IN THE MIDDLE OF NOWHERE: HOW A TEXTBOOK CAN POSITION THE MATHEMATICS LEARNER

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We outline a framework for investigating how a mathematics textbook positions the mathematics learner. We use tools and concepts from discourse analysis, a field of linguistic scholarship, to illustrate the ways in which a can textbook position people in relation to mathematics and how the text can position the mathematics learner in relation to classmates and to the world outside of the classroom. We conclude with a general consideration of alternative language forms, which, we suggest, ought to include linguistic recognition of the moves associated with mathematisation.

INTRODUCTION

> The equations below express the relationship between a person's height, H, and femur length, F. Males: H = 27.5 + 2.24F. Females: H = 24 + 2.32F. […] If a man's femur is 20 inches long, about how tall is he? (Lappan *et al*, 1998, p. 21)

A student could respond in many ways to this prompt from a mathematics textbook. She might just follow the instructions, or she might pose further questions. Her questions could be mathematical or *about* mathematics, such as the following: Who found these equations? How? Why would someone want to use these equations? For whom and why am I finding the man's height? Should my answer depend on the situation?

Questions such as these will be addressed in this report's investigation of mathematics textbooks and how they position students. In this paper, we use tools and concepts from discourse analysis, a field of linguistic scholarship, to demonstrate a framework for examining the way a textbook can influence a mathematics learner's sense of mathematics. We are interested in the way textbooks position people in relation to mathematics and how the text positions mathematics learners in relation to their classmates and teachers and to their world outside of the classroom.

CONTEXT AND METHOD OF THIS INVESTIGATION

To illustrate our textbook analysis framework, we draw examples from the *Connected Mathematics Project* (CMP) (Lappan *et al*, 1998). This middle school mathematics curriculum was developed and piloted in the 1990s in the United States with funding from the National Science Foundation and was recently classified as 'exemplary' by the U.S. Department of Education's Mathematics and Science Expert Panel (1999). CMP textbooks were written to embody the mathematics *Standards* (1989, 1991) published by the National Council of Teachers of Mathematics, an organization comprising mathematics educators from across Canada and the USA.

Following the suggestions in the *Standards* documents, this CMP curriculum uses problem solving contexts to introduce students to big ideas in mathematics and the curriculum encourages exploration and discussion. Because it makes extensive use of problem solving contexts, this text material is significantly more verbose than more conventional mathematics textbooks.

There are many aspects of this text material that we appreciate. The materials are engaging, mathematically rich and based on constructivist principles within which learners are encouraged to develop and discover big mathematical ideas. The text series' wordiness makes linguistic analysis much easier in many ways – because there are many sentences to analyse – but also much more complex. As researchers who are interested in discourse, ideology, and power, of which, student positioning is a part, we assert the importance of analysing the way students are positioned by this textbook series and other textbooks.

We chose to use CMP to demonstrate our framework because we both, independent from each other, have been involved in detailed critical linguistic analyses of its curriculum materials. (See, for example, Herbel-Eisenmann, 2004.) Indeed, the series' author team has demonstrated its interest in healthy classroom discourse by asking for this linguistic analysis. In our analyses, we examined the materials as a whole made up of parts. We examined the text word-by-word, sentence-by-sentence, and section-by-section to identify and classify particular linguistic forms. We also considered linguistic forms as parts of a whole (i.e., each word is part of a sentence as well as part of a section within a unit) and interpreted the function of language forms in the context of their location within the text. We particularly attended to words and phrases that constructed roles for the reader, relationships with the author and mathematics from a particular epistemological stance.

Due to space limitations, for this report, we draw on our analyses of one unit in the curriculum: *Thinking with Mathematical Models (TMM)* – a 64-page soft-bound booklet. This particular unit focuses on mathematical modelling through the use of experiences that require data collection (e.g., measuring the breaking weight of a set of paper bridges based on the number of pennies each could hold) as well as predefined modelling situations (e.g., different equations that model the economic impact of raising the price of cookies in a bakery). We draw on our textual analyses to show how the unit positions the mathematics learner.

PERSONAL POSITIONING IN RELATION TO MATHEMATICS

To develop an understanding of how the people are positioned with respect to mathematics, we focus on two language forms: personal pronouns and modality. With these forms we see who the text recognises as the people associated with mathematics and how they stand in relation to the mathematics.

