UNIVERSITY STUDENT PERCEPTIONS OF CAS USE IN MATHEMATICS LEARNING

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While research has shown many potential benefits of computer algebra system (CAS) calculators in the learning of mathematics, it has also identified a number of obstacles to attaining them. Most research has been conducted with secondary school students, but this study considered the manner in which beginning university students perceive the benefits and difficulties associated with CAS use. The results describe a number of obstacles to such use and the ways students employed them in their study.

BACKGROUND

One of the technologies that has been of special interest over the past decade is the implementation of CAS on calculators. There has been enthusiasm in many quarters for the possibilities that this technology presents, but some research has shown that the potential is not always realised. For example, Hong, Thomas, and Kiernan (2000) have shown there is a problem when students come to rely on CAS since it can then undermine their learning, preventing them from learning important concepts and procedures. Another important area, and one addressed in this study, is the negative reaction of certain students to the use of CAS, as recorded by Bergsten (1996). While for some this attitude stems from their background in mathematics (where, as Artigue (2002) explains, by-hand techniques may have an elevated status), for others it is learned. It may originate in the challenge of the mathematical interface difficulties (particularly in terms of input and output formats), and the 'black box' syndrome, where students are unable to attach meaning to processes carried out within the CAS. While the release of the CAS potentiality depends on both instrumentalisation and instrumentation of the CAS tool (Rabardel & Samurcay, 2001), or how one adapts to the tool and how one adapts the tool to one's mathematical needs, how these processes should proceed is still unclear, as neither of the steps is inherently simple.

A common idea is that because mathematically experienced users can see wide-ranging benefits to CAS use in learning that students will also gain from these. For example Thomas, Monaghan and Pierce (2004) record perceived benefits including the multi-representational nature of CAS, the idea that CAS can reduce user time on procedures freeing them to concentrate more on learning concepts, a propensity for experimentation and generalisation, and a strong focus on algebraic insight, functions and parameters. However, there are obstacles in the path of students engaging in instrumentalisation and instrumentation of CAS in order to get to the point where they can access the benefits of the instrument. Some of the obstacles to use are associated with the tool itself, such as the constraints of the input and the difficulty in translating output to mathematical notation (Drijvers, 2002) and the influence of the social environment (teacher and peers) on student choices.

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Describing various categories of student CAS use they found, Thomas and Hong (2004) mention direct, straightforward procedures, direct complex procedures, checking procedural by-hand work, procedures within a complex process, and investigating conceptual ideas. However, they report finding little of the last kind of activity, and a lot of the first three, supporting the idea that instrumentalisation and instrumentation of CAS, where students incorporate CAS into their mathematical thinking, can be a slow process. It takes time and teacher direction for students to learn to decide what CAS is useful for, and what might be better done by hand, and how to integrate the two. Thomas and Hong (*ibid*) maintain that teachers using CAS in the classroom have to be both aware of the possibilities provided by the technology, and be confident in the roles they decide to implement. They should engage students in a discussion of the meaning of the CAS techniques and the conceptions that are being developed (Drijvers, 2002). While most CAS research has concentrated on the secondary school classroom, the nature of the didactic contract (Brousseau, 1997) in the lecture theatre and tutorial room is often quite different from the school situation, and hence instrumentalisation and instrumentation of CAS may proceed differently. In this study we were interested in university students' perceptions of the factors influencing their use or non-use of CAS in a first year mathematics course, the CAS instrumentation, and the attitudes towards it.

METHOD

This research took place in June 2004 and comprised an initial case study of first year Maths 108 mathematics and science students from The University of Auckland. Maths 108 covers both calculus and elementary linear algebra and TI-89 CAS calculators were recently been introduced as an optional component of the course. For the first time in 2004, the department included detailed instructions about use of CAS TI-89 calculators as part of the Maths 108 coursebook. At the end of the last tutorial of the course each student was given a questionnaire, and of the 1014 students, mostly 19-22 years old, enrolled in the paper 167 students (16.5%), from among those attending tutorials, agreed to take part in the study. Of these, 24 (5 females, 18 males, 1 unknown) had been using the TI-89 during the course, while the others had not. This mirrored the total CAS take-up for the course of around 15% of the students. None of the students had ever used a CAS calculator before this course. The questionnaire (see Figure 1—format changed) was divided into two sections (A & B) for those using and those not using the CAS. It comprised both 3-point Likert scale and open questions and addressed student attitudes toward the use of the CAS calculator, the value of the CAS, when they decided to use it and how, and their possible reliance upon it. For those without a calculator we wanted to know why they had decided not to purchase one, and their perspective on its potential value in learning mathematics. Since the input from teachers is a crucial variable in attitudes to technology use we also asked the lecturers from the six streams of the course for observations on the student use of the CAS.

