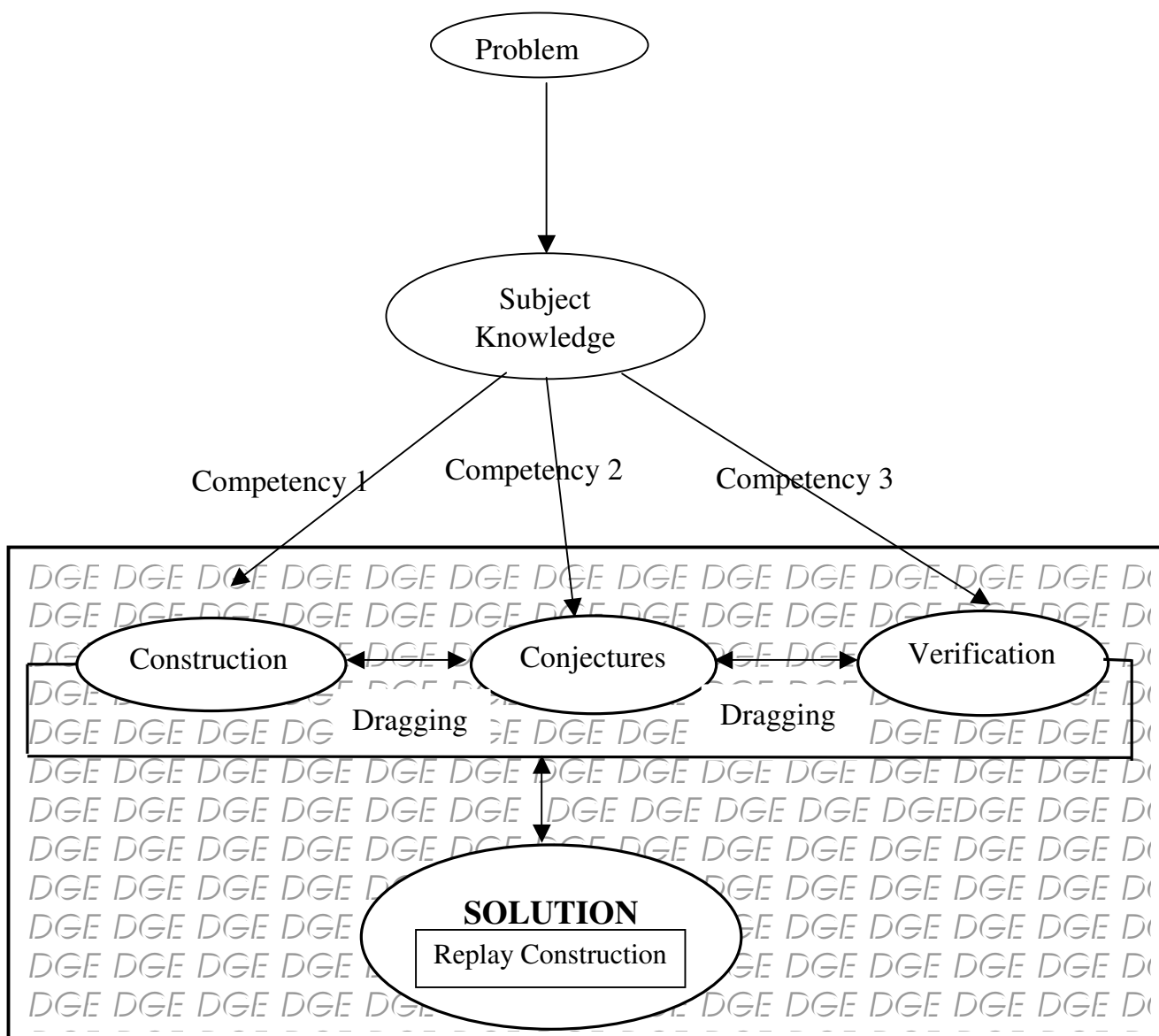
Figure 1: Knowledge-in-Action Design.

