Eleventh International Conference on
Geometry, Integrability and Quantization
June 5–10, 2009, Varna, Bulgaria
Ivaïlo M. Mladenov, Gaetano Vilasi
and Akira Yoshioka, Editors
Avangard Prima, Sofia 2010, pp 97–107

SEIBERG-WITTEN EQUATIONS ON $\mathbb{R}^6$

NEDIM DEĞIRMENCI and ŞENAY KARAPAZAR

Department of Mathematics, University of Anadolu
Eşkisehir 26470, Turkey

Abstract. It is known that Seiberg-Witten equations are defined on smooth four dimensional manifolds. In the present work we write down a six dimensional analogue of these equations on $\mathbb{R}^6$. To express the first equation, the Dirac equation, we use a unitary representation of complex Clifford algebra $\mathbb{C}l_{2n}$. For the second equation, a kind of self-duality concept of a two-form is needed, we make use of the decomposition $\Lambda^2(\mathbb{R}^6) = \Lambda^2_1(\mathbb{R}^6) \oplus \Lambda^2_6(\mathbb{R}^6) \oplus \Lambda^2_8(\mathbb{R}^6)$. We consider the eight-dimensional part $\Lambda^2_8(\mathbb{R}^6)$ as the space of self-dual two-forms.

1. Introduction

The Seiberg-Witten equations defined on four-dimensional manifolds yield some invariants for the underlying manifold. There are some generalizations of these equation to higher dimensionsinal manifolds. In [2, 7] some eight-dimensional analogies were given and a seven-dimensional analog was presented in [5]. In this work we write down similar equations to Seiberg-Witten equations on $\mathbb{R}^6$.

2. spin^c–structure and Dirac Operator on $\mathbb{R}^{2n}$

Definition 1. *A* spin^c*-structure on the Euclidian space* $\mathbb{R}^{2n}$ *is a pair* (S, Γ) *where* S *is a* 2^n-*dimensional complex Hermitian vector space and* $\Gamma : \mathbb{R}^{2n} \to \mathrm{End}(S)$ *is a linear map which satisfies*

$$\Gamma(v)^* + \Gamma(v) = 0, \qquad \Gamma(v)^*\Gamma(v) = |v|^2 1$$

for every $v \in \mathbb{R}^{2n}$.

The 2^n-dimensional complex vector space S is called spinor space over $\mathbb{R}^{2n}$.
From the universal property of the complex Clifford algebra $\mathbb{C}l_{2n}$ the map Γ can be extended to an algebra isomorphism Γ: $\mathbb{C}l_{2n} \to \mathrm{End}(S)$ which satisfies $\Gamma(\tilde{x}) =$