Zbl 1184.42010

Matei, Basarab; Meyer, Yves

A variant of compressed sensing. (English)

Rev. Mat. Iberoam. 25, No. 2, 669-692 (2009). ISSN 0213-2230
http://projecteuclid.org/euclid.rmi/1255440070
http://projecteuclid.org/rmi
http://www.uam.es/departamentos/ciencias/matematicas/ibero/irevista.htm

Given a bounded set K in $\mathbb{R}^n$, denote by L^2_K the set of all L^2-functions which vanish a.e. outside K. Let $\Lambda \subset \mathbb{R}^n$ be a uniformly discrete set. The following two problems are classical:

Sampling Problem: When can every function $F \in L^2_K$ be reconstructed in a stable way from the values of its Fourier transform $\hat{F}$ on Λ? If this is possible, Λ is called a set of sampling for L^2_K.

Interpolation Problem: When does there exist for every sequence $\{c(\lambda)\} \in l^2(\Lambda)$ some $F \in L^2_K$ such that $\hat{F}(\lambda) = c(\lambda), \lambda \in \Lambda$? If this is possible, Λ is called a set of interpolation for L^2_K.

When $K = [a, b] \subset \mathbb{R}$ is a single interval, Beurling and Kahane established that the problems can be essentially solved in terms of appropriate densities of Λ: (i) If $D^-(\Lambda) > b - a$ then Λ is a sampling set for $L^2_{[a,b]}$. If $D^-(\Lambda) < b - a$, then it is not. (ii) If $D^+(\Lambda) < b - a$ then Λ is a set of interpolation for $L^2_{[a,b]}$. If $D^+(\Lambda) > b - a$, then it is not. Here $D^\pm$ are the the upper and the lower uniform densities, respectively. *H. J. Landau* [Acta Math. 117 Acta Math. 117, 37–52 (1967; Zbl 0154.15301)] discovered that the necessary conditions for sampling in (i) and interpolation in (ii) still hold in the most general situation. However, finding sufficient conditions for sampling and interpolation is a hard problem, since in general such conditions cannot be obtained in terms of densities of Λ. This is where the geometric structure of Λ comes into play.

A. Olevskii and *A. Ulanovskii* [C. R., Math., Acad. Sci. Paris 342, No. 12, 927–931 (2006; Zbl 1096.94017) and Geom. Funct. Anal. 18, No. 3, 1029–1052 (2008; Zbl 1169.42014)] put forward the following

Universality problem: Does there exist a set Λ which provides sampling/interpolation for every space L^2_K of sufficiently small/large measure?

They proved that in general the answer is negative. However under some topological restrictions on K it is positive: They constructed a universal set Λ of given positive uniform density which provide both sampling for every compact set K in $\mathbb{R}$ satsfying meas$(K) < D(\Lambda)$ and interpolation for every open set K such that meas$(K) > D(\Lambda)$. The density of these sets Λ is optimal, as follows from Landau's result.

The authors of the present paper suggest another very elegant construction of universal sampling/interpolation sets Λ of optimal density for spaces L^2_K, where K is every compact/open subset of the unite square $[0, 1]^2$. These sets Λ belong to $\mathbb{Z}^2$ and can be written explicitly. The method can be traced back to [*Y. Meyer*, Nombres de Pisot, nombres de Salem et analyse harmonique. Lecture Notes in Mathematics. 117. Berlin-Heidelberg-New York: Springer-Verlag (1970; Zbl 0189.14301)]. The authors also present a result on reconstruction of positive functions and its stability and discuss sampling/interpolation in some other classical spaces of function and distributions on $[0,1]^2$. All the results are presented in the two-dimensional setting, but the same method works in any number of dimensions. The case where $K, \Lambda \subset \mathbb{R}^n$, is treated by the authors in [C. R., Math., Acad. Sci. Paris 346, No. 23–24, 1235–1238 (2008; Zbl 1154.42006)].

Alexander Ulanovskii (Stavanger)

Keywords : Fourier expansions; compressed sensing; irregular sampling; uniform sampling and interpolation

Classification :

∗42B10 Fourier type transforms, several variables
42B99 Fourier analysis in several variables
42C30 Completeness of sets of functions of non-trigonom. Fourier analysis

---

Zbl 1172.01011

Kahane, Jean-Pierre (Baker, Roger C. (ed.); Meyer, Yves)

Selected works. Edited by Roger C. Baker. Preface by Yves Meyer. (English)

Heber City, UT: Kendrick Press. xiv, 688 p. $ 130.00 (2009). ISBN 0-9793183-9-4/hbk

This is a collection of 63 selected papers of a living mathematician, who is one of the most influential analysts of his generation. His interests range from Fourier analysis to Brownian motion and multiplicative processes. As a student at the Ecole Normale Supérieure from 1946 to 1949 he was influenced by R. Salem with whom he wrote many papers which are not in this selection, because published in Salem's collected works. Kahane completed his doctorate under S. Mandelbrojt in 1954, was professor in Montpellier 1954-1961 and since in Paris. He was elected a full member of the French academy of sciences in 1998. During his career he assumed responsibility for mathematics also in positions such as the President of the International Commission on Mathematical Instruction (1982-1990).

Reinhard Siegmund-Schultze (Kristiansand)

Keywords : Jean Pierre Kahane (born 1926); Fourier analysis; harmonic analysis

Classification :

∗01A65 Development of contemporary mathematics
01A75 Collected or selected works
42-03 Historical (Fourier analysis)
43-03 Historical (abstract harmonic analysis)

Zbl 1158.43002

Deng, Donggao; Han, Yongsheng (Meyer, Yves)

Harmonic analysis on spaces of homogeneous type. With a preface by Yves Meyer. (English)

Lecture Notes in Mathematics 1966. Berlin: Springer. xii, 154 p. EUR 29.95/net; SFR 50.00; $ 44.95; £ 23.99 (2009). ISBN 978-3-540-88744-7/pbk

http://dx.doi.org/10.1007/978-3-540-88745-4

The book reflects recent trends in modern harmonic analysis on spaces of homogeneous type. The main topics are: Calderón-Zygmund operators on spaces of homogeneous type, the boundedness of Calderón-Zygmund operators on wavelet spaces, wavelet expansions on spaces of homogeneous type, wavelets and spaces of functions and distributions, Littlewood-Paley analysis on non-homogeneous spaces. An excellent preface by Yves Meyer is worth being read by every analyst.

Boris Rubin (Baton Rouge)

Keywords : spaces of homogeneous type; Calderón-Zygmund operators; Littlewood-Paley analysis; wavelet expansions; non-homogeneous spaces

Classification :

- ∗43-02 Research monographs (abstract harmonic analysis)
- 22-02 Research monographs (topological groups)

Zbl 1154.42006

Matei, Basarab; Meyer, Yves

Quasicrystals are sets of stable sampling. (English. Abridged French version)

C. R., Math., Acad. Sci. Paris 346, No. 23-24, 1235-1238 (2008). ISSN 1631-073X
http://dx.doi.org/10.1016/j.crma.2008.10.006
http://www.sciencedirect.com/science/journal/1631073X

Summary: Irregular sampling and “stable sampling” of band-limited functions have been studied by *H.J. Landau* [Acta Math. 117, 37–52 (1967; Zbl 0154.15301)]. We prove that quasicrystals are sets of stable sampling.

Classification :

- ∗42C15 Series and expansions in general function systems
- 94A20 Sampling theory

Zbl pre05316266

Meyer, Y.; Yang, Q.X.

Continuity of Calderón-Zygmund operators on Besov or Triebel-Lizorkin spaces. (English)

Anal. Appl., Singap. 6, No. 1, 51-81 (2008). ISSN 0219-5305
http://dx.doi.org/10.1142/S0219530508001055
http://www.worldscinet.com/aa/aa.shtml

Classification :

- ∗42B20 Singular integrals, several variables
- 42B30 Hp-spaces (Fourier analysis)

Zbl 1133.93300

Meyer, Yves

Signal processing. (Traitement du signal.) (French)

Gaz. Math., Soc. Math. Fr. 112, 28-30 (2007). ISSN 0224-8999
http://smf.emath.fr/en/Publications/Gazette/

Summary: Les découvertes faites par Emmanuel Candès et Terence Tao en traitement du signal et de l'image s'inscrivent dans la problématique des problèmes inverses mal posés. Il s'agit de reconstruire un objet inconnu alors qu'on ne dispose à son sujet que de quelques informations fragmentaires. Fournir dans ces conditions un résultat exact, comme si l'on disposait d'une information complète, est impossible et l'on doit pallier l'information manquante par une hypothèse a priori. Pour que la démarche proposée soit intéressante, il convient que cette connaissance a priori soit suffisamment souple et n'exclue aucun des signaux ou des images que l'on cherche à reconstruire.

Classification :

∗93-03 Historical (systems and control)
94-03 Historical (information and communication)

Zbl 1118.68176

Garnett, John B.; Le, Triet M.; Meyer, Yves; Vese, Luminita A.

Image decompositions using bounded variation and generalized homogeneous Besov spaces. (English)

Appl. Comput. Harmon. Anal. 23, No. 1, 25-56 (2007). ISSN 1063-5203
http://dx.doi.org/10.1016/j.acha.2007.01.005
http://www.sciencedirect.com/science/journal/10635203

The paper is devoted to the decomposition of an image into a piecewise-smooth component and an oscillatory component, in a variational approach. [*Y. Meyer*, Oscillating patterns in image processing and nonlinear evolution equations. The fifteenth Dean Jacqueline B. Lewis memorial lectures. University Lecture Series. 22. Providence, RI: American Mathematical Society (AMS). (2001; Zbl 0987.35003)] proposed refinements of the total variation model [*L. I. Rudin, S. Osher, E. Fatemi*, Physica D 60, No. 1–4, 259–268 (1992; Zbl 0780.49028)] that better represent the oscillatory part: the weaker spaces of generalized functions $G = div(L^\infty)$, $F = \mathrm{div}(\mathrm{BMO})$ and $E = \dot{B}^{-1}_{\infty,\infty}$ have been proposed to model the oscillatory part, instead of the standard L^2 space, while keeping the piecewise-smooth part in BV, a function of bounded variation. Such new models separate better geometric structures from oscillatory structures, but it is difficult to realize them in practice. In the paper a generalization of Meyer's (BV, E) model is proposed, using homogeneous Besov spaces $\dot{B}^{\alpha}_{p,q}$, $-2 < \alpha < 0$, $1 \le p, q \le \infty$ to represent the oscillatory part. Theoretical, experimental results and comparisons to validate the proposed methods are presented.

Agnieszka Lisowska (Sosnowiec)

Keywords : image denoising; Besov spaces; bounded variation functions
Classification :
∗68U10 Image processing
94A08 Image processing
49J10 Free problems in several independent variables (existence)

Zbl 1117.42001

Chuong, N.M. (ed.); **Egorov, Yu.V. (ed.)**; **Khrennikov, A. (ed.)**; **Meyer, Y. (ed.)**; **Mumford, D. (ed.)** (Nguyen Minh Chuong (ed.))