METHODOLOGY

The subjects of this experiment were four secondary PGCE mathematics students (3 males PST1, PST2, PST3 and 1 female PST4) of University of London. None of the PSTs had used Cabri before taking part in this experiment but they had studied geometry at the high school/O level. The experiment was conducted on one- to-one basis on different days in January 2004 at the Institute of Education, London. The experiment was recorded.

Constructive Experiment: Discussion

The constructive experiment designed for the study is an open-ended problem involving Varignon's quadrilateral and medial triangle. I analyze the Varignon's quadrilateral using non - conventional quadrilaterals (crossed/ bow tie) in DGE background, which is otherwise not possible with pencil and paper. Consider a quadrilateral ABCD (figure 2a) with the midpoints of its sides E, F, G and H. In the experiment, PSTs conjectured that EFGH is a parallelogram. By dragging the vertices of the quadrilateral ABCD they observed that EFGH remained a parallelogram no matter what type of quadrilateral ABCD was. And this result is valid for a crossed quadrilateral too (figure 2b). Also the area of the Varignon parallelogram EFGH is exactly half of the area of the original quadrilateral ABCD; even in the case of the crossed quadrilateral.
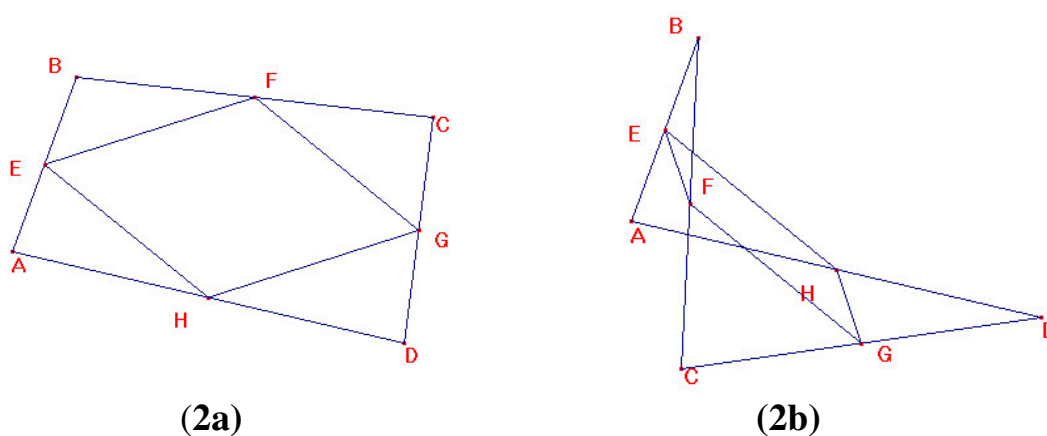

(**2a**)   (**2b**)

Figure 2: Varignon's Quadrilateral.

Now let us consider three-sided polygon (n=3) in the above example; the quadrilateral is then replaced by a triangle. I see that the medial triangle formed by joining the adjacent midpoints is similar to the original triangle. PSTs explored this task with special cases: equilateral triangle, right triangle and isosceles triangle. Now, the midpoints of the sides of a triangle form a triangle DEF (see Fig 3) that is exactly one quarter of the area of the original triangle ABC.
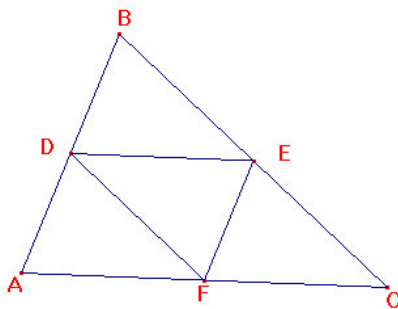
Figure 3: Medial Triangle.

Also the medial triangle is not only congruent to the other three small triangles (ADF, BDE, EFC) but also similar to ABC and the line segment connecting the midpoints of two sides of a triangle is parallel to the third side and is congruent to one half of the third side.

