Zbl 1172.37003

Einsiedler, Manfred; **Lindenstrauss, Elon**; **Michel, Philippe**; **Venkatesh, Akshay**

Distribution of periodic torus orbits on homogeneous spaces. (English)

Duke Math. J. 148, No. 1, 119-174 (2009). ISSN 0012-7094
http://dx.doi.org/10.1215/00127094-2009-023
http://www.dukemathjournal.org
http://projecteuclid.org/handle/euclid.dmj

The authors obtain results toward the equidistribution of certain families of periodic torus orbits on homogeneous spaces. They focus on the case of diagonal torus actions on quotients of $PGL_n(\mathbb{R})$. By attaching to each periodic orbit an integral invariant (the discriminant), they obtain the result that certain standard conjectures about the distribution of such orbits hold up to exceptional sets of at most $O(\Delta^\epsilon)$ orbits of discriminant at most Δ. The proof relies on the facts that periodic orbits are well-separated and that torus actions are 'measure-rigid'. Examples of sequences of periodic orbits of these actions that fail to become equidistributed (even in higher rank) are given. Also, the authors give an application of their results to sharpen a theorem of Minkowski on ideal classes in totally real number fields of cubic and higher degrees.

Bernd O. Stratmann (St. Andrews)

Keywords : homogeneous spaces; Diophantine approximations; equidistribution

Classification :

∗37A17 Homogeneous flows
37A45 Relations of ergodic theory with number theory and harmonic analysis
11E99 Forms and linear algebraic groups

Zbl pre05646212

Bourgain, Jean; **Lindenstrauss, Elon**; **Michel, Philippe**; **Venkatesh, Akshay**

Some effective results for $\times a \times b$. (English)

Ergodic Theory Dyn. Syst. 29, No. 6, 1705-1722 (2009). ISSN 0143-3857; ISSN 1469-4417
http://dx.doi.org/10.1017/S0143385708000898
http://journals.cambridge.org/action/displayJournal?jid=ETSbVolume=y

Classification :

∗37A05 Measure-preserving transformations
28D20 Entropy and other measure-theoretic invariants

Zbl 1160.37006

Lindenstrauss, Elon; **Mirzakhani, Maryam**

Ergodic theory of the space of measured laminations. (English)

Int. Math. Res. Not. 2008, Article ID rnm126, 49 p. (2008). ISSN 1073-7928; ISSN 1687-0247
http://dx.doi.org/10.1093/imrn/rnm126
http://imrn.oxfordjournals.org/

Authors' abstract: We classify locally finite invariant measures and orbit closure for the action of the mapping class group on the space of measured laminations on a surface. This classification translates to a classification of measures and orbit closures on the the space of quadratic differentials invariant under the horospheric foliation.

Alexander Kachurovskij (Novosibirsk)

Keywords : invariant measures; orbit closure; measured laminations; quadratic differentials; horospheric foliation
Classification :
∗37C40 Smooth ergodic theory, invariant measures
37D40 Dynamical systems of geometric origin and hyperbolicity

---

Zbl 1146.37006

Einsiedler, Manfred; Lindenstrauss, Elon

On measures invariant under diagonalizable actions: the rank-one case and the general low-entropy method. (English)

J. Mod. Dyn. 2, No. 1, 83-128 (2008). ISSN 1930-5311; ISSN 1930-532X
http://aimsciences.org/journals/JMD/jmd$_o nline.jsp$

Many important problems in dynamical systems and its interactions with number theory involve classifying measures invariant under the action of a real-diagonalizable group on some locally compact quotient of a real algebraic group. All the results in this spirit have involved some positive entropy hypothesis, and some of the results have required other mixing hypotheses on the measure.

Two important steps in the direction of controlling the hypotheses needed are a "high-entropy" method, introduced by *M. Einsiedler* and *A. Katok* [Isr. J. Math. 148, 169–238 (2005; Zbl 1097.37017)] (which requires a genuine action of a multi-dimensional group and more than simply positive entropy for a single element) and a "low-entropy" method, introduced by *E. Lindenstrauss* [Ann. Math. (2) 163, No. 1, 165–219 (2006; Zbl 1104.22015)] (in which information about the measure is extracted from properties of a single transformation). Because a typical situation involves a completely unknown measure, these two approaches are complementary, and indeed were both brought to bear in the work of the authors and *A. Katok* on Littlewood's problem [Ann. Math. (2) 164, No. 2, 513–560 (2006; Zbl 1109.22004)].

In this paper the low-entropy method is substantially generalized, with the development of rigidity properties for the measurable factors of the action of a single positive entropy element (for an unknown measure) on certain locally homogeneous spaces $\Lambda\backslash G$. This is used to classify positive entropy measures invariant under a one-parameter group with an extra recurrence condition when G is a product with a rank-one algebraic group.

Thomas Ward (Norwich)
Keywords : torus action; homogeneous space; invariant measure; entropy
Classification :
∗37A35 Invariants of ergodic theory
37D40 Dynamical systems of geometric origin and hyperbolicity
37A45 Relations of ergodic theory with number theory and harmonic analysis
11H46 Products of linear forms
22D40 Ergodic theory on groups
28D20 Entropy and other measure-theoretic invariants

Zbl 1161.37301

Lindenstrauss, Elon
Some examples how to use measure classification in number theory. (English)
Granville, Andrew (ed.) et al., Equidistribution in number theory, an introduction. Proceedings of the NATO Advanced Study Institute on equidistribution in number theory, Montréal, Canada, July 11–22, 2005. Dordrecht: Springer. NATO Science Series II: Mathematics, Physics and Chemistry 237, 261-303 (2007). ISBN 978-1-4020-5403-7/pbk; ISBN 978-1-4020-5402-0/hbk
http://dx.doi.org/10.1007/978-1-4020-5404-4$_1$3

Summary: We give examples of how classifying invariant probability measures for specific algebraic actions can be used to prove density and equidistribution results in number theory.
The paper was written for graduate students interested in the interplay between ergodic theory and number theory. Chapter headings are as follows:
1. Introduction
2. Dynamical Systems: Some Background
3. Equidistribution of $n^2\alpha$ mod 1
4. Unipotent Flows and Ratner's Theorems
5. Entropy of Dynamical Systems: Some More Background
6. Diagonalizable Actions and the Set of Exceptions to Littlewood's Conjecture
7. Applications to Quantum Unique Ergodicity

Classification :
∗37A45 Relations of ergodic theory with number theory and harmonic analysis
11K31 Special sequences

Zbl 1137.22011

Lindenstrauss, Elon; **Venkatesh, Akshay**
Existence and Weyl's law for spherical cusp forms. (English)
Geom. Funct. Anal. 17, No. 1, 220-251 (2007). ISSN 1016-443X; ISSN 1420-8970
http://dx.doi.org/10.1007/s00039-006-0589-0
http://link.springer.de/link/service/journals/00039/