Harmonic, wavelet and p-adic analysis. Based on the summer school, Quy Nhon, Vietnam, June 10–15, 2005. (English)

Hackensack, NJ: World Scientific. ix, 381 p. $ 108.00; £ 59.00 (2007). ISBN 978-981-270-549-5/hbk

Contents: Part A: Wavelet and Harmonic Analysis – Chapter I: Wavelet and Expectations
Karl Gustafson, Wavelets and expectations: a different path to wavelets (5–22); Xiaoyan Liu, Construction of univariate and bivariate exponential splines (23–36); Peter R. Massopust, Multiwavelets: some approximation-theoretic properties, sampling on the interval, and translation invariance (37–57); Reinhold Schneider and Toralf Weber, Multiscale approximation schemes in electronic structure calculation (59–81); Elena Cordero, Luigi Rodino and Karlheinz Gröchenig, Localization operators and time-frequency analysis (83–110).
Part A: Wavelet and Harmonic Analysis – Chapter II: Harmonic Analysis
Yu. V. Egorov and Yavdat Il'yasov, On multiple solutions for elliptic boundary value problem with two critical exponents (113–139); Yavdat Il'yasov, On calculation of the bifurcations by the fibering approach (141–155); Bui An Ton, On a free boundary transmission problem for nonhomogeneous fluids (157–174); Vu Kim Tuan and Amin Boumenir, Sampling in Paley-Wiener and Hardy spaces (175–209); Do Ngoc Diep, Quantized algebras of functions on affine Hecke algebras (211–227); Vo Van Tan, On the C-analytic geometry of q-convex spaces (229–265).
Part B: P-adic and Stochastic Analysis – Chapter III: Over p-adic Field
Nguyen Minh Chuong, Nguyen Van Co and Le Quang Thuan, Harmonic analysis over p-adic field. I: Some equations and singular integral operators (271–290); Andrei Khrennikov, p-adic and group valued probabilities (291–309).
Part B: P-adic and Stochastic Analysis – Chapter IV: Archimedean Stochastic Analysis
Takeyuki Hida, Infinite dimensional harmonic analysis from the viewpoint of white noise theory (313–330); Shigeyoshi Ogawa, Stochastic integral equations of Fredholm type (331–342); Situ Rong [Rong Situ], BSDEs with jumps and with quadratic growth coefficients and optimal consumption (343–361); Arturo Kohatsu-Higa and Makoto Yamazato, Insider problems for markets driven by Lévy processes (363–381).
The articles of this volume will be reviewed individually.

Classification :
∗42-06 Proceedings of conferences (Fourier analysis)
43-06 Proceedings of conferences (abstract harmonic analysis)

00B25 Proceedings of conferences of miscellaneous specific interest
42C40 Wavelets
35-06 Proceedings of conferences (partial differential equations)
60-06 Proceedings of conferences (probability theory)

Zbl 1113.49037

Haddad, A.; Meyer, Y.

An improvement of Rudin–Osher–Fatemi model. (English)

Appl. Comput. Harmon. Anal. 22, No. 3, 319-334 (2007). ISSN 1063-5203
http://dx.doi.org/10.1016/j.acha.2006.09.001
http://www.sciencedirect.com/science/journal/10635203

Summary: We investigate some mathematical properties of a new algorithm proposed by Meyer [*Y. Meyer*, Oscillating patterns in image processing and in some nonlinear evolution equations, The Fifteenth Dean Jacqueline B. Lewis Memorial Lectures, University Lectures Series, vol. 22, Amer. Math. Soc., Providence, RI, 2001] to improve the Rudin–Osher–Fatemi model (ROF) [*L. Rudin, S. Osher* and *E. Fatemi*, Nonlinear total variation based noise removal algorithms, Physica D 60, 259–268 (1992; Zbl 0780.49028)] in order to separate objects and textures contained in an image. He pointed out the crucial role played by a certain norm called the G-norm or "dual norm," denoted $\| \|_*$, and the main drawback for the ROF model: any image is considered to have a textured component. We are then interested in minimizing the functional $\|u\|_{BV} + \lambda\|v\|_*$. The main Theorem 6.1 is about invariance and stability properties of the new algorithm. It was first implemented by Osher and Vese [*L. Vese, and S.J. Osher, Modeling textures with total variation minimization and oscillating patterns in image processing, J. Sci. Comp. 19, No. 1–3, 553–572 (2003; Zbl 1034.49039)]. In particular, we point out the role played by particular functions called extremal functions and characterize them.*

Keywords : function of bounded variations; textures; oscillating patterns; extremal functions; ROF model; BV-G model; stability

Classification :
∗ *49M30 Methods of successive approximation, not based on necessary cond.*
94A08 Image processing
68U10 Image processing

Zbl 1172.01309

Meyer, Yves

How can surfaces be measured? *(French)*

Gaz. Math., Soc. Math. Fr. 109, 23-36 (2006). ISSN 0224-8999
http://smf.emath.fr/en/Publications/Gazette/

The paper is devoted to the modern history of the answers to the title question. It starts from Archimedes. Then the theory of Lebesgue is presented. Lebesgue posed also the

problem: how to measure surfaces of 3-dimensional objects? The main part of the paper is devoted to the presentation and discussion of solutions of this problem proposed by Hausdorff, Besicovitch, Tonelli and De Giorgi.

Roman Murawski (Poznań)
Keywords : Hausdorff, Besicovitch; Tonelli; De Giorgi.
Classification :
∗ *01A60 Mathematics in the 20th century*

Zbl 1142.46014

Bourdaud, Gérard; Meyer, Yves
Sublinear functional calculus in homogeneous Besov spaces. *(French)*
Rev. Mat. Iberoam. 22, No. 2, 725-746 (2006). ISSN 0213-2230
http://projecteuclid.org/euclid.rmi/1161871354
http://projecteuclid.org/rmi
http://www.uam.es/departamentos/ciencias/matematicas/ibero/irevista.htm

The first author and several other collaborators, including the second author, have proven the existence of a sublinear functional calculus in some fractional Sobolev spaces [see for instance G. Bourdaud and M. E. D. Kateb, Math. Z. 210, No. 4, 607–613 (1992; Zbl 0759.46032)]. All these interesting results could be summarized as follows: Let $1 \leq p \leq \infty$, $0 < s < 1 + (1/p)$, *and* $f \in BH(\mathbb{R})$ *such that* $f(0) = 0$, *where* $BH(\mathbb{R})$ *is the set of continuous functions whose second derivative, in the sense of distributions, is a bounded measure. There exists a constant* $c > 0$, *depending on* s, p, *and* n, *such that*

$$\|f \circ g\|_{W_p^s(\mathbb{R}^n)} \leq c\|f\|_{BH}\|g\|_{W_p^s(\mathbb{R}^n)}, \quad \forall g \in W_p^s(\mathbb{R}^n).$$

Here, $W_p^s(\mathbb{R}^n)$ *could be either the Besov space* $B_p^{s,q}(\mathbb{R}^n)$ *or the Lizorkin-Triebel space* $F_p^{s,q}(\mathbb{R}^n)$, $1 \leq q \leq \infty$.
In this paper, the authors prove that the preceding result also holds in homogeneous Besov spaces, $\dot{B}_p^{s,q}(\mathbb{R}^n)$, *where* $1 \leq p < \infty$, $q \in [1, +\infty]$, $0 < s < 1 + (1/p)$ *and where the function* f *belongs to* $U_p^1(\mathbb{R})$, *a larger space than* $BH(\mathbb{R})$. *As the authors point out these homogeneous spaces are of great interest because first, the usual Besov spaces are subspaces of the homogeneous Besov spaces, and second, they are preferred for the applications of Besov spaces in Fluid Dynamics and Image Processing.*
An application to Image Processing is also given in the last section. More precisely, the authors show that a step function belongs to the homogeneous Besov space $\dot{B}_1^{1,\infty}(\mathbb{R}^2)$ *if and only if it belongs to* $BV(\mathbb{R}^2)$, *the space of functions of bounded variation.*

Fabrice Colin (Sudbury)
Keywords : homogeneous Besov spaces; composition operators; bounded variation
Classification :
∗ *46E35 Sobolev spaces and generalizations*
47H30 Particular nonlinear operators

Zbl 1100.42031

Meyer, Yves

From wavelets to atoms. *(English)*

Jensen, Gary R. (ed.) et al., 150 years of mathematics at Washington University in St. Louis. Sesquicentennial of mathematics at Washington University, St. Louis, MO, USA, October 3–5, 2003. Providence, RI: American Mathematical Society (AMS). Contemporary Mathematics 395, 105-117 (2006). ISBN 0-8218-3603-X/pbk

The paper is an extended version of a talk given at the conference "150 years of Mathematics at Washington University". It gives a historical view on the development of wavelet analysis, starting with work by Wigner around 1930. The focus is on the relationship between wavelet bases and unconditional bases in Banach spaces, mainly Hardy spaces.

Ole Christensen (Lyngby)

Keywords : wavelets; atoms; atomic decomposition; unconditional basis; Hardy space

Classification :

- ∗*42C40 Wavelets*
- *42-03 Historical (Fourier analysis)*
- *01A60 Mathematics in the 20th century*
- *01A61 21st century*

Zbl pre05551757

Meyer, Yves

Oscillating patterns in some nonlinear evolution equations. *(English)*

Cannone, Marco (ed.) et al., Mathematical foundation of turbulent viscous flows. Lectures given at the C.I.M.E. summer school held in Martina Franca, Italy, September 1–5, 2003. Berlin: Springer. Lecture Notes in Mathematics 1871, 101-187 (2006). ISBN 3-540-28586-5/pbk

http://dx.doi.org/10.1007/11545989_4

Summary: My interest in Navier-Stokes equations arose from the wavelet revolution. I was puzzled by (1) a series of talks and preprints by Marie Farge and (2) an intriguing paper by Paul Federbush entitled 'Navier and Stokes meet the wavelets'. Both Marie Farge and Paul Federbush were convinced that wavelets could play an important role in fluid dynamics.

Classification :

- ∗*76D03 Existence, uniqueness, and regularity theory*
- *35Q30 Stokes and Navier-Stokes equations*

Zbl 1103.94302

Meyer, Yves

Compression of fixed images. (Compression des images fixes.) *(French)*

Gaz. Math., Soc. Math. Fr. 103, 9-23 (2005). ISSN 0224-8999
http://smf.emath.fr/en/Publications/Gazette/

Keywords : JPEG2000,wavelet compression

Classification :

- ∗ *94A08 Image processing*
- *68U10 Image processing*
- *65T60 Wavelets*

Zbl 1087.37057

Wesfreid, Eva; Billat, Véronique L.; Meyer, Yves

Multifractal analysis of heartbeat time series in human races. *(English)*

Appl. Comput. Harmon. Anal. 18, No. 3, 329-335 (2005). ISSN 1063-5203
http://dx.doi.org/10.1016/j.acha.2004.12.005
http://www.sciencedirect.com/science/journal/10635203

Summary: We present a multifractal analysis of heartbeat and heart rate time series in human races. In order to improve the training of athletes, we compare heart rate multifractal behavior in free and constant speed 10,000 m runnings. We analyze also marathon races, free pace 42.195 km running, we compare the first and the second half heartbeat signals to measure the effect of fatigue. We find that freedom for choosing the own pace variation could be the racing condition for keeping good health conditions in an exhausting exercise.

