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THE BIHARMONIC STRESS-ENERGY TENSOR AND THE GAUSS MAP

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Abstract. We consider the energy and bienergy functionals as variational problems on the set of Riemannian metrics and present a study of the biharmonic stress-energy tensor. This approach is then applied to characterize weak conformality of the Gauss map of a submanifold. Finally, working at the level of functionals, we recover a result of Weiner linking Willmore surfaces and pseudo-umbilicity.

1. Introduction

The guiding principle of variational theory is that geometric objects can be selected according to whether or not they minimize certain functionals and, since Morse theory, critical points can prove sufficiency. Once this criterion is chosen, the adequate Euler–Lagrange equation will characterise maps particularly well adapted to our geometric framework. However, roles can be reversed and metrics can be viewed as variables and required to fit with a map and complete the picture. Other than the duality of these approaches, the theory of general relativity has put metrics firmly in centre of the stage and the characterisation of Einstein metrics as (constrained) critical points of the total curvature has created a new viewpoint on the usual functionals, in particular the various energies defined for maps between manifolds.

Let $\phi : (M,g) \to (N,h)$ be a smooth map between Riemannian manifolds of dimension m, respectively n. Assuming that M is compact we can define the