# INVESTIGATION ON AN ELEMENTARY TEACHER'S MATHEMATICS PEDAGOGICAL VALUES THROUGH HER APPROACH TO STUDENTS' ERRORS

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*Based on a three-year case study on the subject teacher (Ms. Lin), this article investigates how an elementary teacher's mathematics pedagogical values are manifested in her approach to students' mathematics errors. Ms. Lin's initial mathematics pedagogical value was "learning the knowledge in the textbook." She evaluated students' performance by whether they can solve the problems correctly and students' errors were taken as mere indications of failure and should be ignored and not discussed in class. After some value-cultivating programs, Ms. Lin's teaching belief and behaviors began to change to "learning a method of thinking and debating." She realized that discussing students' errors was important for clarification of concepts and improvement on the ability to reflect. In the end, the article points out that it would be a feasible way for teacher educators to probe into teachers' mathematics pedagogical values by studying how teachers' approach to students' mathematics errors.*

## INTRODUCTION

The teaching of mathematics involves three core elements: mathematics, teachers and students. These three elements are not value-free; in fact, they are value-carriers. In the aspect of mathematics, it changes with time and it carries certain contemporary values. One example is the elementary mathematics curriculum reform in Taiwan in 1996. The Old Elementary Mathematics Curriculum (OEMC) emphasized on acquisition of mathematics knowledge and mastery of calculation skills. The New Elementary Mathematics Curriculum (NEMC), however, not only focuses on " … to guide children to obtain mathematics knowledge from their daily life experiences" but also emphasizes on "… to develop the consciousness to communicate, discuss, rationalize and criticize in mathematics language" and the problem solving skills (ME, 1993, p.91).

The OEMC and NEMC in Taiwan's elementary mathematics curriculum reflects the two education views proposed by Borasi(1996): transmission model and inquiry approach. The transmission model regards mathematical knowledge as "a body of established facts and techniques that are hierarchically organized, context-free, value-free, and thus able to be broken down and passed along by experts to novices." The teaching approach of transmission model is " (a) direct transmission of knowledge

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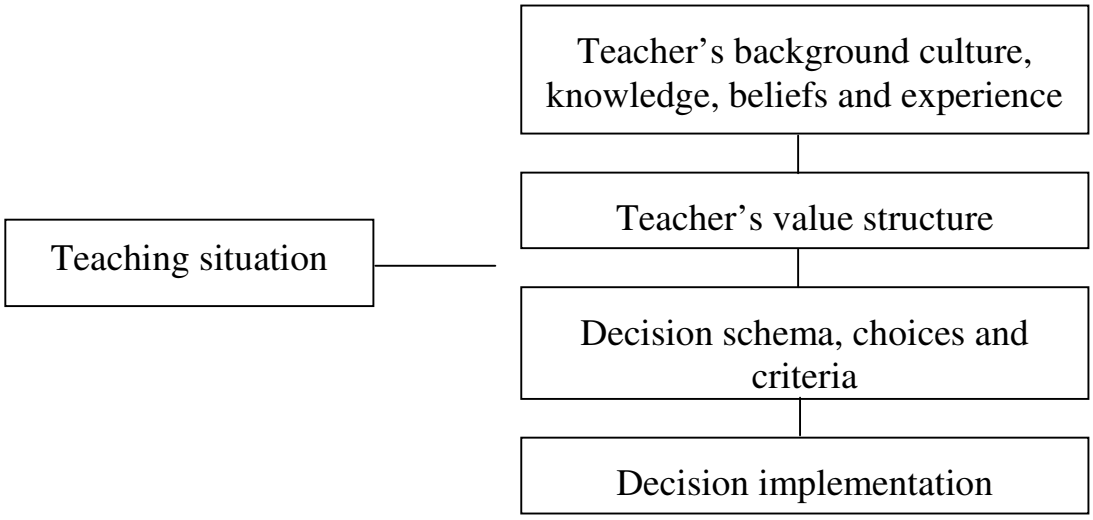
that can be achieved effectively as long as the teacher provides clear explanations and the students pay attention to them and follow them with memorization and practice. " On the contrary, the inquiry approach views mathematics knowledge as "a humanistic discipline" and the construct of knowledge is through "a process of inquiry where uncertainty, conflict and doubt provide the motivation for the continuous search for a more and more refined understanding of the world." The teaching approach of the inquiry approach is "(to) stimulate and support the students' own inquiry and establishing a learning environment conducive to such inquiry."