Keywords : Multifractal analysis; Wavelets; Heartbeat signals

Classification :

- ∗ *37N25 Dynamical systems in biology*
- *92C50 Medical appl. of mathematical biology*
- *37M10 Time series analysis*
- *42C40 Wavelets*

Zbl 1089.01522

Meyer, Yves

Jean Leray and the search for the truth. (Jean Leray et la recherche de la vérité.) *(French)*

Guillopé, L. (ed.) et al., Proceedings of the colloquium dedicated to the memory of Jean Leray, Nantes, France, June 17–18, 2002. Paris: Société Mathématique de France. Séminaires et Congrès 9, 1-12 (2004). ISBN 2-85629-160-0/pbk

Keywords : Biography

Classification :

- ∗ *01A70 Biographies, obituaries, personalia, bibliographies*
- *76-03 Historical (fluid mechanics)*
- *35-03 Historical (partial differential equations)*

Zbl 1127.35317

Meyer, Yves; Rivière, Tristan

A partial regularity result for a class of stationary Yang-Mills fields in high dimension. *(English)*

Rev. Mat. Iberoam. 19, No. 1, 195-219 (2003). ISSN 0213-2230
http://projecteuclid.org/euclid.rmi/1049123085
http://projecteuclid.org/rmi
http://www.uam.es/departamentos/ciencias/matematicas/ibero/irevista.htm

Summary: We prove, for arbitrary dimension of the base $n \geq 4$, that stationary Yang-Mills fields satisfying some approximability property are regular apart from a closed subset of the base having zero $(n-4)$-Hausdorff measure.

Keywords : Yang-Mills Fields; Gauge Theory; Nonlinear elliptic regularity theory; Interpolation inequalities; Morrey Spaces

Classification :

∗ *35D10 Regularity of generalized solutions of PDE*
35J60 Nonlinear elliptic equations
35Q40 PDE from quantum mechanics
53C07 Special connections and metrics on vector bundles
58E15 Appl. of variational methods to extremal problems in sev.variables

---

Zbl pre02134856

Meyer, Yves

Kato's conjecture [after Pascal Auscher, Steve Hofmann, Michael Lacey, John Lewis, Alan McIntosh and Philippe Tchamitchian]. (La conjecture de Kato [d'après Pascal Auscher, Steve Hofmann, Michael Lacey, John Lewis, Alan McIntosh et Philippe Tchamitchian].) *(French)*

Bourbaki seminar. Volume 2001/2002. Exposes 894–908. Paris: Société Mathématique de France. Astérisque 290, 193-206, Exp. No. 902 (2003). ISBN 2-85629-149-X/pbk
numdam:SB_2001-2002__44__193_0

Classification :

∗ *35J15 Second order elliptic equations, general*
47B44 Accretive operators, etc. (linear)
47F05 Partial differential operators

Zbl 1080.35063

Brandolese, Lorenzo; Meyer, Yves

On the instantaneous spreading for the Navier-Stokes system in the whole space. *(English)*

ESAIM, Control Optim. Calc. Var. 8, 273-285 (2002). ISSN 1292-8119; ISSN 1262-3377
http://dx.doi.org/10.1051/cocv:2002021
numdam:COCV_2002__8__273_0
http://www.edpsciences.org/journal/index.cfm?edpsname=cocv

Summary: We consider the spatial behavior of the velocity field $u(x,t)$ of a fluid filling the whole space $\mathbb{R}^n$ $(n \geq 2)$ for arbitrarily small values of the time variable. We improve previous results on the spatial spreading by deducing the necessary conditions $\int u_h(x,t)u_k(x,t)\,dx = c(t)\delta_{h,k}$ under more general assumptions on the localization of u. We also give some new examples of solutions which have a stronger spatial localization than in the generic case.

Keywords : Navier-Stokes equations; space-decay; symmetries
Classification :
- ∗ *35Q30 Stokes and Navier-Stokes equations*
- *35B40 Asymptotic behavior of solutions of PDE*
- *76D05 Navier-Stokes equations (fluid dynamics)*
- *76D03 Existence, uniqueness, and regularity theory*

Zbl 1032.35002

Meyer, Yves

The role of oscillations in some nonlinear problems. *(English)*

Casacuberta, Carles (ed.) et al., 3rd European congress of mathematics (ECM), Barcelona, Spain, July 10-14, 2000. Volume I. Basel: Birkhäuser. Prog. Math. 201, 75-99 (2001). ISBN 3-7643-6417-3/hbk; ISBN 3-7643-6419-X/set

The author surveys recent progress concerning wavelet decompositions of function spaces. The new methods can be used to address problems from image compression and nonlinear PDEs in remarkably similar ways. One difficult aspect of image compression is how to distinguish 'features' from 'texture' from 'noise'. To do this, one expresses the image f as $f = u + v$ in which u encapsulates the features and v encapsulates the remainder. One characterizes u as a superposition of certain 'atomic' features quantified in terms of their norm in the space BV of functions of bounded variation. The ability of wavelets to pick out u from f, then, is quantified in terms of a theorem stating that if $f \in BV(\mathbb{R}^2)$ then the wavelet coefficients of f define a sequence in weak-ℓ^1. Meyer refers to this fact as the "first outstanding application of wavelet techniques inside mathematics." Meyer discusses several spaces that contain BV and have similar wavelet characterizations as candidates for quantifying $v = f - u$. However, those spaces all fail to distinguish 'texture' from 'noise'. Finally, it is implied that a certain geometric mean of BV and the space B of distributions that contain no large 'bumps' provides a good measure for distinguishing texture from noise.
In the second part of the paper Meyer argues that similar thinking leads to invention of norms in which solutions of certain nonlinear PDE – including the nonlinear heat equation and Navier-Stokes equations – do or should remain bounded for all time. The

principle at work here is that "blowup does not happen when the initial condition is oscillating". The aforementioned 'bumps' are not oscillatory.

Joseph Lakey (Las Cruces)

Keywords : wavelet decompositions; nonlinear PDE; bounded variation; nonliner heat equation; Navier-Stokes equations
Classification :
- ∗*35-02* *Research monographs (partial differential equations)*
- *42C40* *Wavelets*
- *35B05* *General behavior of solutions of PDE*
- *35K55* *Nonlinear parabolic equations*
- *94A08* *Image processing*

Zbl 1030.42027

Coifman, R.; **Geshwind, F.**; **Meyer, Y.**

Noiselets. *(English)*

Appl. Comput. Harmon. Anal. 10, No.1, 27-44 (2001). ISSN 1063-5203
http://dx.doi.org/10.1006/acha.2000.0313
http://www.sciencedirect.com/science/journal/10635203

Noiselets are functions which are totally uncompressible by orthogonal wavelet packet methods. In this paper the authors construct complex-valued noiselets for which all Haar-Walsh wavelet packet coefficients have the same modulus (hence totally uncompressible by Haar-Walsh wavelet packets). Two methods starting with the Haar function are used for the constructions. One is the wavelet packet recursive procedure (with complex coefficients) related to a complexification of the Thue-Morse sequence. The other is carried out with a complexification of the Rudin-Shapiro sequence. Both methods (and their combinations) provide noiselets which give bases for spaces of the Haar multiresolution analysis. They also have fast computational structure, hence will be useful for some applications.

Ding-Xuan Zhou (Hong Kong)

Keywords : noiselets; wavelet packet; Haar multiresolution analysis
Classification :
- ∗*42C40* *Wavelets*

Zbl 0987.35003

Meyer, Yves

Oscillating patterns in image processing and nonlinear evolution equations. The fifteenth Dean Jacqueline B. Lewis memorial lectures. *(English)*

University Lecture Series. 22. Providence, RI: American Mathematical Society (AMS). x, 122 p. £ 25.00 (2001). ISBN 0-8218-2920-3/pbk

Three seemingly unrelated scientific problems are beautifully tied together via the "oscillating patterns" that are within the structure of the problems.

The first problem constitutes the contents of Chapter I. It studies image processing and more precisely algorithms of image compression and denoising. It enters into the problem with a wonderful discussion of wavelet analysis and still image compression. Research sources are indicated to further study the applications. An application indicated is to further study the FDA (Federal Drug Administration) approval given for wavelet implementation for medical devices. Also the FBI (Federal Bureau of Investigation) adopted a wavelet transform technique for digitized finger print records and finally a discussion on the JPEG-2000 standard for still image compression. It then gives a very clear and concise treatment for the wavelet techniques. It offers the reader a wonderful connection between several standard models and their interrelation to wavelets.
The second chapter addresses the role of oscillations in some nonlinear partial differential equations. Again wavelet-based methods are applied to the celebrated Gagliardo-Nirenberg as well as the Poincaré inequalities. The results implementing wavelet techniques cannot be obtained by standard conventional techniques. The nonlinear heat equation, the Navier-Stokes equations as well as the nonlinear Schrödinger equation are then analyzed with these new mathematical results.
The third chapter is devoted to frequency-modulated signals, also named "chirps" or "Doppler". Some nice results surrounding tempered distributions are included in this chapter.
The book is a joy to read and contains so much information where wavelets are presently being used as well as a resource on technical mathematical techniques.

J.Schmeelk (Richmond)
Keywords : image compression and denoising; wavelet techniques; role of oscillations in some nonlinear partial differential equations; frequency-modulated signals; tempered distributions
Classification :
- ∗35-02 Research monographs (partial differential equations)
- 43-02 Research monographs (abstract harmonic analysis)
- 35B05 General behavior of solutions of PDE
- 35Q30 Stokes and Navier-Stokes equations
- 65T60 Wavelets
- 76D05 Navier-Stokes equations (fluid dynamics)
- 94A08 Image processing
- 68U10 Image processing

Zbl 0970.42020

Jaffard, Stéphane; Meyer, Yves; Ryan, Robert D.
Wavelets. Tools for science and technology. *(English)*
Philadelphia, PA: SIAM. xiii, 256 p. £ 62.00 (2001). ISBN 0-89871-448-6/hbk

This book is an updated and extended version of [Y. Meyer, "Wavelets: algorithms and applications" (1995; Zbl 0821.42018)]. The new book contains extra chapters on wavelets and turbulence, wavelets and fractals, data compression, and wavelets in astronomy. Rather than beeing a mathematical introduction to wavelets, the book exhibits the connection between wavelets and several topics in applied science. It is a very good

source of information for as well applied scientists as mathematicians who want to know about applications of wavelet theory. Furthermore, the books gives a nice historical presentation of ideas from the last century which eventually lead to wavelets.
Chapter 1 is an introduction to the subjects that are developed in the later chapters. The basic concepts (signals, wavelets, representation of signals, optimal representation) are introduced in an easy understandable way, and analogies are drawn whenever possible. By example, it is made clear that "optimal representation" depends on the information one is interested in. Chapter 2 is devoted to the historical perspective. Most sources only mention Fourier and Haar as the ancestors of wavelets, but here the developement is connected with, e.g., Brownian motions, Littlewood-Paley theory, and the work by Lusin and Franklin. Chapter 3 is about quadrature mirror filters and its close relationship to Daubechies' pioneering work. Chapter 4 discusses pyramid algorithms and their applications to image processing. A short introduction to multiresolution analysis and biorthogonal wavelets is included here. Chapter 5 deals with time-frequency analysis. An example explains why the Fourier transform is not the optimal tool for analyzing music (the time-localization of the notes is lost). Time-frequency atoms (corresponding to dilations, modulations, and translates of a single function g) of the type $W_{h,w,t_0}(t) = h^{-1/2}e^{-iwt}g(\frac{t-t_0}{h})$ *are introduced. The special case* $h = 1$ *corresponds to a Gabor system, while the case* $w = 0$ *gives a wavelet system. The Wigner-Ville transform is defined and related to Mallat's matching pursuit algorithm and pseudodifferential calculus. Chapter 6 concerns Malvar-Wilson bases consisting of translated versions of a window multiplied with certain sine and cosine functions. Bases of this type are for instance used to overcome the problems caused by Balian-Low's theorem. An algorithm for finding the optimal Malvar-Wilson basis is described. This is strongly related to Chapter 7 about wavelet packets. Chapter 8 describes work by Marr, in particular his conjecture about zero-crossings (a mathematical description of an image with sudden intensity changes) and the more precise version by Mallat. Counterexamples are given in Chapter 8 and Appendix C. Chapter 9 is about turbulence and the need for simultaneous space and scale representations, making wavelets an obvious tool. Chapter 10 (about wavelets and multifractal functions) gives an interesting proof of the non-differentiability of the Weierstrass function which highlights the importance of selecting the right wavelet, and analyses Riemann's function. It is also shown how to find the Hölder exponents of a multifractal function using wavelet analysis. Chapter 11 defines sparse wavelet expansions and discusses its role in data compression and signal transmission. The relationship between wavelets, Besov spaces, and nonlinear approximation is examined. In Chapter 12 a problem from astronomy (recovering of an object from data) is formulated as an inverse convolution problem, and wavelets are used to attack it. It is also explained that astronomy leads to very large data sets, and therefore compression is an important issue here, too.*