The asymptotics for the number of the Laplacian eigenvalues for compact manifolds was proved by Weyl. For a noncompact manifold of finite volume, it is expected that similar results hold for the number of discrete spectra of the Laplacian.
The work of *R. Phillip* and *P. Sarnak* [J. Anal. Math. 59, 179–187 (1992; Zbl 0809.30035)] indicates this for a special case. On the other hand, *A. Selberg* [J. Indian Math. Soc., n. Ser. 20, 47–87 (1956; Zbl 0072.08201)] has shown this for the quotient space of the upper half plane by a congruence subgroup of $\mathrm{PGL}(2, \mathbb{Z})$. It has been conjectured by *P. Sarnak* [Isr. Math. Conf. Proc. 3, 237–250 (1990; Zbl 0707.11040)] that the same holds in the general setting of congruence quotients of noncompact symmetric spaces. In his thesis, *S. D. Miller* [J. Reine Angew. Math. 533, 127–169 (2001; Zbl 0996.11040)] established this conjecture for $G = \mathrm{PGL}(3)$. More recently, *W. Müller* [C. R. Math. Acad. Sci. Paris 338, No. 5, 347–352 (2004; Zbl 1062.11028)] has established this conjecture for $\mathrm{PGL}(n)$.
For a general split adjoint semisimple group, the asymptotics for the number of cusp forms with Laplacian eigenvalues was proved by *H. Donnelly* [J. Differ. Geom. 17, 239–253 (1982; Zbl 0494.58029)]. The works of Selberg, Miller, and Müller rely essentially on the theory of Eisenstein series. In this paper, the authors give a simple proof of the Wey law for cusp forms, valid for any split adjoint group G over Q.

Jiuzu Hong (Rehovot)

Keywords : cusp forms; Weyl law; trace formulas; congruence quotients

Classification :

∗22E55 Repres. of Lie and linear algebraic groups over global fields
32N15 Automorphic functions in symmetric domains
11F03 Modular and automorphic functions
11F72 Spectral theory

Zbl 1119.43001

Bourgain, Jean; Furman, Alex; Lindenstrauss, Elon; Mozes, Shahar

Invariant measures and stiffness for non-Abelian groups of toral automorphisms. (English)

C. R., Math., Acad. Sci. Paris 344, No. 12, 737-742 (2007). ISSN 1631-073X
http://dx.doi.org/10.1016/j.crma.2007.04.017
http://www.sciencedirect.com/science/journal/1631073X

Summary: Let Γ be a non-elementary subgroup of $\mathrm{SL}_2(\mathbb{Z})$. If μ is a probability measure on $\mathbb{T}^2$ which is Γ-invariant, then μ is a convex combination of the Haar measure and an atomic probability measure supported by rational points. The same conclusion holds under the weaker assumption that μ is ν-stationary, i.e. $\mu = \nu * \mu$, where ν is a finitely supported probability measure on Γ whose support $\operatorname{supp}\nu$ generates Γ. The approach works more generally for $\Gamma < \mathrm{SL}_d(\mathbb{Z})$.

Classification :

∗43A05 Measures on groups, etc.

Zbl 1118.37008

Einsiedler, Manfred; Lindenstrauss, Elon

Joinings of higher-rank diagonalizable actions on locally homogeneous spaces. (English)

Duke Math. J. 138, No. 2, 203-232 (2007). ISSN 0012-7094
http://dx.doi.org/10.1215/S0012-7094-07-13822-5
http://www.dukemathjournal.org
http://projecteuclid.org/handle/euclid.dmj

H. Furstenberg [Math. Syst. Theory 1, 1–49 (1967; Zbl 0146.28502)] introduced the notion of joinings between measure-preserving actions and described situations in which individual elements of a group action may be far from rigid (that is, may possess many closed nontrivial invariant sets and many nonatomic invariant measures) while the whole action exhibits rigidity. Many of the important subsequent developments link rigidity of measures to descriptions of the space of joinings, and exhibit measure rigidity for large classes of group actions under some additional hypothesis (typically the assumption that some elements of the action give the measure in question positive entropy). In this paper rigidity of joinings between a large class of higher-rank diagonalizable actions on locally homogeneous spaces is shown, without any additional entropy hypothesis. These results are used to deduce a classification of the measurable factors of such actions following a method initiated by *D. Witte* [Trans. Am. Math. Soc. 345, No. 2, 577–594 (1994; Zbl 0831.28010)], and further results on equidistribution of orbits following a method of Furstenberg. *B. Kalinin* and *R. Spatzier* [Ergodic Theory Dyn. Syst. 25, No. 1, 175–200 (2005; Zbl 1073.37005)] gave results on the structure of joinings of such actions under the assumption that the joining made certain one-parameter subgroups ergodic. Here entropy methods related to those used by *M. Einsiedler* and *A. Katok* [Isr. J. Math. 148, 169–238 (2005; Zbl 1097.37017)] and *M. Einsiedler* and the reviewer [J. Reine Angew. Math. 584, 195–214 (2005; Zbl 02191025)] are used to end up in a situation where *M. Ratner*'s theorem [Ann. Math. (2) 134, No. 3, 545–607 (1991; Zbl 0763.28012)] can be applied.

Thomas Ward (Norwich)

Keywords : higher-rank actions; equidistribution

Classification :

$*$37A17 Homogeneous flows
22E46 Semi-simple Lie groups and their representations

Zbl 1186.58022

Lindenstrauss, Elon

Adelic dynamics and arithmetic quantum unique ergodicity. (English)

Jerison, David (ed.) et al., Current developments in mathematics, 2004. Somerville, MA: International Press. 111-139 (2006). ISBN 978-1-57146-105-6/hbk

Let $\mathbb{A}$ be the ring of Adeles over $\mathbb{Q}$, let $A(\mathbb{A})$ denote the diagonal subgroup of $\mathrm{PGL}(2, \mathbb{A})$,

and let $X = \mathrm{PGL}(2, \mathbb{Q})\backslash\mathrm{PGL}(2, \mathbb{A})$.
The purpose of the paper is to prove that any $A(\mathbb{A})$-invariant probability measure on X is also $\mathrm{PGL}(2, \mathbb{A})$-invariant. This theorem is proved with the techniques used by the author to partially classify invariant measures satisfying a recurrence condition, which was used to give an almost complete answer to the quantum unique ergodicity conjecture for non-compact hyperbolic surfaces with finite volume.