The present research is intended to assist elementary school teachers on professional development so they can implement the new curriculum successfully into their classrooms under the elementary mathematics curriculum reform in Taiwan. The specific goal of this research is to understand whether the mathematics pedagogical values of elementary teachers have changed under the curriculum reform. Since students usually make various kinds of errors when they are learning mathematics, one manifestation of a teacher's mathematics pedagogical values is his/her approach to students' errors. Under Borasi's transmission model, errors can only provokes teacher and students' negative feelings but errors are seen as "a prototypical example of an anomaly" under the inquiry approach (Borasi, 1996). The present research is to access and to investigate an elementary teacher's mathematic pedagogical values through her approach on students' errors.

## THEORETICAL BACKGROUND

Mathematics curriculum is realized by mathematics instruction, and mathematics teaching carries implicit and explicit values (Bishop, 1988; Bishop, FitzSimons, Seah, & Clarkson, 2001; Chin, & Lin, 2001). Bishop (2001, p.241) proposed a diagram to illustrate how teacher's value structure affects mathematics instruction (i.e. decision implementation).




According to the diagram, Bishop(2001) stated "the teacher's value structure monitors and mediates the on-going teaching situation, constructing options and choices together with criteria for evaluating them. The teacher thus is able to

implement the decisions in a consistent manner." Consequently, mathematics teaching is not value-free and researchers can retrace a teacher's value structure through his/her mathematics instructions (i.e. decision implementation).

The statement made by Bishop (2001) has been supported by research data. For an instance, Ms. Chen, the sample teacher under Leu & Wu's experiment (Leu & Wu, 2002), would hint her students that they made some error(s) when they solved a mathematics problem wrong but she would not point out explicitly their error(s). It's her intention to develop students' practice of self-reflect and self-correct. Ms. Chen's particular teaching behavior can be explained by one of her main mathematics pedagogical values, that is, "the purpose of education is to reinstate students' enlightment."

In this study, the valuing theory (Raths, Harmin, & Simon, 1987) served as the foundation for exploring teachers' value-driven mathematical teaching. Raths, et al. defined values as "any beliefs, attitudes, activities or feelings that satisfy the following three criteria: choosing, prizing and acting." The criterion of choosing includes choosing freely, choosing from alternatives and choosing after thoughtful consideration of the consequences of each alternative. The criterion of prizing includes prizing, cherishing and affirming. The criterion of acting includes acting upon choices and repeating.

## METHODOLOGY

In this case study, data were collected by once-a-week, whole-unit and un-scheduled observations and interviews. Various methods over a variety of schedules and topics were observed and interviewed to prevent unjustified influence of any single method, mathematics topic or instructional event and to make claims across multiple sources and to allow the triangulation of data.

The research subject of this case study is Ms. Lin who has had nine years of teaching experiences in elementary school at the time she joined the research in April 1999. During the three years of research, she taught $5^{th}$ and $6^{th}$ graders. For the first year, she taught the OEMC and the following two years she taught NEMC.

Based on Raths et al's theory, researchers used classroom observations to notice repeated behavioral patterns during her mathematics lessons. The purpose of the interviews is to recognize the reasons why Ms. Lin developed these behavioral patterns and to formulate some value indicators, as well as to examine if the value indicators met the criteria of "choosing" and "prizing". Over the course of three years, the researchers did 37 classroom observations and 36 interviews with the teacher.

During the first year of the research, the goal is to find out Ms. Lin's teaching method and mathematics pedagogical values under OEMC and to start some program to assist Ms. Lin's professional development in mathematics teaching. The design of the value-cultivating program was to select teaching videos where the demonstration

teachers have distinctively different pedagogical values and teaching strategies from Ms. Lin and where group discussion is used as a main classroom activity. After viewing the videos, Ms. Lin was asked to evaluate and comment on the advantages and disadvantages of the presented pedagogical values and strategies.

The research took a teacher's approach to students' errors in real classroom instruction as a probe to the underlying mathematics pedagogical values. It is to investigate whether some behavioral and belief changes has occurred on her approach to students' errors as researchers assisted the subject teacher on her professional development.

## RESEARCH RESULTS

### The initial Meaning of Error

We recorded a whole mathematics unit on circumference during June 1999 and during a total of 4 periods (160 minutes in total), students made 8 errors as they explained their problem solving strategies on the blackboard. However, Ms. Lin regarded these errors as the result of carelessness or forgetfulness. She didn't make use of students' errors as discussion prompts to further explain/clarify the possible underlying concepts behind these errors. In below dialogs, "T" is for Ms. Lin (the teacher), "S" represents individual student.

Example 1:

(The question is a word problem that reads "What is the area of a circle with a diameter of 20 cm?" Ms. Lin assigned S1 to solve the problem on the blackboard but S1 did it wrong. S1's answer was 20 X 2 X 3.14 = 125.6).