Ole Christensen (Lyngby)
Keywords : wavelets; signals; image processing; quadrature mirror filter; Gabor wavelet; multifractals; signal processing; wavelet packet; turbulence; computer vision; data compression; astronomy; Malvar-Wilson basis; multiresolution analysis
Classification :
∗*42C40 Wavelets*
42-02 Research monographs (Fourier analysis)

94A12 Signal theory

Zbl 1149.42309

Meyer, Yves

Wavelets and functions with bounded variation from image processing to pure mathematics. *(English)*

Atti Accad. Naz. Lincei, Cl. Sci. Fis. Mat. Nat., IX. Ser., Rend. Lincei, Mat. Appl. 2000, Suppl., 77-105 (2000). ISSN 1120-6330; ISSN 1720-0768
http://www.ems-ph.org/journals/journal.php?jrn=rlm
http://www.lincei.it/pubblicazioni/rendicontiFMN/cliccami.htm

Summary: JPEG-2000 has the potential to be the most significant advance in still image compression since the introduction of JPEG over a decade ago. JPEG-2000 is a wavelet based algorithm and it relies on new estimates on wavelet coefficients of functions of bounded variation. These new estimates have far reaching implications in pure mathematics.

Keywords : wavelets; image processing; functions with bounded variation
Classification :

∗42C40 Wavelets
65T60 Wavelets
94A08 Image processing

Zbl 1131.94301

Meyer, Yves

Approximation by wavelets and nonlinear approximation. (Approximation par ondelettes et approximation non-linéaire.) *(French)*

Charpentier, Éric (ed.) et al., Leçons de mathématiques d'aujourd'hui. Paris: Cassini. Le Sel et le Fer, 193-222 (2000). ISBN 2-84225-007-9/pbk

The paper under review forms an essay which presents some ideas of a filter bank approach to image preprocessing for readers who need not to be specialists in the field of non-linear approximation protocols. Starting off with the problem of visual perception in cognitive neuroscience [D. Marr, E. Hildreth, "Theory of edge detection", Proc. R. Soc. Lond. B 207, 187–217 (1980); D. Marr, Vision: A Computational Investigation into the Human Representation and Processing of Visual Information (W. H. Freeman, New York) (1982); G. J. Mitchison, G. Westheimer, "The perception of depth in simple figures", Vision Res. 24, 1063–1073 (1984)], the conception of the essay inevitably leads to the problem of image reconstruction performed by the human retina which forms an image preprocessing outpost of the brain [G. Westheimer, "Sensitivity for vertical retinal image differences", Nature 307, 632–634 (1984); J. E. Dowling, Retina: An Approachable Part of the Brain (The Belknap Press of Harvard University Press, Cambridge, Massachusetts, London) (1987); D. H. Hubel, Eye, Brain, and Vision (Scientific American Library, New York) (1988)].

The goal of the paper is to provide an introduction to some of the function spaces of approximation theory such as Besov and BMO (bounded mean oscillation) spaces [R. A. Adams, Sobolev spaces (Academic Press, New York, San Francisco, London) (1975; Zbl 0314.46030), 2nd ed. (2003; Zbl 1098.46001); E. M. Stein, Singular integrals and differentiability Properties of Functions (Princeton University Press, Princeton, New Jersey) (1970; Zbl 0207.13501)] by adopting the point of view of image processing. Vision or visual perception is fundamentally an information-processing task which is based on the concepts of rational approximation and filter bank [P. P. Vaidyanathan, Multirate systems and filter banks (Prentice Hall PTR, Englewood, New Jersey) (1993; Zbl 0784.93096); A. Mertins, Signal Analysis: Wavelets, Filter Banks, Time-Frequency Transforms and Applications (John Wiley & Sons, Chichester, New York, Weinheim) (1999; Zbl 0934.94001); E. Binz, W. Schempp, "Information technology: the Lie groups defining the filter banks of the compact disc", J. Comput. Appl. Math. 144, 85–103 (2002; Zbl 1016.94517)].
Visual information processing admits a dual procedure, the internal representation of information or visualization [D. Marr, "Visual information processing: The structure and creation of visual representations", Phil. Trans. R. Soc. Lond. B 290, 199–218 (1980); C. Blakemore (ed.) Vision: Coding and Efficiency (Cambridge University Press, Cambridge, New York, Port Chester) (1990); N. Dodgson, M. S. Floater, M. A. Sabin (eds.) Advances in Multiresolution for Geometric Modelling (Springer-Verlag, Berlin, Heidelberg, New York) (2005; Zbl 1047.65001)] which challenges the performances of the neurofunctional and morphological imaging protocols of the human retina and the visual cortex cerebri.
The somewhat unbalanced discussion of visual information compression and the biased exposition of non-linear approximation protocols in the cognitive neuroscience part of the present paper unfortunately excludes the dual cross-sectional visualization modalities such as laser scanning tomography, optical coherence tomography (OCT), multislice helical computer tomography (X-ray CT), clinical magnetic resonance imaging (MRI), positron emission tomography (PET) and neurofunctional combined PET/CT imaging which nevertheless are of overwhelming importance in the field of cognitive neuroscience [R. O. W. Burke, K. Rohrschneider, H. E. Völcker, "Posterior segment laser scanning tomography: Contour line modulation in optic disc analysis", Proc. SPIE 1357, 228–235 (1990); M. R. Hee, J. A. Izatt, E. A. Swanson, D. Huang, J. S. Schuman, C. P. Lin, C. A. Puliafito, J. G. Fujimoto, "Optical coherence tomography of the human retina", Arch. Ophthalmol. 113, 325–332 (1995); M. I. Posner, M. E. Raichle, Images of Mind (Scientific American Library, New York) (1994); C. T. W. Moonen, P. A. Bandettini (eds.), Functional MRI (Springer-Verlag, Berlin, Heidelberg, New York) (1999)].

Walter Schempp (Siegen)
Classification :
- ∗ *94A08 Image processing*
- *92C55 Tomography*
- *42C40 Wavelets*
- *94A12 Signal theory*
- *41A20 Approximation by rational functions*
- *41A10 Approximation by polynomials*

Zbl 0986.42016

Meyer, Yves

Signal processing and mathematical analysis. (Le traitement du signal et l'analyse mathématique.) *(French)*

Ann. Inst. Fourier 50, No.2, 593-632 (2000). ISSN 0373-0956
numdam:AIF_2000__50_2_593_0
http://aif.cedram.org/
http://annalif.ujf-grenoble.fr/aif-bin/feuilleter

This is a review of certain mathematical aspects of signal processing from a historical perspective. Section 1 discusses the "numerical revolution", the early contributions of Wiener, von Neumann, Shannon, Rosenblueth, Wigner, Brillouin, Gabor, Wilson, Morlet and Mallat, and ends with a statement of Daubechies' theorem on the existence of orthonormal wavelet bases of compact support. Section 2 introduces the concepts of analysis and synthesis in signal processing. Sections 3 and 4 deal with Shannon's theorem and its generalizations. Section 5 discusses the time–frequency plane and mentions Gabor wavelets, Wilson bases and Dolby filtering, Heisenberg's uncertainty principle, the time–frequency plane, and Gibbs cells. Section 6 contains a definition of Riesz basis and a brief description of windowed Fourier analysis and Gabor wavelets, including the Balian–Low theorem and its relationship to critical sampling. Wilson bases are described in Section 7. Sections 8 and 9 are titled "Search for the optimal basis" and "Chirps in mathematics and signal processing". Section 10 deals with frequency–modulated Wilson bases. After a brief historical discussion, Section 11 gives an elementary proof of the differentiability at $x = 1$ of Riemann's series $\sigma(x) = \sum_{n=1}^{\infty} n^{-2}\sin(\pi n^2 x)$. The purpose of this proof is to illustrate the author's position that when studying a given property, certain series expansions are more relevant than others. In the case at hand, the property is differentiability at a given point. The expansion of $\sigma(x)$ in a Fourier series does not yield the desired proof. On the other hand, the proof of the assertion readily follows by expanding $\sigma(x)$ in a series of chirps. Sections 12 and 13 discuss chirp functional spaces and analysis by r–regular chirp wavelets. Finally, Section 14 gives an application to the detection of gravitational waves and briefly discusses the work of B. Torrésani and his colleagues. The bibliography is not exhaustive and is intended to be a guide for further reading.

Richard A.Zalik (Auburn University)

Keywords : wavelet; time-frequency atom; Fourier analysis; chirp; Riesz basis
Classification :
∗*42C40 Wavelets*
94A12 Signal theory

Zbl 0968.46023

Jaffard, Stéphane; Meyer, Yves

On the pointwise regularity of functions in critical Besov spaces. *(English)*

J. Funct. Anal. 175, No.2, 415-434 (2000). ISSN 0022-1236

http://dx.doi.org/10.1006/jfan.2000.3605
http://www.sciencedirect.com/science/journal/00221236

Authors' abstract: We bound the spectrum of singularities of functions in the critical Besov spaces, and we show that this result is sharp, in the sense that equality in the bounds holds for quasi-every function of the corresponding Besov space.

Josef Wloka (Kiel)

Keywords : critical indexes; Hölder regularity; spectrum of singularities; fractal sets; Hausdorff dimension; multifractal functions; Baire categories; Besov spaces
Classification :
∗*46E35 Sobolev spaces and generalizations*

Zbl 0945.42015

Meyer, Yves; Coifman, Ronald
Wavelets: Calderón-Zygmund and multilinear operators. Transl. from the French by David Salinger. Paperback ed. *(English)*
Cambridge Studies in Advanced Mathematics. 48. Cambridge: Cambridge University Press. xix, 314 p. £24.90; £ 39.95 (2000). ISBN 0-521-79473-0/pbk

This book is the English translation of the volumes II and III of the book "Ondelettes et operateurs", originally published in French by Herman, Paris, in 1990 (Zbl 0745.42011) and 1991 (Zbl 0745.42010), respectively.
See also the review of Vol. I of the French original (1990) in Zbl 0694.41037. An English translation of Vol. I has appeared in 1995 (cf. Zbl 0819.42016).