Jesus A. Álvarez López (Santiago de Compostela)

Keywords : Adeles; invariant measure; quantum unique ergodicity; recurrent measure; transient measure
Classification :
∗58J51

Zbl 1121.37028

Einsiedler, Manfred; Lindenstrauss, Elon

Diagonalizable flows on locally homogeneous spaces and number theory. (English)

Sanz-Solé, Marta (ed.) et al., Proceedings of the international congress of mathematicians (ICM), Madrid, Spain, August 22–30, 2006. Volume II: Invited lectures. Zürich: European Mathematical Society (EMS). 1731-1759 (2006). ISBN 978-3-03719-022-7/hbk

Authors' abstract: We discuss dynamical properties of actions of diagonalizable groups on locally homogeneous spaces, particularly their invariant measures, and present some number theoretic and spectral applications. Entropy plays a key role in the study of these invariant measures and in the applications.

Pei-Chu Hu (Jinan)

Keywords : invariant measure; locally homogeneous spaces; Littlewoods's conjecture; quantum unique ergodicity; distribution of periodic orbits; ideal classes; entropy
Classification :
∗37D40 Dynamical systems of geometric origin and hyperbolicity
37A45 Relations of ergodic theory with number theory and harmonic analysis
11J13 Simultaneous homogeneous approximation, linear forms
81Q50 Quantum chaos

Zbl 1109.22004

Einsiedler, Manfred; Katok, Anatole; Lindenstrauss, Elon

Invariant measures and the set of exceptions to Littlewood's conjecture. (English)

Ann. Math. (2) 164, No. 2, 513-560 (2006). ISSN 0003-486X; ISSN 1939-0980
http://dx.doi.org/10.4007/annals.2006.164.513
http://annals.math.princeton.edu/annals/about/cover/cover.html
http://pjm.math.berkeley.edu/annals/about/journal/about.html
http://www.jstor.org/journals/0003486X.html

There is a well-known and long-standing conjecture of Littlewood: $\forall u, v \in \mathbb{R}, \liminf_{n\to\infty} n\langle nu\rangle\langle nv\rangle$ 0, where $\langle w\rangle = \min_{n\in\mathbb{Z}} |w-n|$ is the distance of $w \in \mathbb{R}$ to the nearest integer. Let A be the group of positive diagonal $k\times k$ matrices on $SL(k,\mathbb{R})/SL(k,\mathbb{Z})$. In the paper under review some results which have implications on Littlewood's conjecture are proven.
Main results of the paper: 1) Let μ be an A-invariant and ergodic measure on $X = SL(k,\mathbb{R})/SL(k,\mathbb{Z})$ for a subgroup of A which acts on X with positive entropy. Then μ is algebraic. 2) Let $\Xi = \{(u,v) \in \mathbb{R}^2 : \liminf_{n\to\infty} n\langle nu\rangle\langle nv\rangle > 0\}$. Then the Hausdorff dimension $\dim_H \Xi = 0$. 3) For any k linear forms $m_i(x_1,\dots,x_k) = \sum_{j=1}^k m_{ij}(x_j)$ and $f_m(x_1,\dots,x_k) = \prod_{i=1}^k m_i(x_1,\dots,x_k)$, where $m = (m_{ij})$ denotes the $k\times k$ matrix whose rows are the linear forms $m_i(x_1,\dots,x_k)$, there is a set $\Xi_k \subset SL(k,\mathbb{R})$ of Hausdorff dimension $k-1$ so that $\forall m \in SL(k,\mathbb{R}) \setminus \Xi_k$, $\inf_{x\in\mathbb{Z}^k\setminus\{0\}} |f_m(x)| = 0$.
The last result has applications to a generalization of Littlewood's conjecture.

Victor Sharapov (Volgograd)

Keywords : Littlewood's conjecture; entropy; Hausdorff dimension; spaces $SL(k;\mathbb{R})/SL(k;\mathbb{Z})$
Classification :
∗22D40 Ergodic theory on groups
11H46 Products of linear forms

Zbl 1104.22015

Lindenstrauss, Elon (Rudolph, D.)

Invariant measures and arithmetic unique ergodicity. Appendix by E. Lindenstrauss and D. Rudolph. (English)

Ann. Math. (2) 163, No. 1, 165-219 (2006). ISSN 0003-486X; ISSN 1939-0980
http://dx.doi.org/10.4007/annals.2006.163.165
http://projecteuclid.org/euclid.annm/1152899198
http://annals.math.princeton.edu/annals/about/cover/cover.html
http://pjm.math.berkeley.edu/annals/about/journal/about.html
http://www.jstor.org/journals/0003486X.html

The main result of this paper is part of a sequence of 'measure rigidity' results of the following general form: under certain additional conditions, probability measures which are invariant under some 'algebraic' group action are themselves 'algebraic'.
The main result of the paper deals with the classification of probability measures μ on homogeneous spaces of the form $\Gamma\setminus G$, $G = \mathrm{SL}(2,\mathbb{R})\times L$, where L is a finite Cartesian product of algebraic groups over $\mathbb{R}$, $\mathbb{C}$ or $\mathbb{Q}_p$ and $\Gamma \subset G$ a discrete subgroup, which are invariant under the right action of the diagonal subgroup $A \subset \mathrm{SL}(2,\mathbb{R})$ and which satisfy a recurrence condition under L. Under an entropy condition (that all A-ergodic components of μ have positive entropies) it is shown that any such measure is a convex combination of probability measures which are invariant under, and carried by orbits of, closed subgroups of G containing $\mathrm{SL}(2,\mathbb{R})$.
This result has several applications, among them 'quantum unique ergodicity', the statement that the sequence of absolutely continuous probability measures whose densities are joint eigenfunctions of the Laplacian and the Hecke operators on a compact surface

of the form $M = \Gamma \setminus \mathbb{H}$, where Γ is a lattice in $\mathrm{SL}(2,\mathbb{R})$, converge weakly to the normalized invariant volume on M. For noncompact M it is shown that (under appropriate conditions on Γ) any limit of these measures is a multiple of the invariant value on M. Another application of this result assumes that $G = \mathrm{SL}(2,\mathbb{R}) \times \mathrm{SL}(2,\mathbb{R})$, $H \subset G$ is isomorphic to $\mathrm{SL}(2,\mathbb{R})$, and $\Gamma \subset G$ a discrete subgroup whose kernels under the two coordinate projections are finite. Then every probability measure μ on $\Gamma \setminus G$ which is invariant under the Cartesian product B of the two diagonal subgroups of $\mathrm{SL}(2,\mathbb{R})$ in G is either algebraic or has zero entropy under every one-parameter subgroup of B.
The approach to measure rigidity developed in this paper, combined with earlier work by M. Einsiedler and A. Katok, has since led to significant progress on the Littlewood conjecture on simultaneous Diophantine approximation of pairs of real numbers [*M. Einsiedler*, *A. Katok* and *E. Lindenstrauss*, Ann. of Math. (2) 164, No. 2, 513–560 (2006; Zbl 1109.22004)].