Now, how did the dynamic figures control the PSTs exploration? In paper-and-pencil geometry, all entities are static, but in dynamic geometry, they behave in a specific way to maintain their geometric properties. For instance, the vertices of quadrilateral ABCD are completely arbitrary and can be moved freely, while E, F, G and H are fixed as midpoints that cannot be altered. This is the same with triangles as well. For instance, in the right triangle the perpendicularity of two lines controlled the activities of PSTs.

Exploration by PSTs as Replay Construction – The Road to the Solution

In this experiment, the *Replay Construction* function in the edit menu of Cabri describes the construction process when a convincing solution was achieved by a set of physical manipulations by the PSTs. The following two illustrations describe step-by-step how each PST works.

Illustration 1: Medians of Isosceles Triangle

All the four PSTs were able to construct the isosceles triangle using different conjectures. PST 1 & PST3 used angle bisectors to construct the triangle while PST2 used the conjecture (*perpendicular bisector of a chord passes through the centre of the circle*) & PST4 used (*isosceles triangle has two equal sides*) circles to construct the triangles. PST2 & PST4 used the radii of circles as the two equal sides of the right triangle (Fig 4).
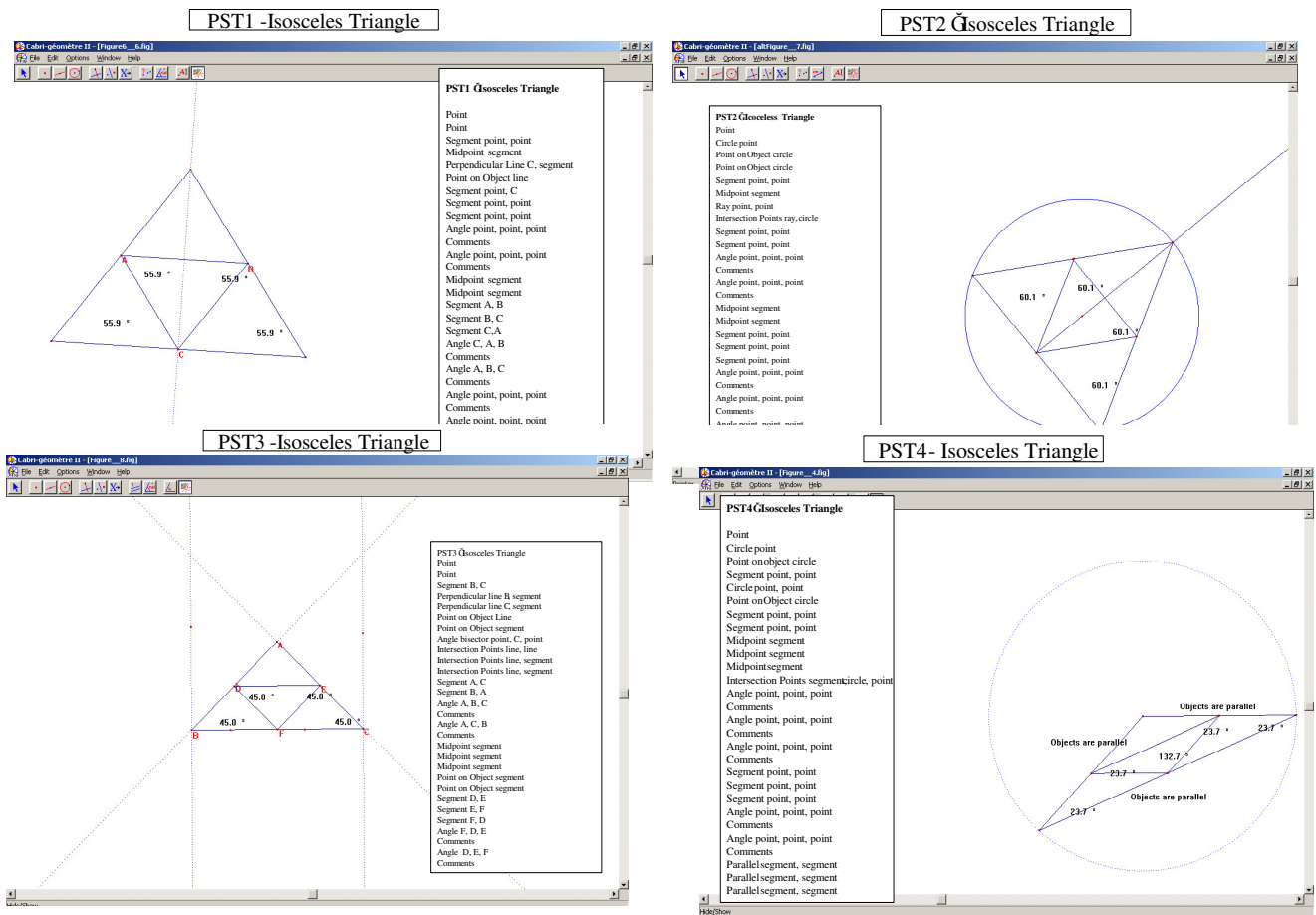
Figure 4: Medians of Isosceles Triangle.

