

JACOBI THETA EMBEDDING OF A HYPERBOLIC 4-SPACE WITH CUSPS*

ROLF-PETER HOLZAPFEL

*Mathematisches Institut, Humboldt-Universität Berlin
26 Rudower Chaussée, 10099 Berlin, Germany*

Abstract. Starting from a fixed elliptic curve with complex multiplication we compose lifted quotients of elliptic Jacobi theta functions to abelian functions in higher dimension. In some cases, where complete Picard–Einstein metrics have been discovered on the underlying abelian surface (outside of cusp points), we are able to transform them to Picard modular forms. Basic algebraic relations of basic forms come from different multiplicative decompositions of these abelian functions in simple ones of the same lifted type. In the case of Gauß numbers the constructed basic modular forms define a Baily–Borel embedding in \mathbb{P}^{22} . The relations yield explicit homogeneous equations for the Picard modular image surface.

1. Introduction: Basic Problems, Motivations

1) Rational Cuboid Problem.

Find a cuboid with rational edges and (face) diagonals. There is no solution until now. By some work of Shiga and others, see [14], the rational cuboids define (and are defined up to similarity by) rational points on a \mathbb{Q} -model of the $K3$ -surface $E \times E / \langle -1 \rangle$, E an elliptic curve with Gauß number multiplication. This $K3$ surface is Picard modular.

2) Hilbert’s 12-th Problem.

Explicit construction of “nice” number fields via special values of transcendental functions with more than one variable. There is no completely understood example until now.

* For the celebration of 200 years since the appearance of Gauß’ *Disquisitiones Arithmeticae*.