Harald Rindler (Wien)

Keywords : rigidity; algebraic group actions; quantum unique ergodicity; entropy Laplacian; Hecke operators

Classification :

∗22E40 Discrete subgroups of Lie groups
11F72 Spectral theory
37A45 Relations of ergodic theory with number theory and harmonic analysis
37D40 Dynamical systems of geometric origin and hyperbolicity

---

Zbl 1155.37301

Lindenstrauss, Elon
Rigidity of multiparameter actions. (English)
Isr. J. Math. 149, 199-226 (2005). ISSN 0021-2172; ISSN 1565-8511
http://dx.doi.org/10.1007/BF02772541
http://www.springerlink.com/content/0021-2172/
http://www.ma.huji.ac.il/ ijmath/

Summary: We survey some of the recent progress in understanding diagonalizable algebraic actions of multidimensional abelian groups, a subject pioneered by *H. Furstenberg* [Math. Syst. Theory 1, 1–49 (1967; Zbl 0146.28502)].

Classification :

∗37A15 General groups of measure-preserving transformation
28D15 General groups of measure-preserving transformations

Zbl 1115.37008

Lindenstrauss, Elon
Invariant measures for multiparameter diagonalizable algebraic actions – a short survey. (English)
Laptev, Ari (ed.), Proceedings of the 4th European congress of mathematics (ECM), Stockholm, Sweden, June 27–July 2, 2004. Zürich: European Mathematical Society

(EMS). 247-256 (2005). ISBN 3-03719-009-4/hbk

The author makes a short survey about the invariant measures for multiparameter diagonalizable algebraic actions.
He begins with the $\times_n$, $\times_m$ action on the one-dimensional torus. He recalls the initial topological result of Furstenberg, the conjecture for the $\times_n, \times_m$-invariant measure and the best known results in this way.
Then he explains how the $\times_n, \times_m$ action on the one-dimensional torus can be seen as an algebraic multiparameter action and how the conjectures of Furstenberg, of Margulis and of Littlewood are related.
In the rest of the paper, Lindenstrauss surveys recent works on the classification invariant measures in the locally homogeneous case and on the joining and isomorphism rigidity of diagonalizable actions on locally homogeneous spaces, especially his works with *M. Einsiedler* and *M. Katok* [Ann. Math. (2) 164, No. 2, 513–560 (2006; Zbl 1109.22004)].

Anne Broise-Alamichel (Orsay)
Keywords : multiparameter diagonalizable algebraic actions
Classification :
∗37A45 Relations of ergodic theory with number theory and harmonic analysis
11J13 Simultaneous homogeneous approximation, linear forms
22E40 Discrete subgroups of Lie groups
37A15 General groups of measure-preserving transformation
37A40 Nonsingular transformations

Zbl 1087.37010

Lindenstrauss, Elon; Schmidt, Klaus
Symbolic representations of nonexpansive group automorphisms. (English)
Isr. J. Math. 149, 227-266 (2005). ISSN 0021-2172; ISSN 1565-8511
http://dx.doi.org/10.1007/BF02772542
http://www.springerlink.com/content/0021-2172/
http://www.ma.huji.ac.il/ ijmath/

If α is an irreducible nonexpansive ergodic automorphism of a compact abelian group X (such as an irreducible nonhyperbolic toral automorphism), than α has no finite or infinite state Markov partitions, and there are no nontrivial continuous embeddings of Markov shifts in X. In spite of this in the paper a symbolic space V and a class of shift-invariant probability measures on V each of which corresponds to an α-invariant probability measure on X is constructed. It is shown, that every α-invariant probability measure on X arises in this way.
The connection between the two-sided beta-shift V_β arising from a Salem number β and the nonhyperbolic ergodic toral automorphism α arising from the companion matrix of the minimal polynomial of β, is invastigated. The connection between the two-sided beta-shifts defined by Pisot numbers and the corresponding hyperbolic ergodic toral automorphisms is studied.

Victor Sharapov (Volgograd)

Keywords : shift-invariant measure; toral automorphisms; symbolic dynamics; entropy
Classification :
∗37B10 Symbolic dynamics
28D10 One-parameter continuous families of measure-preserving transforms
37A15 General groups of measure-preserving transformation

Zbl 1130.11039

Kleinbock, Dmitry; Lindenstrauss, Elon; Weiss, Barak

On fractal measures and Diophantine approximation. (English)

Sel. Math., New Ser. 10, No. 4, 479-523 (2004). ISSN 1022-1824; ISSN 1420-9020
http://dx.doi.org/10.1007/s00029-004-0378-2
http://link.springer.de/link/service/journals/00029/

The paper under review is concerned with an extension of Mahler's problem in metric number theory, originally resolved by *V. G. Sprindzuk* ["Mahler's problem in metric number theory," Nauka i Teknika, Minsk (1967; Zbl 0168.29504)]. Mahler conjectured that almost no points with respect to the natural measure on the curve $(x, x^2, \dots, x^n)$ should be no better approximable by rationals with the same denominator than is the case for generic points in $\mathbb{R}^n$. Here the approximation is measured in the sup-norm. A subset E of $\mathbb{R}^n$ supporting a natural measure is said to be *extremal* if it satisfies this property. If we instead consider multiplicative approximation, where the distances of the individual coordinates are multiplied, we arrive at the notion of *strong extremality*, which implies extremality. In a far-reaching extension of Mahler's conjecture, *D. Y. Kleinbock* and *G. A. Margulis* [Ann. Math. (2) 148, No.1, 339–360 (1998; Zbl 0922.11061)] showed that manifolds satisfying a certain non-degeneracy condition are strongly extremal.

The present paper extends and resolves Mahler's conjecture further. We say that a measure is (strongly) extremal if the support of the measure satisfies the (strong) extremality property. A measure μ on $\mathbb{R}^n$ is said to be *friendly* if it is doubling (or Federer) almost everywhere, if the μ-measure of any affine hyperplane is zero, and if the measure satisfies a certain technical decay condition. The main result of the paper under review is that friendly measures are strongly extremal.

The class of friendly measures include the volume measure on non-degenerate manifolds, and so the result of *Kleinbock* and *Margulis* [loc.cit.] is contained in the main result. In fact, this is part of a larger class of push-forwards of the so-called absolutely friendly measures. Other examples are also given. The Hausdorff s-measure restricted to an s-dimensional attractor of an iterated function system of affine contractions satisfying the open set condition is friendly. The same holds for direct products of friendly measures.

The main theorem is proved by extending the method of *Kleinbock* and *Margulis* [loc.cit.], re-interpreting the strong extremality property as a certain quantitative non-divergence property for a certain flow in the homogeneous space $\mathrm{SL}(n, \mathbb{R})/\mathrm{SL}(n, \mathbb{Z})$. It is then shown that the above measures are all friendly. The paper is concluded with a section on related problems, results and conjectures.

