THE EFFECT OF MATHMAGIC ON THE ALGEBRAIC KNOWLEDGE AND SKILLS OF LOW-PERFORMING HIGH SCHOOL STUDENTS

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Algebra is considered one of the most important areas of school mathematics. Despite its importance, students find it difficult to understand simple algebraic concepts such as variables, expressions, and equivalence. Although basic algebraic concepts are introduced at the elementary and middle school levels, some high school students cannot understand algebra because they find it abstract and difficult. Based on the study of an algebra class of 23 high school freshman students, this paper claims that student understanding of basic algebraic knowledge and skills is enhanced when it is taught using mathmagic. It is suggested that the teaching of algebra should provide an opportunity for students to engage in mathmagic activities and connect them to the learning of variables, expressions, and equations.

INTRODUCTION AND THEORETICAL FRAMEWORK

Teachers, mathematics educators, and mathematicians consider algebra to be one of the most important areas of school mathematics. Despite the importance placed on algebra in school mathematics curricula, many students find it abstract and difficult to comprehend (Witzel, Mercer, & Miller, 2003). They cannot understand simple algebraic concepts such as variables, expressions and equivalence. A substantial amount of research has been devoted to the learning and teaching of algebra at the elementary and secondary levels (Carraher, 2001; Kieran, 1992; Malara, 2003). A quick survey of the PME proceedings in the last 10 years provides evidence that algebra has been one of the most widely discussed topics in its annual conferences. A considerable amount of research in PME and other literature is focused on how students can transit from arithmetic to algebra. Some researchers argue that many students at the elementary level face "difficulties in moving from an arithmetic world to an algebraic world" mainly because they cannot understand the structure and patterns of arithmetic (Warren & Cooper, 2003, p. 10). In order to connect number patterns and structures to more abstract concepts in algebra, the National Council of Teachers of Mathematics (NCTM, 2000) recommends that children in elementary and middle grades should be provided an opportunity to describe, extend, and generalize numeric patterns.

A substantial amount of research in learning and teaching of algebra is focused on the effectiveness of manipulatives (Raymond & Leinebach, 2000; Witzel et al., 2003) and computer applications (Glickman & Dixon, 2002; O'Callaghan, 1998) on student

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ability to generalize patterns and solve algebraic relationships. Although these studies have produced mixed results, the majority of them favor the use of manipulatives or computer applications in developing student abilities and interest in solving algebraic problems. Although the research on the effect of manipulatives and computer applications is not hard to find, the research on the effectiveness of number games such as mathmagic in the learning and teaching of algebra is scarce.

Mathmagic is a game in which students are invited to play with numbers in which the students "think of a number", "add 10", "multiply it by 3", and so on (Koirala & Goodwin, 2000). Utilizing basic algebraic knowledge, the mathmagician then figures out the final number that a student is thinking of. Provided below is an example of a mathmagic.

Think of a number.

Add 10.

Multiply by 3.

Subtract 3.

Divide by 3.

Subtract 5.

Subtract your original number.

Map the digit to a letter in the alphabet 1=A, 2=B, 3=C, etc…

Pick a name of a country in Europe that begins with that letter.

Take the second letter in the country name and think of an animal that begins with that letter.

Think of the color of that animal.

The mathmagician then predicts that the students would be thinking of a "Grey Elephant from Denmark." When this magic is completed the students attempt to make a link between working principles of the magic and algebra. Students need to understand the concept of variables in algebra to be able to complete the magic successfully. They need to translate the sentence "think of a number" to a variable, for example n, and then extend it to expressions such as $n+10$ and $3(n+10)$. The directions for mathmagic and the corresponding algebraic expressions are shown in Table 1.

A few studies have reported that this simple magic generates tremendous amount of excitement and interest in students (Lovitt & Clarke, 1988; Koirala & Goodwin, 2000). Despite a strong potential to motivate students towards the learning of algebra, the research on the effect of mathmagic on student achievement and attitude is inadequate. More systematic studies are needed to determine the effectiveness of mathmagic on student learning of algebra. This paper contributes to the existing literature by adding research on the effectiveness of mathmagic on the learning and teaching of algebra. Based on quantitative and qualitative data, it provides insights into students' attitude and achievement towards algebra.

