Seventh International Conference on
Geometry, Integrability and Quantization
June 2–10, 2005, Varna, Bulgaria
Ivaïlo M. Mladenov and Manuel de León, Editors
SOFTEX, Sofia 2006, pp 280–291

PAINLEVÉ ANALYSIS AND EXACT SOLUTIONS OF NONINTEGRABLE SYSTEMS

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Abstract. Here we consider the cubic complex Ginzburg–Landau equation. Applying the Hone's method, based on the use of the Laurent-series solutions and the residue theorem, we have proved that this equation has no elliptic standing wave solutions. This result supplements Hone's result, that this equation has no elliptic travelling wave solutions. It has been shown that the Hone's method can be applied to a system of polynomial differential equations more effectively than to an equivalent differential equation.

1. Introduction

Nonlinear dynamic systems and evolution equations actively used in physics are often nonintegrable in the sense that it is impossible to find their general solutions using known methods. At the same time knowledge of special solutions with some given properties is sufficient for construction of physical models. At present time methods for construction of special solutions in terms of elementary (degenerated elliptic) and elliptic functions are well developed [3, 8–11, 19–21, 24, 26, 29–31, 34, 36] (see also [22] and references therein). Some of these methods are intended for the search of elliptic solutions only [3, 20, 29], others allow to find either solutions in terms of elementary functions [8, 9, 36] or both types of solutions [10, 11, 19, 21, 24, 26, 30, 31, 34]. Note that both elliptic and degenerate elliptic functions are solutions of some first order polynomial differential equations.

Elliptic and degenerate elliptic solutions of some differential equation can exist only if there exist formal Laurent series solutions of them. One can construct such formal solutions using the Ablowitz–Ramani–Segur algorithm of the Painlevé test [1] (see also [32]). In [24] the method of construction of analytic special solutions of nonintegrable systems with the help of the Laurent series solutions