T: Let's see. The question states that there is a circle with a diameter is 20 cm and S1 just calculated out the area of that circle. S4 (one random student), what is the formula of the area of a circle?

S4: Radius X Radius X 3.14.

T: Very Good. Please sit down. S1, look at your answer, what were you thinking? Did you use the formula? NO! You didn't. This is the diameter and you multiplied the diameter by 2! What did you think it was? What were you calculating? You thought 20 cm was the radius and you calculated the circumference. Am I right? (Ms. Lin marks a cross on S1's calculation.) The least thing we want (when you are doing mathematics problems) is calculation mistakes. BE CAREFUL.

From the above incidence, Ms. Lin didn't analyze why students got confused between the area formula and circumference formula but she regarded this kind of error as calculation mistake and asked students to correct immediately. When the researchers asked Ms. Lin why she didn't make use of students' error as a discussion prompt, she answered

… it's because when I presented the wrong structure of a Chinese character on the blackboard (to ask students not to make the same mistake) during my Mandarin lessons, students would just pick up the wrong version instead of preventing the error. That is

> why I think that it's better to let students find out their mistakes and correct their errors privately. Otherwise other students will focus on the error(s) a*nd cause some negative effects.*

As a result, various errors were merely mistakes to Ms. Lin. To her, errors didn't reflect different levels of students' conceptual understanding nor did errors function as a source of concept clarification. Under Ms. Lin's value of "learning the knowledge in the textbook"(Leu & Wu, 2002), errors are only indications of failures with no any other functions.

**Loose up the Old Beliefs**

Regarding to the issue of the meaning of errors in problem solving, Ms. Lin's beliefs loosened prior to her behavioral changes. At the beginning of the research, Ms. Lin had some doubts about using paper-pencil tests as the sole indication of "learning the knowledge." She stated that

> I was impressed by one of the questions you asked, 'Do you think the result of paper-pencil tests can really represent students' understanding?' In the past, I would say if students could do it right on a test, everything would be OK. …But now I wonder whether student really understand or they are just a copy machine that reproduced what the teacher has taught or corrected from repeated practice and rote memorization.

Based on this doubt, Ms. Lin started to discuss some wrong problem solving strategies in class. The following is an example excerpted from a workbook problem after Ms. Lin had finished teaching the unit on fraction.

> Example 2
>
> (The question is a picture with a basket of 8 eggs in total and 5 of them are hatched into chicks. When students were doing this problem, some got confused with the unit.)
>
> T: The question asks, "A hen has a basket of eggs. How many chicks are hatched?" Some students' answer was 5/8 chicks. Let me ask you a question. When you were born, was there a fraction of you that were born and the rest of you was left in you mother's tummy? When you were writing this answer, have you not noticed that your answer is wrong?
>
> Ss: No!
>
> T: What is the meaning of 5/8 chicks? A chick is divided into 8 pieces. … You can tell it's wrong by the first sight. It must be wrong. The topic of this unit is fraction so that your answers are all in fraction. S8, please answer me, how many eggs are hatched into chicks in this basket?
>
> S8: 5 chicks.
>
> T: … Good. This is a basket of egg, right? So what can be the unit of our answer? A basket. The unit is in basket. All right. Now there are 8 eggs in the basket and 5 of them are hatched, right? How many baskets of chicks are hatched?
>
> Ss: 5/8.
>
> *(2000.11.17)*

In Example 2, Ms. Lin would indicate which problem solving strategy was wrong and lead students to identify the errors and come up with the correct problem solving strategy step by step. However, from the excerpted dialogue, Ms. Lin and the students were speaking by turns and she was actually saying much more words than the students. At this stage, Ms. Lin started to discuss students' errors but she did not really know how to lead a classroom discussion. Moreover, Ms. Lin was not confident and assertive about the meaning of discussion on errors. She stated that

> Frankly speaking, I am still in the experiment stage as I dare to present errors to my students during this semester. I am not sure whether the result will come out definitely positive, but it is OK so far. Kids have their ways through.

Even Ms. Lin had been still experimenting and waiting for some positive results from conducting discussions on students' errors, from her words, it's clear that Ms. Lin discovered that discussion on students' errors would not cause more students make the same error and the stereotype and negative image of errors she had established from correcting students' Chinese characters has dissolved.

## FIRST CHANGES IN BELIEF THEN CHANGES IN BEHAVIORS

From the interviews about the meaning of errors with Ms. Lin in May 2001, the researchers found that Ms. Lin had more concrete ideas and positive attitudes about discussing students' errors. She stated that,

> I would point out errors on purpose and help students better understand the concept through group interaction. This way of teaching is very different what I used to. I want to train my students to have the ability to digest different opinions. They should be able to compare the differences and transfer the differences into a more justified and refined thinking process. This ability not only can be applied to mathematics, but also covers a wide range, such as correcting his/her own behaviors and communication skills.