Simon Kristensen (Aarhus)

Keywords : Diophantine approximation; extremality; friendly measures; dynamical systems.
Classification :
∗11J83 Metric theory of numbers

Zbl 1086.37002

Lindenstrauss, Elon

Recurrent measures and measure rigidity. (English)

Maass, Alejandro (ed.) et al., Dynamics and randomness II. Lectures given at the 2nd conference, Santiago, Chile, December 9–13, 2002. Dordrecht: Kluwer Academic Publishers. Nonlinear Phenomena and Complex Systems 10, 123-145 (2004). ISBN 1-4020-1990-4/hbk

Summary: We study maps which preserve a foliation and a metric on this foliation. Such maps arise when studying multiparameter abelian actions, and also in the study of arithmetic quantum unique ergodicity. We also discuss measurable dynamics in which neither the measure nor the measure class is preserved, but nonetheless the system has complicated orbit structure.

Keywords : automorphic forms; quantum unique ergodicity; Hecke recurrences; leaves; conditional measures; ergodic theorems; foliation; multiparameter abelian actions; complicated orbit structure
Classification :
∗37A17 Homogeneous flows
11F72 Spectral theory
22F30 Homogeneous spaces
37A15 General groups of measure-preserving transformation
81Q50 Quantum chaos

Zbl 1076.28014

Lindenstrauss, Elon; Schmidt, Klaus

Invariant sets and measures of nonexpansive group automorphisms. (English)

Isr. J. Math. 144, 29-60 (2004). ISSN 0021-2172; ISSN 1565-8511
http://dx.doi.org/10.1007/BF02984405
http://www.springerlink.com/content/0021-2172/
http://www.ma.huji.ac.il/ ijmath/

This paper studies the properties of nonexpansive, ergodic and totally irreducible automorphisms α of a compact connected abelian group X, and their invariant measures. It is well known that for such an $\alpha : X \to X$, the Haar measure λ_X is invariant and $(X, \alpha, \mathcal{B}_X, \lambda_X)$ is isomorphic to a Bernoulli shift. The results of this paper can be thought of as generalizations of *D. J. Rudolph's* theorem concerning measures on the circle that are both 2×- and 3×-invariant [Ergodic Theory Dyn. Syst. 10, No. 2, 395–

406 (1990; Zbl 0709.28013)] and the results of *A. Katok* and *R. J. Spatzier* [Ergodic Theory Dyn. Syst. 16, No. 4, 751–778 (1996; Zbl 0859.58021); correction ibid. 18, No. 2, 503–507 (1998)].
In this paper it is shown that all α-invariant probability measures μ, singular with respect to λ_X, have the property that the conditional measure ρ_x on the central leaf through x is finite for almost every $x \in X$.

Geoffrey R. Goodson (Towson)

Keywords : nonexpansive group automorphisms; Ruldoph's theorem
Classification :
$*$28D15 General groups of measure-preserving transformations
22D40 Ergodic theory on groups
37A05 Measure-preserving transformations
37A45 Relations of ergodic theory with number theory and harmonic analysis

---

Zbl 1068.37002

Einsiedler, Manfred; **Lindenstrauss, Elon**

Rigidity properties of $\mathbb{Z}^d$-actions on tori and solenoids. (English)

Electron. Res. Announc. Am. Math. Soc. 9, 99-110, electronic only (2003). ISSN 1079-6762
http://dx.doi.org/10.1090/S1079-6762-03-00117-3
http://www.emis.de/journals/ERA-AMS/2003-01-013/2003-01-013.html
http://www.ams.org/era/
http://www.emis.de/journals/ERA-AMS/

Summary: We show that Haar measure is a unique measure on a torus or more generally a solenoid X invariant under a not virtually cyclic totally irreducible $\mathbb{Z}^d$-action by automorphisms of X such that at least one element of the action acts with positive entropy. We also give a corresponding theorem in the nonirreducible case. These results have applications regarding measurable factors and joinings of these algebraic $\mathbb{Z}^d$-actions.

Keywords : Entropy; invariant measures; invariant σ-algebras; measurable factors; joinings; toral automorphisms; solenoid automorphism
Classification :
$*$37A15 General groups of measure-preserving transformation
37C85 Dynamics of group actions other than $\mathbb{Z}$ and $\mathbb{R}$, etc.

Zbl 1018.58019

Bourgain, Jean; **Lindenstrauss, Elon**

Entropy of quantum limits. (English)

Commun. Math. Phys. 233, No.1, 153-171 (2003). ISSN 0010-3616; ISSN 1432-0916
http://dx.doi.org/10.1007/s00220-002-0770-8
http://link.springer.de/link/service/journals/00220/
http://projecteuclid.org/DPubS?service=UIversion=1.0verb=Displaypage=pasthandle=euclid.cm

Summary: We show that any measure arising as a weak* limit of microlocal lifts of eigenfunctions of the Laplacian on certain arithmetic manifolds have dimension at least 11/9, and in particular all ergodic components of this measure with respect to the geodesic flow have positive entropy.

Keywords : eigenfunctions; Laplacian; arithmetic manifolds; geodesic flow; positive entropy
Classification :
∗58J50 Spectral problems; spectral geometry; scattering theory

---

Zbl 1018.37003

Ledrappier, François; Lindenstrauss, Elon

On the projections of measures invariant under the geodesic flow. (English)

Int. Math. Res. Not. 2003, No.9, 511-526 (2003). ISSN 1073-7928; ISSN 1687-0247
http://dx.doi.org/10.1155/S1073792803208114
http://imrn.oxfordjournals.org/

Let M be a compact, 2-dimensional Riemannian manifold equipped with a probability measure μ on the unit tangent bundle SM that is invariant under the geodesic flow. The authors are interested in the image of μ and its dimension under the projection $\pi : SM \to M$. They define the information dimension of μ, which is closely related to the the entropy of μ, as follows: $\dim_X \mu = \lim_{\varepsilon\to 0} \log \mu\{B(x,\varepsilon)\}/\log\varepsilon$, $x \in SM$, and $\dim\mu = \text{ess-inf}\dim_X \mu$. Here $B(x,\varepsilon)$ denotes the ball of radius ε centered at x. The main result of the paper is the following Theorem: Let M, μ, SM and $\pi : SM \to M$ be as above. Then 1) If $\dim\mu \leq 2$, then $\dim\mu = \dim\pi(\mu)$. 2) If $\dim\mu > 2$, then $\pi(\mu)$ is absolutely continuous with respect to the Lebesgue measure vol_M on M. As a corollary it follows that case 2) applies if $h(x) > (1/2)\lambda(x) > 0$ for x a.e. Here $h(x)$ denotes the entropy of the ergodic component containing x, and $\lambda(x)$ denotes the Lyapunov exponent at x (defined for x a.e.). If in addition μ has locally finite s-energy for $s > 2$, then the Radon Nikodym derivative $d\pi(\mu)/d\text{vol}_M$ lies in $L^2(M)$. The authors observe that the problem of this article is related to the theory of Kakeya sets. They also note that the direct extension of their approach to manifolds M of dimension ≥ 3 yields results that are not likely to be sharp.

