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WEITZENBÖCK FORMULAS ON POISSON PROBABILITY SPACES

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Abstract. This paper surveys and compares some recent approaches to stochastic infinite-dimensional geometry on the space Γ of configurations (i. e. locally finite subsets) of a Riemannian manifold M under Poisson measures. In particular, different approaches to Bochner–Weitzenböck formulas are considered. A unitary transform is also introduced by mapping functions of n configuration points to their multiple stochastic integral.

1. Weitzenböck Formula under a Measure

Let M be a Riemannian manifold with volume measure $\mathrm{d}x$, covariant derivative ∇, and exterior derivative d. Let ∇_μ^* and d_μ^* denote the adjoints of ∇ and d under a measure μ on M of the form $\mu(\mathrm{d}x) = \mathrm{e}^{\phi(x)}\,\mathrm{d}x$. The classical Weitzenböck formula under the measure μ states that

$$\mathrm{d}_\mu^*\,\mathrm{d} + \mathrm{d}\,\mathrm{d}_\mu^* = \nabla_\mu^*\nabla + R - \mathrm{Hess}\,\phi\,,$$

where R denotes the Ricci tensor on M. In terms of the de Rham Laplacian $H_R = \mathrm{d}_\mu^*\,\mathrm{d} + \mathrm{d}\,\mathrm{d}_\mu^*$ and of the Bochner Laplacian $H_B = \nabla_\mu^*\nabla$ we have

$$H_R = H_B + R - \mathrm{Hess}\,\phi\,.$$

In particular the term $\mathrm{Hess}\,\phi$ plays the role of a curvature under the measure μ.

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