First person pronouns (*I* and *we*) indicate an author's personal involvement with the mathematics. Textbook authors can use the pronoun *I* to model an actual person

doing mathematics, and can draw readers into the picture by using the pronoun *we* (though there is some vagueness with regard to whom *we* refers). The use of the second person pronoun *you* also connects the reader to the mathematics because it speaks to the reader directly, though it can be used in a general sense, not referring to any person in particular (see Rowland, 2000)

In *TMM*, first person pronouns were entirely absent. Morgan (1996) notes that such an absence obscures the presence of human beings in a text and affects "not only […] the picture of the nature of mathematical activity but also distances the author from the reader, setting up a formal relationship between them" (p. 6).

The second person pronoun *you* appears 263 times in *TMM*. Two of these forms, in particular, are relevant here: 1) **you + a verb** (165 times); and 2) **an inanimate object + an animate verb + *you* (as direct object)** (37 times). The most pervasive form, ***you + a verb***, includes such phrases as 'you find', 'you know' and 'you think'. In these statements, the authors tell the readers about themselves, defining and controlling what linguists Edwards and Mercer (1987) call the 'common knowledge'. The textbook authors use such control to point out the mathematics they hope (or assume) the students are constructing.

Linguists use the term *nominalisation* to describe language structure that obscures human agency. In *TMM*, the other common *you*-construction (**an inanimate object + an animate verb + *you* (as direct object)**) provides a striking example of nominalisation in the text. In this text, inanimate objects perform activities that are typically associated with people, masking human agency – for example, 'The graph shows you…' and 'The equation tells you...'. In reality graphs cannot show you anything; it is the person who is reading the graph who determines what the graph 'shows'. This type of nominalisation depicts an absolutist image of mathematics, portraying mathematical activity as something that can occur on its own, without humans. Scattered throughout these instances of nominalisation, however, the text refers to human actors in the mathematical problems. These actors are named by occupation, by group (e.g. 'riders on a bike tour'), or by fictional name (e.g. 'Chantal'). The combination of nominalisation alongside these human actors may send mixed messages to the reader about the role of human beings in mathematics.

The modality of a text also points to the text's construction of the role of humans in relation to mathematics. The modality of the text includes "indications of the degree of likelihood, probability, weight or authority the speaker attaches to an utterance" (Hodge and Kress, 1993, p. 9). Modality can be found in the "use of modal auxiliary verbs (*must, will, could*, etc.), adverbs (*certainly, possibly*) or adjectives (e.g., *I am sure that…*)" (Morgan, 1996, p. 6). With such forms, the text suggests that mathematics is something that people can be sure about (for example, with the adverb *certainly*) or that people can imagine possibilities within mathematics (for example, with the adverb *possibly*).

Linguists use the term *hedges* to describe words that point at uncertainty. For instance, because of the hedge *might* in 'That function *might* be linear', there is less certainty than in the unhedged 'That function is linear'. The most common hedge in the *TMM* text is *about* (12 instances), followed by *might* (7 instances) and *may* (5 instances). This kind of hedging could raise question about the certainty of what is being expressed and could also, as Rowland (1997) asserts, open up for readers an awareness of the value of conjecture.

Modality also appears in verb choice. The modal verbs in this text include *would* (55 times), *can* and *will* (40 times each), *could* (13 times), and *should* (11 times). The frequency of these different modal verbs indicate an amplified voice of certainty because the verbs that express stronger conviction (*would*, *can*, and *will*) are much more common than those that communicate weaker conviction (*could* and *should*). The strong modal verbs, coupled with the lack of hedging, suggest that mathematical knowledge can be and ought to be expressed with certainty.

HOW TEXT CONSTRUCTS AND POSITIONS A MODEL READER

In the second half of this report, we draw attention to how the *TMM* booklet speaks to and constructs what Eco (1994) calls the *model reader*. Any text gives linguistic and other clues about the audience-in-mind, which can be described both as the intended reader or the reader the text is trying to create. While textbooks are written for both teachers and students to read, we concentrate only on the positioning of the student–reader, whom we call the text's *model student*. To do this, we focus on the language 'choices' made by the authors (though we recognise that most choices are subconscious). Morgan (1996, 1998), in her extensive study of mostly student-authored mathematical writing, also focuses on author choices in her analyses:

> Whenever an utterance is made, the speaker or writer makes choices (not necessarily consciously) between alternative structures and contents. Each choice affects the ways the functions are fulfilled and the meanings that listeners or readers may construct from the utterance. […] The writer has a set of resources which constrain the possibilities available, arising from her current positioning within the discourse in which the text is produced. (Morgan, 1996, p. 3)

Language indirectly indexes particular dispositions, understandings, values, and beliefs (Ochs, 1990). By examining the language choices authors make, we can see how they construct the model reader and position mathematics students. From this, we can infer the student's experience of mathematics.