Section A: For each statement below please circle the number which most closely corresponds to your own view. **(For TI-89 users only)**

1. The TI-89 CAS calculators do not improve my understanding of mathematics.
2. I waste a lot of time trying to get the TI-89 CAS calculator going.
3. I am glad that I can use the TI-89 CAS calculator during the exam.
4. TI-89 CAS calculators help me to visualise the problems.
5. I can solve problems using TI-89 CAS calculators even though I don't understand the theory.
6. My answers are usually different from the answers that the TI-89 CAS calculator gives me.
7. I think the TI-89 manual at the back of my book is very helpful.
8. I often check my answers using the TI-89 CAS calculator
9. I would like to learn more about the TI-89 CAS calculators, so I can use them fully.
10. I believe technology is the way to go to learn mathematics.
11. I hope to use my TI-89 CAS calculator in other courses when applicable
12. My lecturers are very supportive and encouraging in using the TI-89 CAS calculators.
13. I explore the TI-89 by myself.
14. I find it difficult to decide when to use the TI-89 in maths problems.
15. Since I have been using the TI-89 CAS calculator, I have forgotten how to do the basic skills.
16. I like to use both TI-89 CAS calculator and pen and paper when working on maths problems.
17. I only use TI-89 CAS calculator when I am stuck using pen and paper for mathematics problems.
18. I bought a calculator at the beginning of the year but never used it, so I think I wasted my money.
19. I find all the TI-89 menus and key presses too difficult to remember.
20. TI-89 CAS calculators make mathematics fun.
21. There is not enough support outside lecture time for using the TI-89 calculator.
22. I believe the TI-89 gives me an unfair advantage in learning mathematics.

Open Questions (For TI-89 users only)

- What do you like using the TI-89 calculator for? (Why?)
- How do you feel about using the TI-89 calculators this year?
- Should the TI-89 calculators be used in the mathematics lectures? If so, how?
- How do you decide when to use the TI-89 calculator?
- Has the TI-89 calculator helped you learn any mathematics? If so, what?
- How much do you feel you rely on your TI-89 calculator? For example, could you still do the problems without having one?
- Do you just try to apply the applications of the TI-89 calculators in the course manual or do you explore for yourself?
- Did you buy the TI-89 calculator at the beginning of the year but never used it? (If yes, why?)
- Why did you buy a TI-89 calculator?
- When did you find out there were notes at the back of your course book (notes on the CD for 150 students) on TI-89 calculators?

Section B: For each statement below please circle the number which most closely corresponds to your own view. **(For those who don't use the TI-89).**

1. I can do everything without a TI-89. In other words I don't need one.
2. The TI-89 CAS calculators are far too expensive.
3. The lecturers are not using them so why should I bother.
4. I would have liked to have one if they were more affordable.
5. I really don't know if they are good or not until I have tried one.
6. I believe students shouldn't be allowed to use them in test and examinations, because it is not fair on those of us who can't afford one.
7. If someone showed me how useful they can be I might consider buying one.
8. I wish I had a TI-89 that I could use.

Please write below any other comments you would like to make.

