# FRACTIONS IN THE WORKPLACE: FOLDING BACK AND THE GROWTH OF MATHEMATICAL UNDERSTANDING

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*This paper presents some initial findings from a multi-year project that is exploring the growth of mathematical understanding in a variety of construction trades training programs. In this paper, we focus on John, an entry-level plumbing trainee. We explore his understandings for fractions and units of imperial measure as he attempts to solve a pipefitting problem. We contend that it cannot be assumed that the images held by adult apprentices for basic mathematical concepts are flexible or deep and that folding back to modify or make new images as needed in particular contexts is an essential element in facilitating the growth of mathematical understanding in workplace training.*

## MATHEMATICAL UNDERSTANDING AND THE WORKPLACE

Although recent years have seen an increase in the attention paid by researchers to mathematics in the workplace there is still only a limited body of work that considers cognition and understanding in a vocational setting. Research that does exist in the area of adult mathematical thinking within a vocational setting is primarily concerned with the use of informal mathematics (e.g. Noss, Hoyles & Pozzi, 2000) or the place of mathematics within the workplace (e.g. Wedege 2000a,b), rather than with the process of coming to understand mathematical concepts for the individual learner in the workplace. In the larger ongoing study we are looking at the growth of mathematical understanding and numeracy in workplace training, and more specifically at some of the difficulties faced by apprentices as they engage with mathematics in workplace tasks. In this paper we focus on John and explore his understandings for fractional units of length.

## MATHEMATICAL UNDERSTANDING AND THE PLACE OF IMAGES

The research reported in this paper is framed by the Pirie-Kieren Theory for the Dynamical Growth of Mathematical Understanding (Pirie & Kieren, 1994). This theory provides a way to look at, describe and account for developing mathematical understanding as it is observed to occur in action. The Pirie-Kieren theory posits eight layers of understanding together with the cognitive activity of ‘folding back’ as crucial to the growth of understanding. Two of the inner layers are defined as *Image Making* and *Image Having*, and it is these layers that are relevant to our discussion in this paper.

At Image Making learners are engaging in specific activities aimed at helping them to develop particular ideas and images for a concept. By “images” the theory means any

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ideas the learner may have about the topic, any "mental" representations, not just visual or pictorial ones. Image Making often involves the drawing of diagrams, working through specific examples or playing with numbers. However, it does not have to have an observable physical manifestation, it is the thinking and acting around the concept that is the actual process of making an image.

By the Image Having stage the learners are no longer tied to actual activities, they are now able to carry with them a general *mental* plan for these specific activities and use it accordingly. This frees the mathematical activity of the learner from the need for particular actions or examples. At this layer, the learner *has* an understanding, although this may still be very specific, mathematically limiting and context dependent, which they are able to employ when working on mathematical tasks.

## FOLDING BACK AND FRACTIONS

Apprentices in workplace training are often re-learning mathematics that they have already met and have existing images and understandings for. As they engage in mathematical activity during training they may need to re-visit these existing understandings and images, make sense of them again in specific trades situated applications, and if necessary construct new understandings. Within the Pirie-Kieren Theory this process is known as 'folding back' implying that when a learner revisits earlier images and understandings he or she carries with them the demands of the new situation and uses these to inform their new thinking at the inner layer. Different images may be required for the same concept when used in the workplace rather than the school classroom (See Forman and Steen, 2000), or a previously held image may need to be modified or broadened. As Martin, Pirie & Kieren (1994) note "fraction learning involves constructing an ever more elaborate, complex, broad and sophisticated fraction world and developing the capacity to function in more complex and sophisticated ways within it. Such an achievement will prove impossible if the foundations laid by the images the learners hold are not adequate to the task" (p.248).