Keywords : Calderón-Zygmund operator; wavelets; Hardy space; bounded mean oscillation; BMO; algebra of operators; Cauchy kernel; Lipschitz curve; pseudo-differential operator; commutator; multilinear operator; paradifferential operator
Classification :
∗*42C40 Wavelets*
42-02 Research monographs (Fourier analysis)
42B20 Singular integrals, several variables
47G10 Integral operators
47G30 Pseudodifferential operators

Zbl 0954.94003

Flandrin, Patrick *(Meyer, Yves)*
Time-frequency/time-scale analysis. With preface by Yves Meyer. Transl. from the French by Joachim Stöckler. *(English)*
Wavelet Analysis and Its Applications. 10. San Diego, CA: Academic Press. xii, 386 p. £ 59.95 (1999). ISBN 0-12-259870-9

Publisher's description: This highly acclaimed work has so far been available only in

French. It is a detailed survey of a variety of techniques for time-frequency/time-scale analysis (the essence of "Wavelet Analysis"). This book has broad and comprehensive coverage of a topic of keen interest to a variety of engineers, especially those concerned with signal and image processing. Flandrin provides a discussion of numerous issues and problems that arise from a mixed description in time and frequency, as well as problems in interpretation inherent in signal theory. Key features: – Detailed coverage of both linear and quadratic solutions. – Various techniques for both random and deterministic signals. Contents: Preface. Foreword. The Time-Frequency Problem. Classes of Solutions. Issues of Interpretation. Time-Frequency as a Paradigm. Bibliography. Subject Index.

Classification :

∗ *94A12 Signal theory*
42A38 Fourier type transforms, one variable
42C40 Wavelets
94A11 Application of orthogonal functions in communication

Zbl 0948.60026

Meyer, Yves; Sellan, Fabrice; Taqqu, Murad S.

Wavelets, generalized white noise and fractional integration: The synthesis of fractional Brownian motion. *(English)*

J. Fourier Anal. Appl. 5, No.5, 465-494 (1999). ISSN 1069-5869
http://dx.doi.org/10.1007/BF01261639
http://link.springer.de/link/service/journals/00041/index.htm

Authors' abstract: We provide an almost sure convergent expansion of fractional Brownian motion in wavelets which decorrelates the high frequencies. Our approach generalizes Lévy's midpoint displacement technique which is used to generate Brownian motion. The low frequency terms in the expansion involve an independent fractional Brownian motion evaluated at discrete times or, alternatively, partial sums of a stationary fractional ARIMA time series. The wavelets fill in the gaps and provide the necessary high frequency corrections. We also obtain a way of constructing an arbitrary number of non-Gaussian continuous-time processes whose second-order properties are the same as those of fractional Brownian motion.

Yimin Xiao (Redmond)

Keywords : fractional ARIMA; midpoint displacement technique; fractional Gaussian noise; fractional derivative; generalized functions; self-similarity

Classification :

∗ *60G18 Self-similar processes*
42C40 Wavelets
41A58 Series expansions
60F15 Strong limit theorems
60J65 Brownian motion

Zbl 0945.35014

Meyer, Yves

Large-time behavior and self-similar solutions of some semilinear diffusion equations. *(English)*

Christ, Michael (ed.) et al., Harmonic analysis and partial differential equations. Essays in honor of Alberto P. Calderón's 75th birthday. Proceedings of a conference, University of Chicago, IL, USA, February 1996. Chicago, IL: The University of Chicago Press. Chicago Lectures in Mathematics. 241-261 (1999). ISBN 0-226-10456-7/hbk

This survey article is devoted to self-similar solutions of the equations of mathematical physics and to the related topics. As a model example the author considers the following nonlinear heat equation in $\mathbb{R}^3$*:*

$$(1) \qquad \frac{\partial u}{\partial t} = \Delta u + u^3, \quad u\big|_{t=0} = u_0(x)$$

The sufficient conditions on u_0 *which imply the global existence of the solution* u *of* (1) *or in the contrary guarantee the blow up in a finite time are reviewed.*

Moreover, the author gives a new approach to study the self-similar solutions u *of* (1) *(i.e.* $\lambda u(\lambda t, \lambda^2 x) \equiv u(t,x)$, $\forall \lambda > 0$*) and shows that the large-time behavior of most of the global solutions of* (1) *are driven by the corresponding self-similar solutions.*

The analogues of these results to the other types of equations such as Navier-Stockes equations and the nonlinear Schrödinger equation are also considered.

Serguei Zelik (Moskva)

Keywords : Navier-Stokes equations; nonlinear Schrödinger equation

Classification :

∗*35B40 Asymptotic behavior of solutions of PDE*

35K55 Nonlinear parabolic equations

Zbl 1069.46503

Cohen, A.; Meyer, Y.; Oru, F.

Improved Sobolev embedding theorem. *(English)*

Sémin. Équ. Dériv. Partielles, Éc. Polytech., Cent. Math., Palaiseau 1997-1998, Exp. No. XVI, 16 p. (1998).

numdam:SEDP_1997-1998____A16_0

This lecture was given by A. Cohen in 1998. Consider an orthonormal wavelet basis of $L^2(\mathbb{R}^n)$, $n \geq 2$, *defined as*

$$\psi_{j,k}(x) = 2^{\frac{nj}{2}} \psi(2^j x - k), \quad j \in \mathbb{Z}, \ k \in \mathbb{Z}^n,$$

where ψ *belongs to a finite collection of* $2^n - 1$ *mother wavelets, which are assumed to belong to* $L^\infty(\mathbb{R}^n)$ *and to be compactly supported. Let* f *be a function whose gradient* ∇f *belongs to* $L^1(\mathbb{R}^n)$, *set* $\beta_{j,k} = 2^{j(1-\frac{n}{2})}\langle f, \psi_{j,k}\rangle$ *and for* $\lambda \in (0,\infty)$ *denote by* E_λ *the*

collection of (j,k) *for which* $|\beta_{j,k}| > \lambda$*. Then, there exists a constant* $C(\psi,n)$ *such that*

(1) $$\#E_\lambda \le C\|\nabla f\|_{L^1}\ \lambda^{-1}.$$

The lecturer presents a complete proof of (1) *which is related to a result by A. Cohen, R. DeVore, P. Petrushev and H. Xu [Am. J. Math. 121, No. 3, 587–628 (1999; Zbl 0931.41019)]. As a direct consequence of* (1)*, he obtains the following improvement of the usual Sobolev inequality*

(2) $$\|f\|_{n^*} \le C_n\|\nabla f\|_{L^1}^{\frac{n-1}{n}}\|f\|_B^{\frac{1}{n}}$$

for all $f \in B = \dot{B}_\infty^{-(n-1),\infty}(\mathbb{R}^n)$*, such that* $\nabla f \in L^1(\mathbb{R}^n)$*, where* $n^* = \frac{n}{n-1}$ *and* B *is the homogeneous Besov space.*

Denise Huet (Nancy)

Keywords : Sobolev embedding theorems; wavelets; homogeneous Besov spaces
Classification :
∗*46E35 Sobolev spaces and generalizations*
26D20 Analytical inequalities involving real functions
42C15 Series and expansions in general function systems
42C40 Wavelets
26D10 Inequalities involving derivatives, diff. and integral operators

---

Zbl 0959.28007

Daoudi, K.; Lévy Véhel, J.; Meyer, Y.

Construction of continuous functions with prescribed local regularity. *(English)*

Constructive Approximation 14, No.3, 349-385 (1998). ISSN 0176-4276
http://dx.doi.org/10.1007/s003659900078
http://link.springer.de/link/service/journals/00365/

Summary: We investigate from both a theoretical and a practical point of view the following problem: Let s *be a function from* $[0,1]$ *to* $[0,1]$*. Under which conditions does there exist a continuous function* f *from* $[0,1]$ *to* $\mathbb{R}$ *such that the regularity of* f *at* x*, measured in terms of the Hölder exponent, is exactly* $s(x)$*, for all* $x \in [0,1]$*?*
We obtain a necessary and sufficient condition on s *and give three constructions of the associated function* f*. We also examine some extensions regarding, for instance, the box or Tricot dimension or the multifractal spectrum. Finally, we present a result on the "size" of the set of functions with prescribed local regularity.*

Keywords : box dimension; Weierstrass functions; Schauder basis; IFS; fractals; Hölder exponent; Tricot dimension; multifractal spectrum; local regularity
Classification :
∗*28A80 Fractals*
28A78 Hausdorff measures

Zbl 0893.42015

Meyer, Yves

Wavelets, vibrations and scalings. *(English)*

CRM Monograph Series. 9. Providence, RI: American Mathematical Society (AMS). ix, 129 p. £ 29.00/hbk (1998). ISBN 0-8218-0685-8

This book intends to give a summary and unification of characterization of special function spaces by wavelet bases.

Usually, the local regularity of a function $f(x)$ can be described by its scaling exponent. The literature provides different definitions of pointwise scaling exponents $\alpha(f, x_0)$. A first group of definitions for $\alpha(f, x_0)$ satisfies the condition

$$c_1|x - x_0|^\alpha \leq |f(x)| \leq c_2|x - x_0|^\alpha \quad \Longrightarrow \quad \alpha(f, x_0) = \alpha.$$

One typical example of this group is the pointwise Hölder exponent.

In a second group of definitions, oscillations of $f(x)$ also play a role. In this case, $\alpha(f, x_0)$ satisfies the condition

$$\alpha(f, x_0) = \alpha \quad \Longleftrightarrow \quad \alpha\left(\frac{\delta f}{\delta x_j}, x_0\right) = \alpha - 1, \qquad 1 \leq j \leq n.$$

A new scaling exponent belonging to the second group is proposed in the first chapter. This so-called weak scaling exponent plays a key role for estimating the size of the wavelet transform of $f(x)$. Moreover, it is closely related to two-microlocal spaces $C^{s,s'}_{x_0}$ which were introduced by J. M. Bony.

While the first chapter of this monograph is concerned with the small scales behavior of a function f, the relation between the large scales behavior of f and the infrared divergence in the wavelet expansion of f ist studied in Chapter 2. Furthermore, the pointwise Hölder exponent is given by estimation of wavelet coefficients of f. Considering the two-microlocal spaces, one is provided with a new family of scaling exponents. The understanding of these special function spaces is further deepened in Chapter 4. Chapter 5 goals to construct a special wavelet analysis adapted to functions vanishing at a given order at x_0. This permits a more exact characterization of functions in $C^{s,s'}_{x_0}$. Finally, using Wilson bases a second solution to the same problem is given.