	How a student's number changes	Algebraic expressions
a. Think of a number.	7	n
b. Add 10.	17	n+10
c. Multiply by 3.	51	3n+30
d. Subtract 3.	48	3n+27
e. Divide by 3.	16	n+9
f. Subtract 5.	11	n+4
g. Subtract your original number.	4	4

Table 1: Mathmagic Directions and Corresponding Algebraic Expressions

Purpose and Research Questions

The purpose of this project was to teach algebra using mathmagic and determine its effectiveness in helping students to understand basic algebraic concepts. More specifically, the project sought the answers to the following research questions: Does mathmagic improve algebraic knowledge and skills of low-performing high school students? In what ways does mathmagic help students to develop their confidence and interest in learning algebraic concepts?

To answer the first question, this study hypothesized that the use of mathmagic would improve algebraic knowledge and skills of low-performing high school students. The second question is answered mostly through the analysis of student work samples, researcher's field notes, and the interviews with the teacher and students.

RESEARCH METHODOLOGY

This is a case study (Yin, 1989) of a high school freshman algebra class. It follows the mixed methods research paradigm as proposed by Johnson and Onwuegbuzie (2004). This paradigm calls for the combination of traditional quantitative and qualitative research.

Participants

A total of 23 ninth grade (14 years old) students with varied mathematical and algebraic experiences participated in this project. According to the classroom teacher, seven of these students were identified with various special needs and the majority of remaining students were considered "at-risk." Eight out of these 23 students had an exposure to pre-algebra from a previous class. The remaining 15 students had no exposure to pre-algebra. According to the classroom teacher, the students were not motivated to learn algebra. Most of these students did not see how algebra would make a difference in their everyday lives.

Data Collection Procedure

Consistent with the mixed method paradigm, this study collected data by using both quantitative and qualitative techniques. Quantitative data were collected through pretest and posttest. Qualitative data were collected through student surveys, work samples, researcher's field notes and interviews with the teacher and selected students. This study took place in late September/early October and students in this class had not begun their work in algebra. They were simply reviewing other concepts such as fractions, decimals, and measurements.

Surveys and Pretest/Posttest

The data collection process began with a student survey. In the survey, students were asked if they had learned any algebra prior to the class and if they liked or disliked the subject. The survey provided background information about students' prior knowledge and attitude towards algebra. They were also given a postsurvey at the end of the study to find out their thoughts about the mathmagic activities that were taught during the study.

Immediately after the initial survey, all the participating students were given a pretest to determine their level of algebraic knowledge and skills. The students were also given a posttest at the end of the study to determine whether or not they gained algebraic knowledge and skills after the use of mathmagic in their class. The questions in the pretest and posttest were similar in terms of their structure and difficulty. Some sample pretest/posttest items are provided below.

Simplify: $6-3(2-5)$

What is a variable? Explain with an example.

Write an algebraic expression for each of the following phrases and statements:

17 MORE THAN 3 TIMES A NUMBER.

Subtract 3 from x and divide the difference by 2.

Simplify the following expressions:

$3(x\text{-}5)$

$9+2(5y+11)$

Factor each expression:

$5m+15$

$100c^2+10c$

Interviews

At the end of the project 8 students were selected for interviews. The students were selected purposively to represent the ability level of the whole class. The students were asked to explain what they thought about mathmagic activities in the class and if they could perform a mathmagic with the interviewer. The interviews allowed for the exploration of students' ability to connect mathmagic activities with their algebra learning. Each interview lasted approximately 15-30 minutes. All of these interviews were audiotaped and transcribed. In addition to the student interviews, the teacher was also interviewed two times during the study.

Classroom Planning and Teaching

The teacher and researcher collaborated in planning lessons and teaching. The researcher observed each class when mathmagic was used and made field notes. All the notes and other aspects of the course were shared, discussed, and reflected with the teacher for further planning.

Data Analysis Procedure

This study used a paired *t*-test of scores to determine the statistical significance of the research hypothesis which assumed that the use of mathmagic would increase student achievement in algebra. The data from field notes and interview transcripts were analyzed qualitatively using the constant comparative method (Guba & Lincoln, 1989).