Figure 1: The Questionnaire

RESULTS

The student results that follow need to be set in the context of the lecturers' attitude to the introduction of the CAS. Lecturing style and approach are not yet prescribed in the department, and in 2004 none of the lecturers used the CAS, probably due to their inexperience with them. This is a crucial point, since as Kendal and Stacey (1999) found, teachers' *privileging* of approaches can differentially affect student learning. It was clear that the Maths 108 lecturers were strongly privileging by-hand work, and so students could be expected to favour this approach. In addition, the majority of tutors on the course were also not familiar with the CAS calculators. However, the first named researcher was a tutor for Maths 108 and was available 4 hours a week in an assistance room to help with the CAS calculators. From the 6 lecturers involved in teaching this course, only 2 (lecturers A and B) replied to our questionnaire. Both said they had encouraged students to consider the calculator, using "written recommendations in study guide, announcements on Cecil [the course management system]" (A) or "No sales pitches here, but reference to manual and encouragement to learn using the beast" (B). Here lecturer B's language clearly shows a stance not entirely in favour of the CAS, although he had encouraged students to look at specific topics and pages in the manual, with announcments such as "learn to use your graphics calculator to find the inverse of a matrix (cf. pp. 284-285)". Both of them claimed, not surprisingly, to have noticed few, if any, examples of students using the TI-89, either in lectures or in small group tutorials. Hence they were not aware of any positive aspects or problems with CAS use. Lecturer A noticed that his students were affected by the price factor (described in more detail below), writing that "Several students have other types of graphics calculator (either from high school or bought elsewhere at half the price). At the start of the semester I promoted the private TI-89 tutorials vigorously, but no student enrolled. I am sure cost is a factor in all this." Put in terms of the didactic contract, Brousseau (1997) explains that, while there are reciprocal obligations, the teacher is the prime mover in the development of the contract. Since the lecturers demonstrated little expectation of CAS use, and some may even have been opposed to its use, this would become part of their contract.

Obstacles to use

A second strong factor to be considered in the development of CAS use is the attitude of the majority of those taking the course, since this forms the social environment in which the use of CAS is situated. The most common reason by far for those not using CAS was the cost. The mean response to the statement "The TI-89 CAS calculators are far too expensive." was a score of 2.78 out of 3. Of the 143 students not using the CAS, 72 wrote something in the open response section, and 22 of these mentioned the high cost as a significant barrier to use. Typical comments included "the price is too high and I can't afford it", "Far too expensive!!!", and "it is very hard for your regular student to shell out hundreds of dollars for a calculator". It appears that a significant number would have liked one had they been cheaper, with a mean response of 2.53 to the statement "I would have liked one if they were more affordable" and 2.21 for "I wish I had a TI-89 that I could use".

Other factors preventing non-users from using the CAS included the view that they are of limited value, especially in terms of helping with understanding of the mathematics, although this may have emanated from the lecturers. The questionnaire statement "I can do everything without a TI-89. In other words I don't need one." elicited a mean agreement score of 2.28, and 12 wrote similar reasons stating that:

- If I can pass the course without one then I don't see the point.
- It is certainly not necessary. Therefore I don't need one.
- I don't think the calculator is very useful because we're familiar with exams without any calculator and we can do nearly all questions without using this.
- If you can use your brain to think and solve the problem, why you have to use calculator.

Another factor was the perceived limited 'shelf life' of the calculator. Students did not want to invest in something that they thought would not be used in future courses: "If I can use TI-89 in all the maths course (not only 108 and 150) exam. I may consider to buy one.", "Not useful after course's finished.", and "It is far too expensive and not worth to be purchased if we only use it for one semester." Interestingly, in contrast to the view that CAS was not useful, there was even stronger agreement with the statement that "I believe students shouldn't be allowed to use them in test and examinations, because it is not fair on those of us who can't afford one." (Mean 2.41). This seems to indicate a clear perception that the CAS was useful, and hence unfair. This was confirmed by 13 of the open responses, including:

- Students should not be encouraged to use TI-89 calculators in exams and tests…It is unfair for those who do not have calculators.
- I really believe that students shouldn't be allowed to use them in test and examinations. It is unfair for us who do not got one.
- I think that these TI-89s should be forbidden to use in tests and exams because with this calculator you basically don't need to know how to do differentiation, integrations, matrices etc. and still do well. Which means the other students will have quite an advantage.

Although, as one person noted, there was also the factor of a perceived time gain: "People without it are at a slight disadvantage in terms of time spent in the exam calculating manually. We'd have less time to do the rest of the test/exam. Few extra mins could be the difference between passing/failing for some people", and another mentioned the ability to check answers "It is kind of unfair, with the advantage of others to be able to check their answers during exams."

A fourth reason, given by 8 of the students, was that they saw the CAS as too complicated to use (maybe they had looked at those belonging to friends or had read the coursebook), and hence too much effort was required to learn its use.