P.Eberlein (Chapel Hill)

Keywords : geodesic flow; invariant measures; information dimension; entropy; Lyapunov exponent; Lebesgue measure; Kakeya sets
Classification :
∗37A05 Measure-preserving transformations
37D40 Dynamical systems of geometric origin and hyperbolicity
53D25 Geodesic flows
37D25 Nonuniformly hyperbolic systems

Zbl 1024.37004

Lindenstrauss, Elon; Peres, Yuval; Schlag, Wilhelm

Bernoulli convolutions and an intermediate value theorem for entropies of k-partitions. (English)

J. Anal. Math. 87, 337-367 (2002). ISSN 0021-7670; ISSN 1565-8538
http://dx.doi.org/10.1007/BF02868480
http://www.springerlink.com/content/120600/

The authors deal with the following question suggested by Ya. G. Sinai, that is: given a probability measure preserving system $\widetilde{X} = (X, \mathcal{A}, \mu, T)$, what are the possible values of the conditional entropy for K-partitions in X? Here the authors give a complete answer to Sinai's question for the case where X is a Bernoulli system, using certain concrete linear filters.

Messoud Efendiev (Berlin)

Keywords : probability measure; conditional entropy; Bernoulli system

Classification :

$*$37A35 Invariants of ergodic theory
37A05 Measure-preserving transformations
37A30 Ergodic theorems, spectral theory, Markov operators
28D05 Measure-preserving transformations
28D20 Entropy and other measure-theoretic invariants

Zbl 1093.11034

Lindenstrauss, Elon

On quantum unique ergodicity for $\Gamma\backslash\mathbb{H} \times \mathbb{H}$. (English)

Int. Math. Res. Not. 2001, No. 17, 913-933 (2001). ISSN 1073-7928; ISSN 1687-0247
http://dx.doi.org/10.1155/S1073792801000459
http://imrn.oxfordjournals.org/

Summary: The purpose of this note is to point out a connection between the Quantum Unique Ergodicity conjecture of *Z. Rudnick* and *P. Sarnak* [Commun. Math. Phys. 161, No. 1, 195–213 (1994; Zbl 0836.58043)] (or, more precisely, natural higher rank generalizations of this conjecture) and conjectures of H. Furstenberg, A. Katok, and R. Spatzier, and G. Margulis regarding the scarcity of measures invariant under natural $\mathbb{R}^d$ actions ($d \geq 2$) in the ergodic theory of Lie groups. Our main tool is a new variant of the micro local lift of A. Schnirelman, Y. Colin de Verdière, and S. Zelditch with additional invariance properties. We also sharpen and generalize a related result of Rudnick and Sarnak on scarring of Hecke eigenforms.

Classification :

$*$11F72 Spectral theory
35P20 Asymptotic distribution of eigenvalues for PD operators

37C40 Smooth ergodic theory, invariant measures
58J50 Spectral problems; spectral geometry; scattering theory
81Q50 Quantum chaos

Zbl 1073.37006

Lindenstrauss, Elon; Weiss, Barak

On sets invariant under the action of the diagonal group. (English)

Ergodic Theory Dyn. Syst. 21, No. 5, 1481-1500 (2001). ISSN 0143-3857; ISSN 1469-4417
http://dx.doi.org/10.1017/S0143385701001717
http://journals.cambridge.org/action/displayJournal?jid=ETSbVolume=y

Summary: We consider the action of the $(n-1)$-dimensional group of diagonal matrices in $\mathrm{SL}(n,\mathbb{R})$ on $\mathrm{SL}(n,\mathbb{R})/\Gamma$, where Γ is a lattice and $n \geq 3$. Far-reaching conjectures of Furstenberg, Katok-Spatzier and Margulis suggest that there are very few closed invariant sets for this action. We examine the closed invariant sets containing compact orbits. For example, for $\Gamma = \mathrm{SL}(n,\mathbb{Z})$ we describe all possible orbit-closures containing a compact orbit. In marked contrast to the case $n = 2$, such orbit-closures are necessarily homogeneous submanifolds in the sense of Ratner.

Classification :
∗37A15 General groups of measure-preserving transformation
22E40 Discrete subgroups of Lie groups
22D40 Ergodic theory on groups

Zbl 1038.37004

Lindenstrauss, Elon

Pointwise theorems for amenable groups. (English)

Invent. Math. 146, No. 2, 259-295 (2001). ISSN 0020-9910; ISSN 1432-1297
http://dx.doi.org/10.1007/s002220100162
http://link.springer.de/link/service/journals/00222/

Introduction: The classical ergodic theory deals with measure preserving $\mathbb{Z}$-actions. Many of the deep results of the classical ergodic theory have been generalized to actions of general amenable groups. A notable exception to this have been pointwise convergence results; indeed, even such basic theorems as the L^1-pointwise ergodic theorem or the Shannon-McMillan-Breiman (SMB) theorem were not known for general amenable groups (or even for discrete groups).
In this paper, we develop general covering lemmas that are powerful enough to allow fairly straightforward generalizations of classical pointwise convergence results to general amenable groups. In particular, using these tools, we prove the pointwise ergodic theorem for general amenable groups and the SMB theorem for discrete amenable groups. These tools can also be used to prove other pointwise results. For example, in

D. Ornstein and *B. Weiss* [Isr. J. Math. 79, 113–127 (1992; Zbl 0802.28014)] generalized J. Bourgain, H. Furstenberg, Y. Katznelson and Ornstein's proof of the return time ergodic theorem [see Publ. Math., Inst. Hautes Étud. Sci. 69, 5–45 (1989; Zbl 0705.28008)] from $G = \mathbb{Z}$ to some amenable groups. Using our methods, their proof can be extended to general discrete amenable groups. Most of the results presented in this paper have been announced in *E. Lindenstrauss* [Electron. Res. Announc. Am. Math. Soc. 5, 82–90 (1999; Zbl 0944.28014)].

Classification :

∗37A15 General groups of measure-preserving transformation
37A30 Ergodic theorems, spectral theory, Markov operators
28D15 General groups of measure-preserving transformations

Zbl 0990.28012

Lindenstrauss, Elon

p-adic foliation and equidistribution. (English)

Isr. J. Math. 122, 29-42 (2001). ISSN 0021-2172; ISSN 1565-8511
http://dx.doi.org/10.1007/BF02809889
http://www.springerlink.com/content/0021-2172/
http://www.ma.huji.ac.il/ ijmath/

This elegant paper extends work of *D. J. Rudolph* [Ergodic Theory Dyn. Syst. 10, No. 2, 395-406 (1990; Zbl 0709.28013)], *A. S. A. Johnson* [Isr. J. Math. 77, No. 1-2, 211-240 (1992; Zbl 0790.28012)] and *B. Host* [Isr. J. Math. 91, No. 1-3, 419-428 (1995; Zbl 0839.11030)] on the problem of measures on the circle invariant under two maps. The main result is the following: Under a technical condition on the sequence (a_i) depending on m, for any measure μ invariant, ergodic and of positive entropy under multiplication by m, the sequence $(a_i x)$ is Lebesgue-uniformly distributed for μ a.e. x. A special case of this is the case $a_i = r^i$ if m does not divide any power of r. The proofs use p-adic analogues of ideas from smooth dynamical systems.