RESULTS

Table 2 displays means and standard deviations of both pretest and posttest scores. The scores indicate that the students performed better in the posttest than in the pretest. The difference between the pretest and the posttest was statistically significant ($t(22)=5.63$, $p<.01$). This result indicates that mathmagic was effective in helping students solve basic algebraic problems.

	n	M	SD
Pre-test	23	4.78	3.21
Pre-test	23	8.33	3.65

Table 2: Means and Standard Deviations of Pretest and Posttest Scores

The researcher's fieldnotes from the classroom indicated that the students were very excited about mathmagic. When the students completed their computations in mathmagic activities they were surprised that they all ended up with the same number. The students then mapped their final numbers with letters in the English alphabet and created different words and phrases for the amazement of the class. The mathmagic activities were highly motivating to students and helpful in making sense of algebraic concepts such as variables, expressions, and the distributive property of multiplication over addition.

The interviews of the teacher and the selected students added further insights into student motivation and learning of algebra. The teacher agreed that mathmagic motivated the students. For example, in one of the interviews the teacher stated:

> I've been pleasantly surprised on how sophisticatedly they were able to think of the algebra skills. To be honest with you, I thought that they would all get very lost and very bored and start to act out more than they were. But many of the students understood how you were getting rid of the initial number they were thinking of. To get rid of that is a fairly complex thinking. Okay, if you multiply your original number by 2 then either you have to subtract off your original number, or somehow you have to make the original number disappear from the equation. I think that is a skill that I would have never guessed that any of those kids was capable of doing.

All of the students who were interviewed agreed that they had fun with mathmagic and six of the eight claimed that mathmagic helped them to learn algebra better. Provided below is a sample interview with one of the students:

Interviewer: In this class you had some opportunity to learn about mathmagic. Can you explain what you thought about mathmagic?

Student: I think it was fun and helpful when learning algebra. It helped me a lot.

Interviewer: Why was it fun?

Student: It is something we are not used to doing everyday. Last year when I was in math class, we didn't do anything like this; so it was new. Plus it was easy for me after seeing it a couple of times on the board. We all got the same number at the end and were wondering how.

Interviewer: Any other reasons?

Student: I like working with other kids a lot. We could play mathmagic with each other.

Interviewer: How did you learn algebra with mathmagic?

Student: Because we had to use a variable and add, subtract, and multiply with numbers and variables.

This student was very excited about mathmagic. Not only was he excited but was also able to successfully perform mathmagic with the researcher during the interview. Other students consistently stated that mathmagic was fun and it should be a part of their algebra class. Only two of the eight students did not believe that mathmagic helped them to learn algebra. It was interesting to note that both of these students had high pretest scores and the their posttest scores did not increase after the study period. Nevertheless, they stated that they liked the activities. Most of the students, including these two, were excited to try mathmagic with their parents, siblings, and friends.

CONCLUSIONS AND IMPLICATIONS

Both the quantitative and qualitative data in this study indicate that mathmagic motivates students to learn basic algebraic concepts. The students who usually struggle with mathematics were enthusiastic to learn mathmagic so that they could try it with their friends. They understood that mathmagic does not work if the arithmetic computations are not correct. They were interested to learn algebraic skills because learning the skills would help them to become a successful mathmagician. Their engagement in mathmagic activities enhanced their understanding of variables and expressions. They were able to add and subtract like terms and use the distributive property. They understood that the use of a variable is central to successful mathmagic. The posttest indicated that they were more successful in manipulating and evaluating algebraic expressions than in the beginning of the study. This study indicates that algebra can be more accessible to students through mathmagic, especially to those who do not perform well in mathematics.

Although mathmagic was successful in bringing positive results in student motivation and achievement there are some limitations with this study. There was no control group so the student achievement could not be compared with another group of students who were taught in a traditional manner. Also, this study was effective as a short-term intervention. The mathmagic activities lasted only 2 weeks, and covered only 5 sessions of approximately 50 minute class period. Although the posttest indicated higher student achievement, the scores were not very high. The mean posttest score was only 8.33 out of possible 15. Many students could not factor the polynomials even after the completion of mathmagic activities.

In order to improve student interest and confidence in algebraic knowledge and skills, mathmagic needs to be directly connected to important topics such as solving equations. Although mathmagic can be used to create and solve equations such as $3(n+10)=17$, the duration of this study was not sufficient to make these connections. Despite these limitations the data in this study implies that mathmagic activities not only motivate but also improve algebraic knowledge and skills of low-performing high school students. As indicated in the literature extensive effort has been made to help students to improve their algebraic knowledge and skills. Yet, many of these efforts have failed to produce desired results. If mathmagic can improve students' attitude and achievement in algebra it needs to be utilized more consistently in their classrooms.

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