Illustration 2: Medians of Quadrilateral

PST2 tried non-convex quadrilateral while others tried the convex. In either case PSTs found that the inner quadrilateral turned out to be a parallelogram (Fig 5). PST1 Used the conjecture *diagonals of a parallelogram bisect each other.* Others used the conjecture *the opposite sides of a parallelogram are parallel and opposite angles are congruent.* PST1 tried to compare the areas of inner and outer quadrilaterals by drawing the diagonals. He found that the sum of the areas of the four triangles was one-half of the area of the entire quadrilateral.

Figure 5: Medians of Quadrilateral.

CONCLUSION

In this experiment, geometrical facts about medial triangles and mid point quadrilaterals are examined by the PSTs. The emphasis was on tracing a solution to the given problem not a formal proof. As PSTs manipulated the dynamic figures, they could make conjectures about the properties of the shapes. Also, PSTs used on-screen measurements for side lengths and angles to transform their intuitive notions into more-precise formal ideas about geometric properties (Battista, 1998). For instance, PSTs used distance and angle measurement to check the geometric properties of medial triangles. I observed that PSTs are motivated to search for explanations driven by 'dragging' that guided them towards the solution. This is substantiated by PSTs' opinion about the experiential learning; to put it in their own words, '*This is a new experience and I thoroughly enjoyed it* (PST1)', '*It is wonderful that I could create my own way of finding a solution* (PST4)', '*I learned* (PST2)' and '*It is useful but time consuming* (PST3)'. Analysis of the PSTs solution process shows that learning took place in the form of construction, conjectures, and verification and thus validates the assumption of the study (Ernest, 1994; von

Glasersfeld, 1995). However the interaction with dynamic software can't be accepted as a formal proof as argued by Saads & Davis (1997). The problem solving behaviour exhibited by the four PSTs correlates with the Knowledge-in-Action design (Fig1) in terms of the components (construction, conjecture, and verification) described in the design.

The construction process showed that PSTs demonstrated adequate geometric reasoning in accordance with the hierarchical levels of Van Hiele viz., Visualization, Analysis, Abstraction, Deduction, and Rigor on the selected tasks. However, this can't be generalized to the PSTs understanding of other geometrical concepts (Mayberry, 1983). Observation of the problem solving processes of the four PSTs while they attempted to solve geometric problems with the help of Cabri suggests that:

a) PSTs' content knowledge are adequate to understand the given problems b) PSTs are competent enough to apply the content knowledge to construct the dynamic figures c) PSTs are competent enough to apply the content knowledge to make conjectures d) PSTs are competent enough to apply the content knowledge to verify the conjectures and e) PSTs are able to use DGE to justify their solution.

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