Student positioning in relation to particular people

Though the *TMM* booklet presents mathematics as impersonal and pre-existent (or, at best, coming from an obscured body of impersonal mathematicians), most students actually learn mathematics within a social environment. Given their experience of this community, we ask how the text positions students in relation to the people around them – their teachers and their peers. Various aspects of the text, including the

graphical work and the ways the authors directly address the reader, position the model student in relation to the people around her.

Pictures alongside verbal text strongly impact the reader's experience of the text. In the analysed text, for example, there are 24 graphical images (in addition to graphical representations of particular mathematical expressions). Of these, 17 are 'generic' drawings and seven are 'particular' photographs. To illustrate, a *drawing* of a boy could represent any boy, whereas a *photograph* of a boy is a representation of only the one boy in the photograph. The textbook's preference for the more generic drawings mirrors its linguistic obfuscation of particular people, as we detailed above.

This obfuscation is heightened by the absence of people in the graphic images. Only one quarter of the images have people in them. In these, we find people playing on teeter-totters and operating cranes (among other things), but only one image of a person doing mathematics. Significantly, this image is a drawing (which makes the person generic) and the image only shows a person's hand conducting a mathematical investigation. This disembodied, generic hand parallels the lost face of the mathematician in agency-masking sentences such as the ones discussed above.

Mathematics addresses the student literally too, with sentences structured in the imperative mood. Morgan (1996) asserts such imperatives tacitly mark the reader as a capable member of the mathematics community. However, we suggest that such positioning is not clear from the mere presence of imperatives. Rotman (1988) distinguishes between what he calls *inclusive* imperatives (e.g. *describe*, *explain*, *prove*), which ask the reader to be a *thinker*, and what he calls *exclusive* imperatives (e.g. *write*, *calculate*, *copy*), which ask the reader to be a *scribbler*. Mathematicians think and scribble.

We find significance in Rotman's terms: *inclusive* and *exclusive*. The thinker imperatives construct a reader whose actions are *included* in a community of people doing mathematics, whereas the scribbler imperatives construct one whose actions can be *excluded* from such a community. The student who 'scribbles' can work independent from other people (including her teacher and peers). A drawing in this textbook seems to capture the text's view of the model student's relation to other people while doing mathematics. A generic boy sits alone in an empty theatre, with his hand up, as if to hold other people away (p. 15).

Though the textbook we are examining has many more exclusive imperatives than inclusive (221 exclusive and 94 inclusive), we caution against premature characterization of the text's preference for scribbling. When we follow the flow of imperatives in a text, we can form a better picture of the model student constructed by it. For example, in this textbook's initial 'investigation', we find the following stream of imperatives (pp. 5, 6): *make* a bridge, *fold* the paper, *suspend* the bridge, *place* the cup, *put* pennies in, *record* the number, *put* strips together, *find* the weight, *repeat*, *do* the experiment, *make* a table, *graph* your data, *describe* the pattern, *suppose* you use. Of these 14 imperatives, the first 12 are exclusive and the last two

inclusive. Does this mean that scribbling is privileged over thinking because of the ratio 12:2? Not necessarily. It is appropriate to scribble before thinking. Furthermore, if a text makes too many thinker demands, it could be tacitly encouraging students to jump from one thought to another without dwelling on any of the thinker demands.

But this raises a further question about the model student's relation to the people around her. The text makes no mention of the other people involved in the thinking imperatives. Looking again at the closing imperatives in the above sequence, we note that a student could ask, 'Describe? Describe to whom?' The same goes for *TMM*'s most prevalent thinker imperative: 42 of the 94 inclusive imperatives are *explain*. If the students are expected to explain their reasoning, they might reasonably expect some direction from the text with regard to their audience. Morgan (1998), in her analysis of students' mathematical writing, notes the significance of the students' sense of who their audience is.