- It's very complicated to use and takes quite a lot of time.
- It takes a long time to learn how to use TI-89 calculators.
- Not easy to use sometimes may make me confused!

- I've got one, but don't know how to use it. Too much time and effort to learn how.

In the light of the lack of compulsion, or privileging of the CAS by the lecturers (described above), it was interesting to note that 5 students picked up on this and interpreted it as meaning that the CAS was not really necessary for the course.

- It is not widely used, and not required, therefore I see no need in getting it.
- If they are good for the course then they should be made compulsory.
- If the lecturers force us to use, I'd like to buy one, otherwise, I prefer thinking by myself.
- It would better that lecturers teach us how to use TI-89, and using in the lecture, that I will think about to buy one.

Positive use of the CAS

In an environment where they were left more or less to their own devices, guided only by the coursebook notes, the 24 students who used the CAS seem to believe that they have got some benefits from it. Table 1 gives their levels of agreement with each of the 22 questionnaire statements (see Figure 1).

Statement	1	2	3	4	5	6	7	8	9	10	11
Mean	1.88	2.00	2.83	2.65	2.21	1.67	2.37	2.50	2.96	2.46	2.92
Statement	12	13	14	15	16	17	18	19	20	21	22
Mean	2.29	2.54	1.83	1.37	2.92	1.78	1.74	1.92	2.17	2.50	1.67

Table 1: Mean level of agreement with the questionnaire statements

Statements 1, 3, 4, 10, 11, 20 and 22 form a subset addressing the value of the CAS. While a few felt that the CAS did not improve their understanding of mathematics (1), and they were unwilling to admit to any unfair advantage (22), they were very glad they could use CAS in the examination (3), believe in the CAS (10) and strongly asserted that they want to use it in future mathematics courses (11). However, statements 9 and 21 give evidence that they want to learn more about the CAS in order to use them more fully. When asked 'How do you feel about using the TI-89 calculators this year?' 11 responded positively, 2 somewhat negatively, 3 were ambivalent because it was "still a learning curve", they lacked confidence or needed more assistance, and 2 claimed to be neutral or "a little dubious".

Considering the picture of the types of use made of CAS given by statements 5, 13, 14, 15, 16 and 17, we find that they agree they can sometimes do the questions without understanding the theory (5), but do not believe that they have lost basic skills by using the CAS (15). It may be that since this course is largely skills-based, and the theory is emphasised less, that the students find this harder. With or without CAS this may well be the case. Statements 13, 14, 16 and 17 consider instrumentation of the CAS, especially in terms of integration with their by-hand working. They agreed that they explore the TI-89 by themselves (13), an essential part of instrumentation, and strongly affirmed that they use both the CAS and pen

and paper when working on mathematics problems (16). When doing so they don't have a problem deciding when to use the CAS (14), neither do they just turn to the CAS when by-hand methods fail (17). The responses to the open questions reveal that 9 of the 17 responding found the CAS helpful. However, 8 of the 24 students primarily used the CAS for checking their by-hand working, "Checking answers, because I often make mistakes with signs, and the TI-89 makes it easy to check." Several stated that they used the CAS when the by-hand calculation was too difficult, for "confirmations and when it is hard done with pen" or "basically doing tough calculations" or "I could do most problems, but it's helpful when it comes to nasty algebra simplification". Both of these categories of use confirm the findings of Thomas and Hong (2004). In addition, 7 of the students valued the CAS for drawing graphs or for "visualising the graph of a function" because "visualisation makes solving problems easier". When asked how they decided to use the CAS, 2 of the students provided interesting insight into the process of instrumentation. They replied "If I'm stuck I'll try to get the answer and from there work backwards." and "Yes, from the answers to guess the way." It seems that these students had found a way of working that involved using the direct answer from the CAS to try to work out the by-hand method of solution. In answer to another question, the first of these gave further insight into what he does, saying CAS had helped him "With differentiation, I have been able to recognise the patterns." So he was assisted with the structure of the differentiation results by considering a number of examples on the CAS.