Ms. Lin not only upheld the positive meaning of conducting discussions on students' errors, she also started to provide discussion opportunities for students in class, as demonstrated in the following example.

Example 3

(It is a question from workbook and it is to draw out the right half of a symmetric figure in Figure 1. The solution provided by S6 is to draw out a right triangle △AMB and measure the lengths of line $\overline{AM}$ and $\overline{MB}$. Then through the reduplication of △AMD to form △AND (△AND ≅ △AMB), point D is found and lines $\overline{AD}$ and $\overline{DC}$ are connected (Figure 2)).

T: Any questions on this figure?

S8: I have a question to ask S6. (S8 stepping forward to point at the blackboard). He said that he drew the line here (Line $\overline{AD}$) and made a symmetric figure. I did the same thing. I drew a rectangular outside but when I drew out the figure it became like this (**i.e.** $\overline{AE}$). It is not symmetrical. What should I do?

S6: This is simple. You just need to find out how long line $\overline{MB}$ is and go here (line $\overline{ND}$) and make a mark (Point D). Then you just connect here to here (Point A to Point D to make Line $\overline{AD}$).

(2001.10.22)

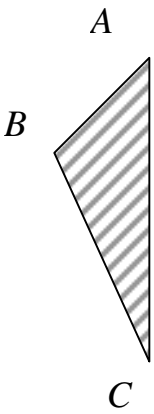


Figure 1

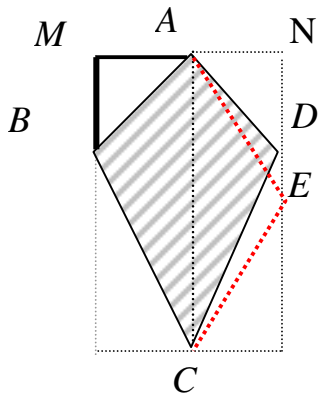


Figure 2

Ms. Lin's thoughts and teaching behaviors reflect her mathematics pedagogical values. It shifts from "learning the knowledge in the textbook" toward "learning a method of thinking and debating". Nevertheless, when facing the pressure such as limited class time constraint and/or students' high score attainment on tests, the practice of conducting a discussion on students' errors would likely to be forfeited. For an instance, when Ms. Lin was interviewed on November 7th, 2001 and was shown the videotape of her class on October 22, she said that she found students could not tell the key points she expected in the free discussion on first problem. That's why she had to lead her students step-by-step in the following discussion on other questions. Ms. Lin admitted that waiting for the expected response was a great challenge for her. She had to keep reminding herself not to interfere and to withhold the urge to tell the answer directly. The confession from Ms. Lin indicated that even though she realized the importance of presenting and discussing students' errors in class, Ms. Lin's initial value of "learning the knowledge in the text book" was still influencing her teaching behaviors, seesawing with the value of "learning a method of critical thinking and debating."

## IMPLICATION

Through Ms. Lin's case study, this paper presents an alternative source to collect a teacher's teaching beliefs and to understand his/her teaching behaviors through his/her approach to students' errors. It can also be used as a probe to investigate a teacher's mathematics pedagogical values. A teacher's approach to students' errors is related to how he/she defines mathematics, knowledge, children's learning and proper teaching strategies. A teacher can find students' errors in doing mathematics problems everywhere, from his/her teaching processes to students' homework. When students' errors are the discussion topic, the interviewed teachers are usually more willing to express openly about their opinions and this allows researchers to perceive teacher's deep values more easily. Besides, during the course of teacher education

programs, the relationship between the meaning of errors and mathematics pedagogical values can be discussed among pre- and/or in-service elementary school teachers and it provides a topic of professional development.

Furthermore, this research presents a fact: it is not easy to change a teacher's mathematics pedagogical values. With the researchers' assistance and value-cultivating programs, Ms. Lin changed her beliefs in two years, under her free will and active choice of teaching beliefs and strategies. Even though Ms. Lin's teaching behaviors did change but they are feeble; they are still influenced by extrinsic factors such as time constraint and/or pressure of students' high score attainment. The question of how to shorten the adjustment time required for changes in mathematics pedagogical values can be the next research topic. In addition, it is found that teaching belief and behaviors do not change concurrently during a teacher's professional development. Changes in teaching behaviors need to be supported by relevant teaching knowledge and this present research finds that belief changes precede knowledge changes. Further researcher is needed on the area of a teacher's professional developmental process and patterns.

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