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MATHEMATICAL ASPECTS IN CELESTIAL MECHANICS, THE LAGRANGE AND EULER PROBLEMS IN THE LOBACHEVSKY SPACE

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Abstract. The generalization of a test particle motion in a central field of the two immovable point-like centers to the case of a constant curvature space, in the space of Lobachevsky, is studied in the paper. The bifurcation set in the plane of integrals of motion was constructed and the classification of the domains of possible motion was carried out. The Lagrange's problem on the pseudosphere: a mass point motion under the action of attracting center field and the analogue of a constant homogeneous field in a constant curvature space, is studied as well.

1. Introduction

For the first time the problem on the motion of dynamical systems in a constant curvature spaces was formulated by N. I. Lobachevsky. He generalized the Newton law of attraction for the space of negative curvature. Later E. Schrödinger found the spectrum of hydrogen atom (the Kepler problem on a three-dimensional sphere $\mathbb{S}^3$). Trajectories and modification of the Kepler laws of the body motion in the field of Newton's potential were investigated by P. W. Higgs [1]. The generalization of the Kepler laws for the classical problem to the spaces $\mathbb{S}^3$ and $\mathbb{H}^3$ (here $\mathbb{H}^3$ is an upper sheet of hyperboloid embedded into the Minkowski space) is given in the work of N. A. Chernikov [2]. The classification of the motion for the plane case was carried out by C. L. Charlier [3]. However his analysis turned out to be incomplete and partially incorrect so it was corrected twice by H. J. Tallqvist [4]and T. K. Badalyan [5] In the paper of V. V. Kozlov and O. A. Harin [6] the full integrability of the