There is an exception to this absence of reference to the people around the model student. At the end of each section, the text includes a page called 'Mathematical Reflections'. In all of these sections, the same sentence follows the prompts for reflection: "Think about your answers to these questions, discuss your ideas with other students and your teacher, and then write a summary of your findings in your journal" (e.g. p. 25). Here the text does recognise the presence of other people around the student. From this, the model student might see herself doing mathematics independent from other people, but thinking *about* her mathematics in community. Only metacognition is interpersonal, it seems.

Student positioning in relation to the world

While we suggest that explicit linguistic reference to the student's audience would be appropriate in this mathematics textbook, we recognise that such reference may raise further questions. As soon as a student is led to explain something to other people, she is likely to require a reason and context for the explanation. She would want to tailor her explanation to the situation and the people's needs within the situation.

Most of the questions and imperatives in the analysed textbook are referred to as 'real life', 'applications' (applications of mathematics to real life, we presume) and 'connections' (connections between mathematics and real life, we presume). Though the textbook consistently places its mathematics in 'real' contexts (with few exceptions), linguistic and other clues still point to an insignificant relationship between the student and her world. When we compare the instances of low modality (expressing low levels of certainty) with those of high modality, we begin to see what experiences the text foregrounds. First, we look at references to the student's past experiences. The text refers with uncertainty to the student's experiences outside the classroom using hedging words like *probably* or *might* – for example, when referring students to their experiences with a teeter-totter the text tells the student "You *might* have found the balance point by trial and error" (p. 28, emphasis ours). However, the text expresses certainty about the student's abstract mathematical experiences, as in

"In your earlier work, you saw that linear relationships can be described by equations of the form $y = mx + b$" (p. 9). This certainty may seem odd, considering that there are no indicators about sequencing in the textbook series, but there are pragmatic reasons for such assertions. Because the authors know what the curriculum offers, they can work under the assumption that the student has learned particular mathematical ideas. Yet, the authors cannot really know who their readers are. *TMM*'s authors hedge statements about students' experiences to acknowledge this. We are left wondering what message the model reader could take away from this. Would the reader think that her *everyday* experiences matter less than her *mathematical* experiences?

This possible suggestion, that everyday experience is less important, is substantiated by the word problem settings in this textbook. Though most problems are situated in everyday contexts, the sequencing of problems jumps from one setting to another. For example, in the first set of 'applications, connections and extensions' (pp. 15f), the student is told to exercise her mathematics on bridge design, bus trip planning, economic forecasting, fuel monitoring, and the list goes on. Similar to our concern for well-placed, inclusive imperatives to promote students' careful thoughtfulness, we suggest that longer series of problems relating to any particular context would guide students to see real life in its complexity, as opposed to operating on the basis of shallow glimpses of small-scoped data sets. Unlike the problem sets, this textbook's 'investigations' do dwell longer on given problem contexts.

REVISIONING MATHEMATICS TEXT

In summary, the analysed textbooks seem to construct a model reader that is independent of personal voices in mathematics. There are no mathematicians but there are people who use mathematics: pure mathematics is impersonal and applied mathematics is more personal. There is an impersonal body of mathematics and there are some children doing some of this mathematics. The model reader is expected to do mathematics independent of the people surrounding her and independent of her classmates and teacher, but to reflect on it within the classroom community. The reader's mathematics is also independent from her environment.

This sense of detachment should come as no surprise because mathematics is characterised by abstraction: Balacheff (1988) has noted the necessity of decontextualisation, depersonalisation and detemporalisation in logical reasoning. We draw attention to his prefix *de* (as opposed to the alternative prefix *a*). Balacheff does not call for apersonal, atemporal, acontextual language. Rather, mathematics requires the *move* from personal to impersonal, and perhaps back. It is the *move* from being situated in a physical and temporal context to finding truth independent of context. Just as mathematics is about the moves from the particular to general and back again, we see mathematisation as the moves between the personal and impersonal, between context and abstraction.

Though our textbooks do not typically recognise the moves we associate with mathematisation, we see room in textbook use for the recognition of mathematisation. In typical classrooms, the textbook is mediated through a person (the teacher) in a conversation amongst many persons (the students). In such a community, even if the textual material at hand does not recognise the role of persons in mathematics, there is room to attend to persons and contexts. There is room to draw awareness to the dance of agency between particular persons (whether they be historical or modern, professional or novice mathematicians) and the apparently abstract, static discipline of mathematics (c.f. Wagner, 2004). For textbook writers, there is much room for change. Considering this possibility, we wonder how students' experiences of mathematics would differ if their textbooks recognised persons, their contexts and mathematisation more.

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