G.Plonka (Rostock)

Keywords : scaling exponents; adapted wavelets; two-microlocal spaces; multifractal signal processing; Wilson basis

Classification :

- ∗*42C40* *Wavelets*
- *42-02* *Research monographs (Fourier analysis)*
- *46E15* *Banach spaces of functions defined by smoothness properties*
- *46F99* *Generalized functions, etc.*

Zbl 1066.46501

Gerard, Patrick; Meyer, Yves; Oru, Frédérique

Sharp Sobolev inequalities. (Inégalités de Sobolev précisées.) *(French)*

Sémin. Équ. Dériv. Partielles, Éc. Polytech., Cent. Math., Palaiseau 1996-1997, Exp. No. IV, 11 p. (1997).
numdam:SEDP_1996-1997____A4_0

In this lecture, given in 1996, Yves Meyer presented improvements of the usual Sobolev inequality. Let $\dot{C}^{-\alpha}(\mathbb{R}^n), \alpha > 0$, be the homogeneous Besov space $B = \dot{B}_\infty^{-\alpha,\infty}(\mathbb{R}^n)$, and $\Lambda = \sqrt{-\Delta}$. In the first part, it is shown, when $n = 3$, by a real interpolation method, the existence of a constant C such that

$$\|f\|_6 \leq C\|\nabla f\|_2^{\frac{1}{3}}\|f\|_*^{\frac{2}{3}}$$

where $\|f\|_*$ is the norm in $\dot{C}^{-\frac{1}{2}}(\mathbb{R}^3)$; it is pointed out the invariance of (1) under the affine and the Weyl-Heisenberg groups. In the second part, when $1 < p < q < \infty$, $s = \alpha(\frac{q}{p} - 1)$, it is obtained, by an elegant proof which does not use interpolation spaces, the existence of a constant $C = C(n, \alpha, p, q)$ such that

$$||f||_q \leq C||\Lambda^s f||_p^{\frac{p}{q}}\|f\|_B^{1-\frac{p}{q}}$$

for all functions $f \in \dot{C}^{-\alpha}(\mathbb{R}^n)$ such that $\Lambda^s f \in L^p(\mathbb{R}^n)$. The core of the proofs is the theory of wavelets, cf. [Y. Meyer, "Ondelettes et opérateurs. I" (Actualités Mathématiques, Hermann, Paris) (1990; Zbl 0694.41037)].

Denise Huet (Nancy)

Keywords : Sobolev inequalities; interpolation spaces; wavelets
Classification :
- ∗46E35 *Sobolev spaces and generalizations*
- 46M35 *Abstract interpolation of topological linear spaces*
- 26D15 *Inequalities for sums, series and integrals of real functions*
- 26D10 *Inequalities involving derivatives, diff. and integral operators*
- 42C40 *Wavelets*

Zbl 0960.94006

Meyer, Yves; Xu, Hong

Wavelet analysis and chirps. *(English)*

Appl. Comput. Harmon. Anal. 4, No.4, 366-379 (1997). ISSN 1063-5203
http://dx.doi.org/10.1006/acha.1997.0214
http://www.sciencedirect.com/science/journal/10635203

Summary: "The authors propose a general model for chirp-like signals. The analytical structure of these chirps is given in the form $|x|^s g(x/|x|^{1+\beta})$, where g is an L^p infinitely oscillating function like $\sin(x)$. Under suitable assumptions, they achieve a characterization of the chirps by means of their wavelet transforms."
Only the central theorems are proved here whereas for detailed and complete proofs the reader is referred to the second author's thesis "Généralisation de la théorie des

chirps à divers cadres fonctionnels et applications à leur analyse par ondelettes", Thèse, l'Université Paris Dauphine.

Keywords : generalized two-microlocal spaces; chirp-like signals; infinitely oscillating function; wavelet transforms
Classification :
- ∗ *94A11 Application of orthogonal functions in communication*
- *42C40 Wavelets*
- *94A12 Signal theory*

Zbl 0926.35115

Meyer, Yves
Wavelets, paraproducts, and Navier-Stokes equations. *(English)*
Bott, Raoul (ed.) et al., Current developments in mathematics, 1996. Proceedings of the joint seminar, Cambridge, MA, USA 1996. Cambridge, MA: International Press. 105-212 (1997). ISBN 1-57146-035-7/hbk

This paper, half review, half contributed, advocates that wavelet transforms provide methods being especially well suited for solving Navier-Stokes equations with high Reynolds number. The main advantage of this approach is that it is an adaptive mesh method in which the mesh is automatically refined depending on the grade of variation of the solution.
This long paper comprises twenty nine sections which can be distributed into three parts as follows. (i) Generalizations of Navier-Stokes equations and wavelets. (ii) Mild solutions of Navier-Stokes equations, Kato's algorithm. It is assumed that the initial condition is a sum of normalized divergence-free wavelets. (iii) Further complements, and mainly self-similarity of the solution to the Navier-Stokes equations.
Last, but not least, wavelets are of valuable help as a visualization tool.

G.Jumarie (Montréal)
Keywords : mild solutions of Navier-Stokes equations; wavelet transforms; Navier-Stokes equations with high Reynolds number; Kato's algorithm; self-similarity; visualization
Classification :
- ∗ *35Q30 Stokes and Navier-Stokes equations*
- *42C40 Wavelets*
- *65T99 Numerical methods in Fourier analysis*

Zbl 0916.42023

Meyer, Yves; Coifman, Ronald
Wavelets: Calderón-Zygmund and multilinear operators. Transl. from the French by David Salinger. *(English)*
Cambridge Studies in Advanced Mathematics. 48. Cambridge: Cambridge University Press. xix, 314 p. £ 59.95; £40.00 (1997). ISBN 0-521-42001-6/hbk

This book is the English translation of the volumes II and III of the book "Ondelettes et

opérateurs", originally published in French by Herman, Paris, in 1990 (Zbl 0745.42011) and 1991 (Zbl 0745.42012), respectively.
See also the review of Vol. I of the French original (1990) in Zbl 0694.41037. An English translation of Vol. I has appeared in 1995 (cf. Zbl 0819.42016).

Keywords : Calderón-Zygmund operator; wavelets; Hardy space; bounded mean oscillation; BMO; algebra of operators; Cauchy kernel; Lipschitz curve; pseudodifferential operator; commutator; multilinear operator; paradifferential operator
Classification :
∗ *42C40 Wavelets*
42-02 Research monographs (Fourier analysis)
42B20 Singular integrals, several variables
47G10 Integral operators
47G30 Pseudodifferential operators

Zbl 0870.94008

Coifman, R.R.; Matviyenko, G.; Meyer, Y.

Modulated Malvar-Wilson bases. *(English)*

Appl. Comput. Harmon. Anal. 4, No.1, 58-61 (1997). ISSN 1063-5203
http://dx.doi.org/10.1006/acha.1996.0198
http://www.sciencedirect.com/science/journal/10635203

Summary: New orthonormal bases of improved time frequency atoms are constructed. These atoms are similar to R. Baraniuk's "chirplets". These new bases are used to unfold frequency modulated signals in the time frequency plane. The selection of the "best basis" amounts to finding an optimal covering with Heisenberg boxes with arbitrary eccentricities and orientations. This analysis is as sharp as the one provided by the Wigner transform.

Keywords : orthonormal bases; time frequency atoms; Heisenberg boxes; Wigner transform
Classification :
∗ *94A12 Signal theory*
42C15 Series and expansions in general function systems

Zbl 0892.46034

Han, Y.-S.; Meyer, Y.

A characterization of Hilbert spaces and the vector-valued Littlewood-Paley theorem. *(English)*

Methods Appl. Anal. 3, No.2, 228-234 (1996). ISSN 1073-2772
http://www.intlpress.com/MAA/
http://projecteuclid.org/maa

It is shown that if the analogue of Littlewood-Paley's theorem holds for some $p, 1 < p <$

∞, and for all strongly p-integrable functions with values in a given Banach space B, then B is isomorphic to a Hilbert space. In the final step of the proof Kwapien's well-known characterization of inner product spaces is used, but the reduction needs hard work.

V.Tarieladze (Tbilisi)

Keywords : Calderón-type reproducing formula; Littlewood-Paley theorem; approximation of the identity
Classification :
∗46E40 Spaces of vector-valued functions
42B20 Singular integrals, several variables
42B25 Maximal functions

Zbl 0873.42019

Jaffard, Stéphane; Meyer, Yves

Wavelet methods for pointwise regularity and local oscillations of functions.

(English)
Mem. Am. Math. Soc. 587, 110 p. (1996). ISSN 0065-9266

This monograph discusses pointwise Hölder regularity of functions, especially when this regularity is not uniform but varies wildly from point to point. The main tool is the use of two-microlocalization spaces which allow to study how singularities deteriorate in a smooth environment and conversely. Such spaces can be characterized in terms of decay conditions in the Littlewood-Paley decomposition or the continuous wavelet transform. These two-microlocalizations are introduced in Chapter I and as an application, the pointwise singularity of elliptic operators is investigated. In Chapter 2, singularities in Sobolev spaces are discussed. Depending on the type of singularity, the Hausdorff or packing dimension of the set where a certain Hölder type condition holds is estimated. Chapter 3 investigates the relation between wavelet expansions and lacunary trigonometric series. Especially selfsimilarity and very strong oscillatory (chirp-like) behaviour is discussed. Such trigonometric or logarithmic chirps are studied in more detail in the subsequent three chapters. There is for example a simple characterization of such chirps in terms of certain conditions that their wavelet transform should satisfy. The chirp-like behaviour of the Riemann function $\sum n^{-2}\sin \pi n^2 x$ is by now well known and is discussed in the last chapter. It has a trigonometric chirp in the rational points $(2p+1)/(2q+1)$ where p and q are integers, and a logarithmic chirp at the quadratic irrationals.

A.Bultheel (Leuven)

Keywords : wavelets; Littlewood-Paley decomposition; two-microlocalization; modulus of continuity; Hausdorff dimension; chirps; selfsimilarity; Riemann function; pointwise Hölder regularity
Classification :
∗42C40 Wavelets
26A16 Lipschitz classes, etc. (one real variable)
28A80 Fractals

26A30 Real functions of one real variable with other special properties
42B25 Maximal functions
42A16 Fourier coefficients, etc.
26A27 Nondifferentiability of functions of one real variable

Zbl 0869.43003

Auslander, Louis; Meyer, Yves

A generalized Poisson summation formula. *(English)*

Appl. Comput. Harmon. Anal. 3, No.4, 372-376 (1996). ISSN 1063-5203
http://dx.doi.org/10.1006/acha.1996.0029
http://www.sciencedirect.com/science/journal/10635203

The authors give two proofs for a generalization of the Poisson formula $\sum_{n\in Z} f(n) = \sum_{n\in Z} \widehat{f}(n)$. Although the generalization involves abelian groups only, the original proofs went by the way of the Heisenberg group and its discrete subgroups.

B.Basit (Claydon)

Keywords : Poisson formula; abelian groups; Heisenberg group

Classification :
∗*43A80 Analysis on other specific Lie groups*

Zbl 0883.35005

Meyer, Y.

n-dimensional chirps and 2-microlocal analysis. (Chirps n-dimensionnels et analyse 2-microlocale.) *(French)*

Sémin. Équ. Dériv. Partielles, Éc. Polytech., Cent. Math., Palaiseau Sémin. 1994-1995, Exp. No.1, 10 p. (1995).