Attending to statements 2, 6, 7, 12 and 19, which examine possible obstacles to CAS use, there is some slight agreement that the complexity of the CAS, including the menus and key presses is a problem, but it is not the major issue that others (Hong, Thomas, & Kiernan, 2000) have found with younger students (2 and 19), and they did not find the format of the CAS output a problem (6). One comment on this was that "It's kind of hard to use because you really need to use and put the brackets, braces at [the] right place." In the open responses 50% said that they explored the CAS by themselves as well as using the coursebook notes, since "both are necessary as the course book manual often lacks details". There is agreement that the CAS manual at the back of the coursebook is very helpful (7), and, surprisingly in view of the above discussion, they thought that the lecturers had been very supportive and encouraging in their use of CAS (12). One student though made the telling comment on the existence of the CAS notes in the back of the coursebook, that "no one told us not even the lecturers, I don't think they even knew about, he was like "oh" when he saw them last week."

In conclusion, we found in this study that the majority of students did not use the CAS because it was too expensive to buy, they saw it as of limited value in doing or understanding the mathematics, the effort to learn it was too great and their was little support in place, and the lecturing staff did not support or promote its use. In spite of this very negative social environment a few positive aspects of CAS use emerged from the small group using them. Even here though most students only employed the

CAS procedurally, using it to check answers, to perform complex but direct calculations, as well as visualise 2-D and 3-D graphs. However, two were able to progress beyond these basic functions to consider conceptually the structure of problem solutions. The students used the CAS alongside their by-hand methods and were making some attempts to integrate it into their learning, although there was an element of resorting to the CAS only when by-hand work was too difficult. Although there are many similarities with results from studies in schools, our results show that university students may be old enough and independent enough for some CAS use to take root even in an adverse social environment. While some obstacles to increased use are currently beyond control (eg price), others, such as the steep functional learning curve, turned out to be less of a problem than non-users anticipated. There would no doubt have been a greater take up of the CAS, improved instrumentation, and hence more prospect of beneficial outcomes, if the department had analysed its support mechanisms and provisions better and put in place systems to coordinate them and foster CAS use. We learn that a piecemeal approach to CAS use, not unexpectedly, produces fragmented results.

References

Artigue, M. (2002). Learning mathematics in a CAS environment: The genesis of a reflection about instrumentation and the dialectics between technical and conceptual work. *International Journal of Computers for Mathematical Learning, 7,* 245-274.

Bergsten, C. (1996). Learning by computing? The case of calculus and a CAS. In L. Lum (Ed.), *Proceedings of the 7th annual international conference on Technology in Collegiate Mathematics* (pp. 36-39). Reading, MA: Addison Wesley.

Brousseau, G. (1997). *Theory of didactical situations in mathematics* (N. Balacheff, M. Cooper, R. Sutherland & V. Warfield: Eds. and Trans.). Dordrecht: Kluwer.

Drijvers, P. (2002). Learning mathematics in a computer algebra environment: Obstacles are opportunities. *Zentralblatt für Didaktik der Mathematik, 34*(5), 221-228.

Hong, Y. Y., Thomas, M. O. J. & Kiernan, C. (2000). Super-calculators and university entrance calculus examinations, In M. Chinnappan & M. O. J. Thomas (Eds.), *Mathematics Education Research Journal Special Issue on Technology in Mathematics Learning and Teaching*, *12*(*3*), 321-336.

Kendal, M., & Stacey, K. (1999). Varieties of teacher privileging for teaching calculus with computer algebra systems, *The International Journal of Computer Algebra in Mathematics Education, 6*(4), 233-247.

Rabardel, P. & Samurcay, R. (2001). *From artifact to instrumented-mediated learning*, New challenges to research on learning, International symposium organized by the Center for Activity Theory and Developmental Work Research, University of Helsinki, March.

Thomas M. O. J. & Hong, Y. Y. (2004). Integrating CAS Calculators into Mathematics Learning: Issues of Partnership, *Proceedings of the 28th Conference of the International Group for the Psychology of Mathematics Education*, Bergen, Norway, *4,* 297-304.

Thomas, M. O. J., Monaghan, J., Pierce, R. (2004). Computer algebra systems and algebra: Curriculum, assessment, teaching, and learning. In K. Stacey, H. Chick, & M. Kendal (Eds.), *The teaching and learning of algebra: The 12th ICMI study* (pp. 155-186). Norwood, MA: Kluwer Academic Publishers.