In the primary school fractions are often taught with reference to parts of a whole circle (usually described as being a pizza or pie) and the image here is quite specific and one based on an area representation. However, when using fractions in the context of measurement, it is more appropriate to see a fraction as a point on the number line – although of course to be able to read a fractional unit of measurement still requires an understanding of the part-whole relationship. This number-line image is particularly important for working with measurements in imperial units, where lengths are stated in fractional units, unlike in metric units where decimals are more commonly utilised. (For example, one would rarely talk about three and seven tenth centimetres, though it is of course mathematically valid and correct). Certainly in the workplace it is the measurement model (or image) for fractions that is likely to prove the more powerful and the one that can allow the development of the kinds of essential skills necessary to function effectively on the job. However, as we shall illustrate through the case of John, it cannot be assumed that apprentices necessarily have and can use this image.

## METHODS AND DATA SOURCES

The larger study, currently underway, is made up of a series of case studies of apprentices training towards qualification in various construction trades in British Columbia, Canada. This paper draws on one of these case studies, and although our conclusions are specific to this case, we would suggest that there are implications that may be relevant to other mathematical concepts and areas of workplace training.

The trainees and their instructor were observed and video-recorded over a number of sessions, as they worked in a classroom and also in a workshop setting. The second author acted as a participant observer in the sessions. The episode on which this paper focuses involved a small group of apprentices in the shop working to calculate the length of a pipe component required for a threaded pipe and fitting assembly to be built to given specifications.

## JOHN AND THE PIPEFITTING TASK

The following extracts of transcript occured a few minutes into the session. John is trying to measure out the length of pipe that he needs, using an imperial units tape measure, prior to cutting it. He is talking to Steve, another apprentice in the group. Although he has the correct length of pipe, he thinks that he is incorrect when measuring it. This is because when needing to count in eighths on the tape measure, he actually counts in sixteenths (also marked on the tape), thus resulting in him reading an incorrect measurement from the tape. The researcher points this out to him:

| | |
|---|---|
| Researcher: | Yeah. Now wait a sec. There's a problem. Make sure you're not counting sixteenths. |
| John: | OK. |
| Steve: | Those are sixteenths. |
| R: | OK. This from here to here is a half, right? *(pointing to the interval on the tape measure bounded by 8" and 8 1/2" )* |
| J: | Right. |
| R: | From here to here is? *(pointing to the interval on the tape measure bounded by 8" and 8 1/4" )* |
| J: | A quarter. |
| R: | A quarter, yeah. From here to here is? *(pointing to the interval on the tape measure bounded by 8" and 8 1/8" )* |
| J: | *(pause)* That's, the little, the little *(pointing to the mark on the tape measure indicating 8 1/8" )*, that's one eighth. |

| | |
|---|---|
| R: | Yeah, like if this is a quarter, an eighth is? |
| J: | I know that's a sixteenth *(pointing to a small vertical line on the tape measure indicating a sixteenth between eighths.)* |
| R: | Half of an eighth. Or, sorry, an eighth is half of a quarter. That's a quarter, then an eighth is from here *(pointing to tape measure)* to here. |
| S: | So every second line is one sixteenth. |
| J: | Right. |
| R: | Every second line. And every single line here is a? |
| J: | Eighth. |
| S: | Thirty second. *(Spoken simultaneously with John)* |
| R: | Every single line? |
| J: | Every line is one is an eighth, is it not? The small line? |
| S: | Every line is a thirty second. |

Here we see John and Steve giving different answers to the questions, using their existing images for fractions. However, while both correctly use the language of fractions in this context, it is not clear at any point that John is actually thinking in terms of equal parts of a whole inch, and thus that to measure involves a comparison with the various fractional units that are superimposed upon one another on the tape. Instead, John seems to have and be using an image based on the fact that measuring tapes use different length lines to represent the different fractional divisions of each inch (i.e. very short vertical lines for thirty-seconds, becoming progressively longer towards the half inch), which does not encapsulate an understanding based on a part-whole fractional relationship. John's confusion is apparent at the end of the extract when he incorrectly states that every single line is equivalent to one eighth, whereas Steve seems more comfortable with the fractional scale and gives the correct answer of one thirty-second. At no time does John talk about an inch being divided into a given number of equal sized parts, which is an image that could help him to more easily work with the complex measuring tape. John continues his explanation to the researcher:

| | |
|---|---|
| John: | Each line here *(pointing to the fine makings on the tape measure)*, see how …. differences between the small and the little taller ones, right? I'm trying to make it as, here we go… *(Grabs a piece of paper and holds it over the tape measure so that only the end points of the marking lines are visible above the paper.)* Here we go, ok. See how every line is different here, right? |
| Researcher: | Mmhu. |
| J: | 'Cause one line that's smaller than the others. So… |
| R: | Yeah. |
| J: | That's one sixteenth, a big one is eighth, correct? |

It is not clear if John recognises the part-whole relationship of fractions, or is simply still making an image based on the length of lines on the measuring tape scale. Sensing this the researcher introduces a set of ruler scales printed on acetate which can be stacked to illustrate how fractions are represented on a measuring tape. Each acetate rule layer has the inches divided up into a different fractional unit, so the first rule has only inches on it, the next shows half inches, the next has quarter inches and so on. This provides a visual image for how a standard tape measure actually incorporates a number of different fractional units superimposed onto one scale. The dialogue continues:

Researcher: So, lets look at your ruler (*the tape measure*) up against this (*the stack of acetates-teaching tool)*. So I'm going to line up the eighth, lay it down flat.

John: OK.

R: Look at where your eighth.

J: So that's one, two, three, (*counting off lines at 8 1/8, 8 3/8 and 8 5/8 on the acetate ruler*) correct? Am I counting that correct?

The set of acetate rulers are used here to offer John a valuable image making tool, allowing him to physically manipulate a set of representations of the different fractional units and to see how these relate, both to each other and to the standard measuring tape. Using the resource also exposes the problem that John is having in working with these fractional units. When he comes to count three-eighths, he points to one-eighth correctly, but then continues to point to the lines of the same length as this, i.e., three-eighths and five-eighths. He does not count two-eighths or four-eighths, as these are equivalent to one quarter and to one half, and thus on the ruler are represented by lines of different lengths.

We see clearly that John did not have an image for a fraction as a location on his measuring tape i.e. as a point on a number line, and does not see the relationship of the numerator and denominator of a written fraction to the part-whole of an actual inch. The researcher responds:

Researcher: No, you're doing spaces. Count spaces.

John: Ok. So that's one.

R: One, Yeah.

J: Two. *(now counting eighth inch intervals on the acetate eighths ruler)*

R: Two, and at the end of that space is your…

J: Is this right here? *(marks 8 3/8 point on his tape measure.)*

R: Right there.

J: Ok. Ok. So, ··· *(Re enacts the stepping process with his pencil tip from 8 inches this time on his tape measure.)* Ok. Didn't see it, here we go. I got yah. *(He explains it back to the researcher.)* Ok, so, this is our one, our

| | |
|---|---|
| | one sixteenth right here, correct? *(pointing to 8 1/8 point on tape measure),* no our one eighth, am I out? |
| R: | Which is it? |
| J: | This one right here, the big one *(pointing to tape measure).* |
| R: | Which is it, eighth or sixteenth? |
| J: | That's eighth. |
| R: | Ok. |
| J: | Ok. This one here is sixteenth *(pointing to 8 1/16 point on tape measure).* |
| R: | Yeah. |
| J: | This one here is thirty two. |
| R: | Thirty-second, yeah. |
| J: | Thirty-second. Am I on the ball with that? |
| R: | Absolutely. |
| J: | Ok. |