Thomas Ward (Norwich)

Keywords : invariant measure; uniform distribution; p-adic dynamics

Classification :

∗28D05 Measure-preserving transformations
37A15 General groups of measure-preserving transformation
37A25 Ergodicity, mixing, rates of mixing

Zbl 0978.54026

Lindenstrauss, Elon; Weiss, Benjamin

Mean topological dimension. (English)

Isr. J. Math. 115, 1-24 (2000). ISSN 0021-2172; ISSN 1565-8511
http://dx.doi.org/10.1007/BF02810577
http://www.springerlink.com/content/0021-2172/

http://www.ma.huji.ac.il/ ijmath/

Given a continuous map f on a compact metric space X, an invariant called mean topological dimension has been recently introduced by M. Gromov. The paper being reviewed develops some basic properties of this invariant. In some cases, it gives a way of distinguishing dynamical systems with infinite topological entropy. Given a finite open cover α of X, its order is one less than the cardinality of any collection of subsets of α with nonempty intersection, and $\mathcal{D}(\alpha)$ is defined to be the minimum order of any finite cover of X that refines α. Under fairly general conditions on α, it is shown that if X has topological dimension at most k, then $\mathcal{D}(\alpha) \leq k$. If $T : X \to X$ is continuous, then the mean dimension of the system (X, T) is the supremum over all finite open covers α of $\lim_{n\to\infty}(\mathcal{D}(\alpha_n)/n)$, where α_n is the common join of the covers $T^{-i}(\alpha)$, $0 \leq i < n$. It follows that if X is finite dimensional, then this mean dimension is 0. Moreover, the mean dimension of the shift dynamical system generated by a k-dimensional, compact space of symbols is at most k. This is used to show that if a minimal system is embeddable in the shift space based on the symbols $0 \leq s \leq 1$, then the mean dimension of the system is at most 1. Examples of minimal systems of mean dimension greater than one are given. Finally, an alternative approach, reminiscent of one definition of topological entropy, is described; here there is explicit dependence upon the metric. It is shown that the new metric version of mean dimension is an upper bound for the original mean dimension. It follows from this that if the topological entropy is finite, then the mean dimension is 0.

Mike Hurley (Cleveland)

Keywords : mean dimension; topological entropy

Classification :

$*$54H20 Topological dynamics

37B40 Topological entropy

Zbl 0978.54027

Lindenstrauss, Elon

Mean dimension, small entropy factors and an embedding theorem. (English)

Publ. Math., Inst. Hautes Étud. Sci. 89, 227-262 (1999). ISSN 0073-8301; ISSN 1618-1913

http://dx.doi.org/10.1007/BF02698858

numdam:PMIHES_1999__89__227_0

http : //link.springer.de/link/service/journals/10240/

The paper under review is an important contribution to the emerging theory of mean dimension of topological dynamical systems. Let (X, T) be a topological dynamical system, where T is a self-homeomorphism of a compact space X. The mean dimension of (X, T) is a new invariant, suggested by Gromov and developed in [*E. Lindenstrauss* and *B. Weiss*, Isr. J. Math. 115, 1-24 (2000; Zbl 0978.54025), see above], aimed at distinguishing between dynamical systems with infinite entropy. Mean dimension is a

variant of the classical concept of Lebesgue covering dimension, taking into account dynamics.
For a finite open cover α, denote by $D(\alpha)$ the minimal order of a finite open cover β refining α. If in addition $n \in \mathbb{N}$, denote $\alpha^n = \alpha \vee T^{-1}\alpha \vee \ldots \vee T^{-n}\alpha$. Now the mean dimension of (X,T) is defined by

$$\operatorname{mdim}(X,T) = \sup_{\alpha} \lim_{n\to\infty} \tfrac{1}{n} D(\alpha^n),$$

where the supremum is taken over all finite open covers of X. Note that if $T = \mathrm{Id}_X$ and the factor $1/n$ is removed, then one obtains the Lebesgue covering dimension. The systems (X,T) with zero mean dimension include (and unify) systems with finite entropy, finite-dimensional phase space, and those admitting at most countably many ergodic invariant measures. However, the main interest of the emerging theory lies namely with systems of positive – or even infinite – mean dimension. (Notice in this connection that for each $t > 0$ there exists a system (X,T) with $\operatorname{mdim}(X,T) = t$.)
In a similar way, building on Bowen's definition of the topological entropy of (X,T), one can define the metric mean dimension of (X,T). This invariant, $\operatorname{mdim}_M(X,T)$, is the infimum, taken over all compatible metrics d on X, of the numbers

$$\operatorname{mdim}_M(X,T,d) := \lim_{\varepsilon\to 0} \frac{\limsup_{n\to\infty} \frac{1}{n} \log sp_n(\varepsilon)}{|\log \varepsilon|},$$

where $sp_n(\varepsilon)$ is the minimal cardinality of a subset $A \subseteq X$ with the property that for every $x \in X$ there is an $y \in A$ such that $\max_{0\le k<n} d(T^k x, T^k y) < \varepsilon$.
One of the first results of the theory asserts that always $\operatorname{mdim}(X,T) \le \operatorname{mdim}_M(X,T)$ (the paper by Lindenstrauss and Weiss [loc. cit.]), and that for systems admitting an infinite minimal factor the two dimensions coincide (the paper under review). The new concepts shed light on the important question of when is a system (X,T) embeddable into a symbolic dynamical system of the form $(([0,1]^d)^{\mathbb{Z}}, \sigma)$, where σ is a shift. Lindenstrauss and Weiss [loc. cit.] have previously shown that a necessary condition for the existence of such an embedding is that $\operatorname{mdim}(X,T) \le d$. The present paper establishes the following result in the converse direction: if (X,T) possesses an infinite minimal factor and $\operatorname{mdim}(X,T) < \frac{1}{36}d$, then (X,T) admits an embedding into the shift system $(([0,1]^d)^{\mathbb{Z}}, \sigma)$. (Moreover, such embeddings are generic in a suitably defined sense.) These results can be considered as providing (an approximation to) a dynamical analogue of the classical Menger-Nöbeling embedding theorem for finite-dimensional compacta.
Recall that for a set $E \subseteq X$ the orbit capacity of E, denoted by $\operatorname{ocap}(E)$, as defined by *M. Shub* and *B. Weiss* [Ergodic Theory Dyn. Syst. 11, No. 3, 535-546 (1991; Zbl 0773.54011)], is the limit, as $n \to \infty$, of the orders of families of sets formed by finite translates of E by the first n powers of T. Subsets E with zero orbit capacity are said to have the small boundary property. Again, it was shown by Lindenstrauss and Weiss [loc. cit.] that if the topology of X has a basis formed by sets with the small boundary property, then $\operatorname{mdim}(X,T) = 0$. The paper under review establishes the converse for those dynamical systems (X,T) admitting infinite minimal factors. These results are a perfect dynamical analogue of Uryson's classical result on the coincidence of the Lebesgue covering dimension and the small inductive dimension for compact metric

spaces.
The paper also contains a number of other results, such as, for instance, a dichotomy between systems with zero mean dimension and those with $\mathrm{mdim}(X, T) > 0$, the concept of the universal zero-mean dimensional factor of X, etc.