The aim of the author is to describe, through 2-microlocal spaces and wavelet analysis, equivalent methods for a precise study of chirps, that is, loosely speaking, of functions that rapidly oscillate when x approaches a given point $x_0 \in \mathbb{R}^n$. In Section 2, he defines a chirp at 0 of type (γ, β), $\gamma > -n$, $\beta > 0$, as a function f, integrable around 0, with the property that there exists $r_0 > 0$ such that for any integer $N \geq 0$, $f(x) = \sum_{|\alpha|\leq N} \partial_x^\alpha U_{\alpha,N}(x)$ in $\mathcal{D}'$, for $x \in \{|x| < r_0\}$, with $|U_{\alpha,N}(x)| \leq C_N |x|^{\gamma+\beta N+|\alpha|}$. A chirp at x_0 will then be a function f such that $f(x+x_0)$ is a chirp at 0. Next, he defines, with $\Omega = \{|x| > R_0\}$, a space of oscillating functions $g \in L^\infty(\Omega)$ such that, for any integer $N \geq 1$, $g(x) = \sum_{|\alpha|=N} \partial_x^\alpha g_\alpha(x)$, where $g_\alpha \in L^\infty(\Omega)$, and establishes the equivalence: a function f, integrable around 0, is a chirp at 0 of type (γ, β) iff $|x|^{\gamma/\beta} f(x|x|^{-(1+\beta)/\beta})$, $|x| > R_0$, belongs to the aforementioned space of oscillating functions. In Sections 3 and 4 he defines r-regular chirps of type (γ, β) $(r > 0)$, characterizes them in terms of Hölder spaces, defines the 2-microlocal spaces $C_{x_0}^{s,s'}$, $s, s' \in \mathbb{R}$, $x_0 \in \mathbb{R}^n$, and proceeds in stating the following
Theorem. If $r, \beta > 0$ and $\gamma > -n$, then $f \in C_0^{s,s'}$ for any s, s' such that $s + s' \leq r$ and

$(\beta+1)s+\beta s' \leq \gamma$, iff $f(x)=u(x)+v(x)$, where u is an r-regular chirp of type (γ,β) at 0, and $v\in C^\infty$ in a neighborhood of 0.
In Section 5, he recalls a theorem by S. Jaffard, which relates the 2-microlocal spaces $C^{s,s'}_{x_0}$ and (local) approximation by polynomials in Hölder norm. In Section 6 he establishes a method for recognizing an r-regular (γ,β) chirp through wavelet analysis and finally, in Section 7, he studies the action on chirps of pseudodifferential operators T_ζ, whose symbols $\tau_\zeta(\xi)$, independent of x, satisfy the homogeneity condition $\tau_\zeta(\lambda\xi)=\lambda^\zeta\tau_\zeta(\xi)$, for any $\lambda>0$ and any $\xi\in\mathbb{R}^n\setminus\{0\}$. One has the following
Theorem. If u is an r-regular chirp of type (γ,β), $\zeta=\sigma+i\omega\in\mathbb{C}$, with $\sigma<\min\{r,(\gamma+n)/(\beta+1)\}$, then $T_\zeta u=u_\zeta+v_\zeta$, where u_ζ is an $r-\sigma$-regular chirp of type $(\gamma-\sigma(\beta+1),\beta)$ and where v_ζ is a smooth function.

A.Parmeggiani (Bologna)

Keywords : 2-microlocal spaces; wavelets; oscillating functions
Classification :
- ∗35A27 Sheaf-theoretic methods (PDE)
- 94A12 Signal theory
- 42C15 Series and expansions in general function systems

Zbl 0881.11059

Meyer, Yves

Quasicrystals, diophantine approximation and algebraic numbers. *(English)*

Axel, Françoise (ed.) et al., Beyond quasicrystals. Papers of the winter school, Les Houches, France, March 7-18, 1994. Berlin: Springer-Verlag. 3-16 (1995). ISBN 3-540-59251-2/pbk; ISBN 2-86883-248-2

The author, whose book [Y. Meyer, Algebraic numbers and harmonic analysis, North-Holland Math. Library 2 (1972; Zbl 0267.43001)] laid the foundation to the field of (mathematical) quasicrystals some 25 years ago, summarizes the structure of model sets (resp. cut and project sets) from the viewpoint of diophantine approximation and harmonic analysis. The focus is on what is nowadays called Meyer sets (here called quasicrystal, which is different in meaning from most other sources) and their connection to special algebraic numbers such as Pisot-Vijayaraghavan or Salem numbers. For further developments the reader could consult various recent articles by R. V. Moody and by J. C. Lagarias.

M.Baake (Tübingen)

Keywords : Pisot-Vijayaraghavan numbers; model sets; diophantine approximation; harmonic analysis; Meyer sets; quasicrystals
Classification :
- ∗11J25 Diophantine inequalities
- 82D25 Crystals
- 11K70 Harmonic analysis and almost periodicity
- 11R06 Special algebraic numbers

Zbl 0864.42014

Coifman, R.R.; Meyer, Y.

Gaussian bases. *(English)*

Appl. Comput. Harmon. Anal. 2, No.3, 299-302 (1995). ISSN 1063-5203
http://dx.doi.org/10.1006/acha.1995.1022
http://www.sciencedirect.com/science/journal/10635203

Consider the functions $\exp(-\zeta x^2)p_j(x)$ *with* $p_j(x) = \sin[\pi/2(x+1)(j+1/2)]$ *and* $Re\zeta > 0$. *It is shown in this note that the family* $h_{j,k}(x) = h_j(x-2k)$, $j = 0,1,\ldots,k \in Z$ *is an unconditional basis for* $L^2(R)$. *Also the dual basis for this is constructed. When* $\zeta > 0$, *this result is implicit in I. Daubechies' "Ten lectures on wavelets" (1992; Zbl 0776.42018), page 120. It is more explicit on page 112, formula (28) in I. Daubechies paper in D. H. Feng et al. (eds.): "Coherent states, past, present and future", World Sci. Publ. 103-117 (1994)].*

A.Bultheel (Leuven)

Keywords : wavelets; series expansion; unconditional basis
Classification :
∗*42C15 Series and expansions in general function systems*

Zbl 0863.42021

Coifman, R.R.; Meyer, Y.; Wickerhauser, V.

Numerical harmonic analysis. *(English)*

Fefferman, Charles (ed.) et al., Essays on Fourier analysis in honor of Elias M. Stein. Proceedings of the Princeton conference on harmonic analysis held at Princeton Univ., Princeton, NJ, USA, May 13-17, 1991 in honor of Elias M. Stein's 60th birthday. Princeton, NJ: Princeton Univ. Press. Princeton Math. Ser. 42, 162-174 (1995). ISBN 0-691-08655-9/hbk

This short paper links harmonic analysis with numerical computation. First the definition of modulated wave form libraries is given. These correspond to short-time or windowed Fourier transform bases. Special attention is paid to the construction of orthogonal bases. The connection is then made with wavelets and wavelet packets. Using the idea of entropy, it is shown that these bases are very efficient to compactly represent the data. Because of compactness, the paper contains not much new for the specialist and is rather uninformative for the non-specialist.

A.Bultheel (Leuven)

Keywords : integral transform; orthogonal bases; wavelets; wavelet packets
Classification :
∗*42C15 Series and expansions in general function systems*
65T99 Numerical methods in Fourier analysis

Zbl 0852.35005

Meyer, Y.

Wavelet analysis, local Fourier analysis, and 2-microlocalization. *(English)*

Marcantognini, S. A. M. (ed.) et al., Harmonic analysis and operator theory. A conference in honor of Mischa Cotlar, January 3-8, 1994, Caracas, Venezuela. Proceedings. Providence, RI: American Mathematical Society. Contemp. Math. 189, 393-401 (1995). ISBN 0-8218-0304-2

The paper studies several equivalent descriptions of some 2-microlocal property of a given function. The equivalence of three different conditions is proved. It is an extension of Jaffard's theorem.

Chen Shuxing (Shanghai)

Keywords : Fourier analysis; 2-microlocalization; wavelet; Jaffard's theorem

Classification :

- ∗ *35A27 Sheaf-theoretic methods (PDE)*
- *42C40 Wavelets*
- *26B35 Special properties of functions of several real variables*

Zbl 0842.35074

Cannone, Marco; Meyer, Yves

Littlewood-Paley decomposition and Navier-Stokes equations. *(English)*

Methods Appl. Anal. 2, No.3, 307-319 (1995). ISSN 1073-2772

http://www.intlpress.com/MAA/

http://projecteuclid.org/maa

The authors prove some existence and uniqueness results for the local strong solutions of the Cauchy problem for the Navier-Stokes equations in $\mathbb{R}^3$. They are primarily interested in strong solutions belonging to $C([0,T);X)$, where X denotes an abstract Banach space of vector distributions on $\mathbb{R}^3$. In contrast to earlier work in which the spaces were adapted to specific methods, the authors present a general approach which can be used for a variety of Banach spaces satisfying a simple sufficient condition. Examples are given. The analysis is partly inspired by the wavelet approach of Federbush, and it rests upon a systematic use of the Littlewood-Paley decomposition.

W. Velte (Würzburg)

Keywords : existence; uniqueness; Cauchy problem; Navier-Stokes equations; wavelet approach of Federbush; Littlewood-Paley decomposition

Classification :

- ∗ *35Q30 Stokes and Navier-Stokes equations*
- *42B25 Maximal functions*

Zbl 0842.42011

Coifman, R.R.; Dobyinsky, S.; Meyer, Y.

Bilinear operators and renormalization. (Opérateurs bilinéaires et renormalisation.) *(French)*

Fefferman, Charles (ed.) et al., Essays on Fourier analysis in honor of Elias M. Stein. Proceedings of the Princeton conference on harmonic analysis held at Princeton Univ., Princeton, NJ, USA, May 13-17, 1991 in honor of Elias M. Stein's 60th birthday. Princeton, NJ: Princeton Univ. Press. Princeton Math. Ser. 42, 146-161 (1995). ISBN 0-691-08655-9/hbk

Dans cet article, les auteurs étudient le produit scalaire de deux champs de vecteurs $E = (E_1, \dots, E_n)$, $B = (B_1, \dots, B_n)$ *dans* $\mathbb{R}^n$ *en "renormalisant" le produit de deux-fonctions u, v en écrivant sous la forme:* $u.v = P(u, v) + R(u, v)$ *où P et R verifient certaines propriétés. Le terme* $R(u, v)$ *est le produit "renormalisé", après soustraction à u.v des termes "genants". Les auteurs donnent ici une renormalisation liée au paraproduit de J. M. Bony. Cette renormalisation donne le fait que si E et B sont dans* L^2 *et si div* $E = 0$ *et Rot* $B = 0$, *alors E.B est dans l'espace de Hardy* $\mathcal{H}^1(\mathbb{R}^n)$. *Ils deduisent aussi, facilement un lemme de Murat-Tartar sur certaines suites* (E_j), (B_j) *de champs de vecteurs, vérifiant l'hypothèse "div-Rot".*
L'article contient des résultats plus precis.

M.Derridj (Paris)

Keywords : Hardy space; renormalization; vector fields; scalar product; paraproduct
Classification :
∗ *42B30 Hp-spaces (Fourier analysis)*
35S99 Pseudodifferential operators

Zbl 0819.42016

Meyer, Yves

Wavelets and operators. Transl. by D. H. Salinger. *(English)*

Cambridge Studies in Advanced Mathematics. 37. Cambridge: Cambridge University Press. 240 p. £16.95 (1995). ISBN 0-521-45869-2/pbk

See the review of the French original edition "Ondelettes et opérateurs. I: Ondelettes (1990) in Zbl 0694.41037. For Parts II and III of this series see Zbl 0745.42011 and Zbl 0745.42012, respectively.

Keywords : BMO; functions of bounded mean oscillation; singular integral operators; Besov spaces; Sobolev spaces; multiresolution analysis; ondelettes; wavelets; Fourier transform; boundedness of a discrete Hilbert transform; inequality of Bernstein; oblique structure; frame Hardy space
Classification :
∗ *42C40 Wavelets*
42-02 Research monographs (Fourier analysis)
42B20 Singular integrals, several variables

Zbl 0882.35090

Cannone, M.; **Meyer, Y.**; **Planchon, F.**
Self-similar solutions of Navier-Stokes equations. (Solutions auto-similaires des équations de Navier-Stokes.) *(French)*
Sémin. Équ. Dériv. Partielles, Éc. Polytech., Cent. Math., Palaiseau Sémin. 1993-1994, Exp. No.8, 10 p. (1994).