In this extract the researcher engages John in a directed image making activity, he tells him to count spaces, and John carries out this physical action. In doing this, we suggest that John has folded back to do something physical that will allow him to modify his inappropriate image for the fractional scale. He returns to Image Making, and is trying to make sense of how the ruler is to be correctly read, and why. After counting with his pen on the ruler and reaching eight inches and three-eighths, he re-enacts this stepping process, suggesting that he is now actively making an appropriate image, and is able to count-on in fractions whilst making the correct one-to-one correspondence with the ruler scale. The physical act of working with a manipulative allows him to understand what he is doing, and towards the end of the extract it seems that he now has an image for a fraction as a point on a continuous number line, as printed on the ruler. He is able to confidently identify the other fractional units, though we do not see him count on in these. Of course we would not suggest that the image John has is now a complete one for measuring using imperial units, and recognise that he may need to fold back again and again to continue to develop his understanding. The researcher probes a little more:

| | |
|---|---|
| Researcher: | Because, that's how many of those little spaces would fit in a whole inch. |
| John: | Right. So if we went one. |
| R: | What are we counting now? What kind of fraction? |
| J: | Eight, eight. Right? |
| R: | Ok. Eighths. |
| J: | So that's one eighth from here *(8 inch point)* to here *(8 1/8 inch point on tape measure).* That's two eighths *(pointing to the interval between 8 1/8* |

*inch to 8 1/4 inch)* and that's three eighths *(pointing to the interval between 81/4 inch to 8 3/8 inch)*. Gotyah. Ok. I understand now. Now that I can actually express it and point it out, ok.

Again in this final extract we see John confidently counting on in eighths, and recognising that he has an understanding of what he is doing. It is not totally clear here whether he fully grasps the idea of "how many of those little spaces would fit in a whole inch" that is articulated by the researcher, but would hope that the future exploration of different fractional units will help him to develop his image to be one that he can confidently use and apply whatever he is asked to measure. We suggest that he has a useful and appropriate image for imperial units of measure, based on a deeper understanding of the part-whole relationship of fractions.

## CONCLUSIONS

It is beyond the scope of this paper to comment in any depth on the complex role that mathematical understandings and images play in the trades training process, but we suggest that trades educators should expect that their trainees may not come with a useful and easily applied range of images for required mathematical concepts needed in their training. It would seem that offering opportunities for apprentices to fold back and to engage in appropriate image making activities for some mathematical concepts would be an appropriate way to occasion their growth of understanding, and enable the development of more widely applicable skills. This is particularly important for those apprentices who perhaps struggled with school mathematics and come to trades training with a very limited set of mathematical images, and also possibly with a negative attitude to mathematics.

Whilst we recognise that in some ways, returning to "play around" with physical manipulatives might seem to be both a backward step and time consuming, we do believe that there is a need to re-engage with some basic mathematical concepts, but within the new context of the workplace. As Forman and Steen (1995) noted, there is a need in the workplace for "concrete mathematics, built on advanced applications of elementary mathematics rather than on elementary applications of advanced mathematics (p.228). Certainly for John, he was being asked to use relatively elementary mathematical concepts but to use these in problem solving contexts that are very different from those in which the concepts will have been taught or used in school. Of course, we also recognise that not all apprentices may need this re-engagement, but given the wide range of needs that exist in trades training, the provision of an opportunity for folding back when necessary is essential.

We see John folding back to image making in the later extracts, and through working with the set of acetate rules and having an opportunity to play around with these (and the mathematics embedded within them) he does seem to have an appropriate image for fractional units at the end of the session. This kind of activity and accompanying learning tools are invaluable for encouraging mathematical understanding that goes beyond being able to merely read a scale or operate on numbers.

Clearly, the measuring tape is a fundamental part of working in pipe trades, and the ability to use this, and to understand the mathematics that is captured by this tool is essential for a worker. Whilst we acknowledge that such understandings are not likely to be made explicit during every task, the possession of a powerful and flexible set of mathematical images related to this offers something to fold back to, should memory fail, or the need arise to work in a new application. We suggest that in the technical training classroom there is a need to re-visit concepts such as fractions and to go beyond learning merely how to operate on and with these.

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