Vladimir Pestov (Wellington)

Keywords : topological entropy; unique ergodicity; symbolic dynamical systems

Classification :

∗54H20 Topological dynamics
37B40 Topological entropy
37C45 Dimension theory of dynamical systems

Zbl 0970.37011

Lindenstrauss, Elon

Measurable distal and topological distal systems. (English)

Ergodic Theory Dyn. Syst. 19, No.4, 1063-1076 (1999). ISSN 0143-3857; ISSN 1469-4417
http://dx.doi.org/10.1017/S0143385799133911
http://journals.cambridge.org/action/displayJournal?jid=ETSbVolume=y

The article is concerned with the investigation of the connection between measurable and topological distal systems. The main result is that every ergodic measurably distal system is isomorphic to a minimal topologically distal system equipped with a Borel measure with full support. Moreover, a partial generalization of a result due to Rudolph for cocycles of a measurable $\mathbb{Z}$-action on a compact metric space to a connected compact group is proved.

Azad Tagizade (Baku)

Keywords : distal system; connected metric group; isometric extension; cocycles

Classification :

∗37B05 Transformations and group actions with special properties

---

Zbl 0963.60042

Lindenstrauss, Elon

Indistinguishable sceneries. (English)

Random Struct. Algorithms 14, No.1, 71-86 (1999). ISSN 1042-9832; ISSN 1098-2418
http://dx.doi.org/10.1002/(SICI)1098-2418(1999010)14:1¡71::AID-RSA4¿3.0.CO;2-9

Keywords : random walk; sceneries; distinguishability; mutually singular measures

Classification :

∗60G50 Sums of independent random variables
82B44 Disordered systems

Zbl 0944.28014

Lindenstrauss, Elon

Pointwise theorems for amenable groups. (English)

Electron. Res. Announc. Am. Math. Soc. 5, No.12, 82-90 (1999). ISSN 1079-6762
http://dx.doi.org/10.1090/S1079-6762-99-00065-7
http://www.ams.org/journal-getitem?pii=S1079-6762-99-00065-7
http://www.ams.org/era/
http://www.emis.de/journals/ERA-AMS/

Summary: We describe proofs of the pointwise ergodic theorem and the Shannon-McMillan-Breiman theorem for discrete amenable groups, along Følner sequences that obey some restrictions. These restrictions are mild enough so that such sequences exist for all amenable groups.

Keywords : pointwise ergodic theorem; Shannon-MacMillan-Breiman theorem; discrete amenable groups; Følner sequences
Classification :
∗28D15 General groups of measure-preserving transformations
37A30 Ergodic theorems, spectral theory, Markov operators
37A45 Relations of ergodic theory with number theory and harmonic analysis
43A07 Means on groups, etc.
60F15 Strong limit theorems

Zbl 0940.28015

Lindenstrauss, Elon; Meiri, David; Peres, Yuval

Entropy of convolutions on the circle. (English)

Ann. Math. (2) 149, No.3, 871-904 (1999). ISSN 0003-486X; ISSN 1939-0980
http://dx.doi.org/10.2307/121075
http://www.math.princeton.edu/ annals/issues/1999/149_3.html
http : //annals.math.princeton.edu/annals/about/cover/cover.html
http : //pjm.math.berkeley.edu/annals/about/journal/about.html
http : //www.jstor.org/journals/0003486X.html

This paper investigates the entropy for convolutions of p-invariant measures on the circle and their ergodic components. In particular the following two theorems are proved:
Theorem 1: Let $\{\mu_i\}$ be a countably infinite sequence of p-invariant ergodic measures on the circle whose normalized base-p measures, $h_i = h(\mu_i, \sigma_p)/\log p$, satisfy $\sum h_i/|\log h_i| = \infty$. Then $h(\mu_1 * \cdots * \mu_n, \sigma_p)$ tends to $\log p$ monotonically as n tends to ∞. In particular $\mu_1 * \cdots * \mu_n$ tends to λ weak*.
Theorem 2: Let $\{\mu_i\}$ be a countably infinite sequence of p-invariant ergodic measures on the circle whose normalized base-p measures satisfy $h(\mu_i, \sigma_p) > 0$. Suppose that $\mu^\wedge$ is a joining of full entropy of $\{\mu_i\}$. Define $\Theta^n : \mathbb{T}^{\mathbb{N}} \to \mathbb{T}$ by $\Theta^n(x) = x_1 + \cdots + x_n$ (mod 1). Then $h(\Theta^n\mu^\wedge, \sigma_p)$ tends to $\log p$ monotonically as n tends to ∞.

Robert Cowen (Gaborone)

Keywords : Furstenberg's conjecture; entropy for convolutions; ergodic measures; joining
Classification :
∗28D20 Entropy and other measure-theoretic invariants
37A35 Invariants of ergodic theory

Zbl 0849.54031

Lindenstraussss, Elon

Lowering topological entropy. (English)

J. Anal. Math. 67, 231-267 (1995). ISSN 0021-7670; ISSN 1565-8538
http://dx.doi.org/10.1007/BF02787792
http://www.springerlink.com/content/120600/

The author considers the topological entropy of (topological) factors of a given dynamical system (X,T) with positive entropy h. He shows that if X is a finite-dimensional space (i.e. a space of finite topological dimension), then there exist factors with all values of entropy between 0 and h. Further, in this situation, he is able to show that given one factor (Y',S) of (X,T), for each value of the entropy between $h_{\text{top}}(S)$ and h, there is a factor (Y,S) of (X,T) such that (Y',S') is a factor of (Y,S) and the factor maps satisfy the equation $\varphi_{Y',Y}\circ\varphi_{Y,X}=\varphi_{Y',X}$. He shows that these results fail in infinite-dimensional systems by exhibiting an infinite-dimensional system which is minimal and of infinite topological entropy (and so has no proper factors and in particular, none of entropy between 0 and ∞).

A.Quas (Cambridge)

Keywords : topological factor; topological entropy
Classification :
∗54H20 Topological dynamics
54C70 Topological entropy