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NEWS ON IMMERSIONS OF THE LOBACHEVSKY SPACE INTO EUCLIDEAN SPACE

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Abstract. An exposition of the new results concerning the nonexistence of local isometric immersions of 3-dimensional Lobachevsky space L^3 into 5-dimensional Euclidean space E^5 with constant curvature of the Grassmannian image metric, on connections between curvatures of asymptotic lines on a domain of $L^3 \subset E^5$, on regularity theorems for surfaces obtained by Backlund transformation of a domain of $L^2 \subset S^3$ and $L^2 \subset E^3$.

Isometric immersions of the domains of the n-dimensional Lobachevsky space L^n into the $(2n-1)$-dimensional Euclidean space E^{2n-1} for $n > 2$ were considered in works by Moore, Tenenblat, Terng, the present author and others. It is well-known, that L^n cannot be locally immersed into E^{2n-2}. So the dimension $(2n-1)$ is the least possible one. In this case there exist relations between the extrinsic and intrinsic properties of the submanifolds. It is possible to prove that on an immersed domain of L^n there exist coordinates of curvature $u^1, \ldots, u^n$ such that the metric of L^n is expressed in the form

$$\mathrm{d}s^2 = \sum_{i=1}^{n} \sin^2 \sigma_i (\mathrm{d}u^i)^2 \tag{1}$$

with the condition

$$\sum_{i=1}^{n} \sin^2 \sigma_i = 1\,. \tag{2}$$