The authors define the self-similar solution of the Navier-Stokes equations and prove a uniqueness theorem of this solution for the initial conditions in a special functional space.

V.A.Sava (Iaşi)
Keywords : Navier-Stokes equation; self-similar solutions; uniqueness
Classification :
- ∗ *35Q30 Stokes and Navier-Stokes equations*
- *76D05 Navier-Stokes equations (fluid dynamics)*

Zbl 0818.94005

Coifman, Ronald R.; **Meyer, Yves**; **Quake, Steven**; **Wickerhauser, M.Victor**
Signal processing and compression with wavelet packets. *(English)*
Byrnes, J. S. (ed.) et al., Wavelets and their applications. Proceedings of the NATO ASI Conference, 16-29 August 1992, Il Ciocco, Italy. Dordrecht: Kluwer Academic Publishers. NATO ASI Ser., Ser. C, Math. Phys. Sci. 442, 363-379 (1994). ISBN 0-7923-3078-1/hbk

Summary: Wavelet packets are a versatile collection of functions generalizing the compactly supported wavelets of Daubechies. They are used to analyze and manipulate signals such as sound and images. We describe a library of such waveforms and demonstrate a few of their analytic properties. We also describe an algorithm to chose a best basis subset, tailored to fit a specific signal or class of signals. We apply this algorithm to two signal processing tasks: acoustic signal compression, and feature extraction in certain images.

Keywords : wavelets; acoustic signal compression; images
Classification :
- ∗ *94A11 Application of orthogonal functions in communication*
- *42C40 Wavelets*
- *94A12 Signal theory*
- *94A13 Detection theory*
- *68U10 Image processing*
- *94A29 Source coding*

Zbl 0948.42500

Meyer, Yves
Wavelets: Their past and their future. *(English)*

Meyer, Yves (ed.) et al., Progress in wavelet analysis and applications. Proceedings of the 3rd international conference on wavelets and applications, Toulouse, France, June 8-13, 1992. Gif-sur-Yvette: Editions Frontières. 9-18 (1993). ISBN 2-86332-130-7/hbk

Keywords : time-scale wavelets; time-frequency algorithms; multiresolution; subband coding schemes
Classification :
∗*42C40 Wavelets*
42-03 Historical (Fourier analysis)
01A65 Development of contemporary mathematics
01A67 Future prospectives in mathematics

Zbl 0878.94010

Coifman, Ronald R.; Meyer, Yves; Quake, Steven; Wickerhauser, M.Victor
Signal processing and compression with wavelet packets. *(English)*
Meyer, Yves (ed.) et al., Progress in wavelet analysis and applications. Proceedings of the 3rd international conference on wavelets and applications, Toulouse, France, June 8-13, 1992. Gif-sur-Yvette: Editions Frontières. 77-93 (1993). ISBN 2-86332-130-7/hbk

From the introduction: The authors describe some new algorithms for signal processing and data compression based on a collection of orthogonal functions which are called wavelet packets. Wavelet packets generalize the compactly supported wavelets described by Daubechies and Meyer. The algorithms given here combine the projection of a sequence onto wavelet packet components, the selection of an optimal orthonormal basis subset, some linear or quasilinear processing of the coefficients, and then reconstruction of the transformed sequence.

Keywords : algorithms; signal processing; data compression; wavelet packets; optimal orthonormal basis; reconstruction
Classification :
∗*94A12 Signal theory*
42C40 Wavelets

Zbl 0864.42009

Coifman, R.; Lions, P.L.; Meyer, Y.; Semmes, S.
Compensated compactness and Hardy spaces. *(English)*
J. Math. Pures Appl., IX. Sér. 72, No.3, 247-286 (1993). ISSN 0021-7824
http://www.sciencedirect.com/science/journal/00217824

The following sample results are, probably, not the most important (and, surely, not the most general or difficult ones) in this very interesting paper. They are choosen only to allow the reader to feel the flavor of the topic.

(a) If $u \in W^{1,N}(\mathbb{R}^N)^N$ *(i.e., u is a vector-valued function with first derivatives in* L^1*), then* $det(\nabla u) \in \mathcal{H}^1(\mathbb{R}^N)$*.*
(b) If $u \in L^p(\mathbb{R}^N)^N$, $v \in L^q(\mathbb{R}^N)^N$ $(p^{-1} + q^{-1} = 1)$ *and* $divu = 0$, $curlv = 0$, *then* $\langle u, v\rangle \in \mathcal{H}^1(\mathbb{R}^N)$*.*
(c) Any function $f \in \mathcal{H}^1(\mathbb{R}^N)$ *can be written in the form* $f = \sum \lambda_k \langle u_k, v_k\rangle$, *where* u_k *and* v_k *are as u and v in (b) and of uniformly bounded norms (moreover,* $p = q = 2$*), and* $\sum |\lambda_k| < \infty$*.*
In general, the authors deal with various nonlinear expressions arising in the so-called compensated compactness theory and show that these quantities belong to the real-variable Hardy classes $\mathcal{H}^p(\mathbb{R}^N)$ *with certain* $p \leq 1$*. Related weak convergence questions are presented. The emphasis is made on the cancellation properties lying beyond the phenomena in question.*

S.V.Kislyakov (St.Peterburg)
Keywords : compensated compactness; Hardy classes; weak convergence; cancellation
Classification :
∗ *42B30 Hp-spaces (Fourier analysis)*
42B25 Maximal functions
35R99 Miscellaneous topics involving PDE

Zbl 0841.00017

Meyer, Yves (ed.); **Roques, Sylvie (ed.)**
Progress in wavelet analysis and applications. Proceedings of the 3rd international conference on wavelets and applications, Toulouse, France, June 8-13, 1992. *(English)*
Gif-sur-Yvette: Editions Frontières. xvi, 785 p. £ 87.00 (1993). ISBN 2-86332-130-7/hbk

The articles of this volume will be reviewed individually. The 2nd Conference has been reviewed (see Zbl 0742.00079).

Keywords : Proceedings; Conference; Wavelet analysis; Toulouse (France)
Classification :
∗ *00B25 Proceedings of conferences of miscellaneous specific interest*
42-06 Proceedings of conferences (Fourier analysis)
42C40 Wavelets

Zbl 0821.42018

Meyer, Yves
Wavelets: algorithms and applications. Transl. and rev. by Robert D. Ryan. *(English)*
Philadelphia, PA: SIAM, Society for Industrial and Applied Mathematics. xi, 133 p. £ 19.50/pbk (1993). ISBN 0-89871-309-9/pbk

This book is a counterpart ot the author's fundamental treatise "Ondelettes et opérateurs" (I: 1990; Zbl 0694.41037, II: 1990; Zbl 0745.42011, III: 1991; Zbl 0745.42012). Whereas

the monograph is written for the mathematical analyst, this book emerged from lectures given to a general scientific audience of different specialities. As a consequence the presentation is very informal and mathematical rigor is only secondary.
The main goal of this book is the exposition of the intuitive ideas of wavelet theory and their practical applications. Each chapter starts with the description of an important problem chosen mostly from signal and image processing. After the introduction of a fundamental idea the mathematical concepts are made to emerge naturally and almost without effort. The core of each chapter is then devoted to a detailed discussion of these concepts. Particular emphasis is put on the resulting algorithms that intervene in the practical applications.
For instance, Chapter 4 deals with image representation and compression. An image contains information on different scales, and going from a fine scale to a coarse one results in a tree structure. Thus it should be possible to calculate an approximation of an image at scale 2^{j+1} from a representation at scale 2^j in a natural and efficient fashion. A more technical formulation of this idea leads to the famous pyramid algorithms of image processing and to the mathematical concept of a multiresolution analysis, which are then developed in detail in this chapter.
The scheme "problem – idea – mathematical concept – development of a numerical algorithm" is executed with great art and persuasion. The book is an excellent model of how an important scientific subject can be communicated to a broader audience. It is very enjoyable to read, and even to the expert it provides new insights and points of view.
The book is divided into eleven chapters. Chapter 1 introduces the problems and objectives of signal and image processing, Chapter 2 traces the history of wavelet theory. Chapter 3 and 4 deal with quadrature mirror filters, subband coding, pyramid algorithms, wavelet bases and multiresolution analysis. Chapters 5 to 7 discuss several ideas from time-frequency analysis, such as the Wigner-Ville transform, Malvar bases, and wavelet packets. The remaining chapters give a fascinating account of some more specific applications of wavelet theory in human and computer vision (Marr's program), the wavelet analysis of fractals, wavelets as a tool in the study of turbulence and of distant galaxies. It is significant that wavelet theory is seen as an interdisciplinary field of research with contributions from and implications for mathematics, physics, and signal processing.

K.Gröchenig (Stoors)

Keywords : signal analysis; image processing; multiresolution analysis; mirror filters; subband coding; pyramid algorithms; time-frequency analysis; Wigner-Ville transform; Malvar bases; wavelet packets; computer vision; fractals; wavelets
Classification :
∗*42C40 Wavelets*
42-02 Research monographs (Fourier analysis)
94A12 Signal theory

Zbl 0810.42015

Meyer, Yves

Wavelets and operators. *(English)*

Daubechies, Ingrid (ed.), Different perspectives on wavelets. American Mathematical

Society short course on wavelets and applications, held in San Antonio, TX (USA), January 11-12, 1993. Providence, RI: American Mathematical Society. Proc. Symp. Appl. Math. 47, 35-58 (1993). ISBN 0-8218-5503-4/hbk

This is an expository paper concerned with such topics as linear operators on the L^2-space, singular integral operators, frames in a Hilbert space (a substitute for approximation purposes of an orthonormal system), pseudo-differential operators, wavelets and their use in constructing approximations for linear (bounded or not) operators. Many other related concepts are involved in the presentation. The focus is on the so-called Calderón-Zygmund operators which are defined by

$$(Tf)(x) = p.v. \int K(x,y)f(y)dy,$$

with $K(x,y)$ satisfying the following conditions:

$$(a) \quad K(y,x) = -K(x,y); \qquad (b) \quad |K(x,y)| \leq C_0|x-y|^{-n};$$

(c) there exists an exponent $\gamma \in (0,1)$ and a constant C such that for $|x'-x| \leq |x-y|/2$,

$$|K(x',y) - K(x,y)| \leq C|x'-x|^{\gamma}|x-y|^{-n-\gamma}.$$

Using wavelets one constructs an approximation for the operator T, say T_m, for which the following estimate is indicated: $\|T - T_m\| \leq Cm^{-\gamma}\sqrt{\log m}$, for each $m \geq 1$, with $C = C(n,\gamma)$.

C.Corduneanu (Arlington)

Keywords : singular integral operators; frames; approximation; pseudo-differential operators; wavelets; Calderón-Zygmund operators

Classification :

∗42C40 Wavelets
35S05 General theory of pseudodifferential operators
44A15 Special transforms
42B20 Singular integrals, several variables

Zbl 0796.42019

Meyer, Yves; Paiva, Freddy

Remarks on the construction of orthogonal wavelets. (Remarques sur la construction des ondelettes orthogonales.) *(French)*

J. Anal. Math. 60, 227-240 (1993). ISSN 0021-7670; ISSN 1565-8538
http://www.springerlink.com/content/120600/

The authors show how to construct the sequence of vector spaces $(V_j)_{j\in\mathbb{Z}} \subset L^2(\mathbb{R})$ of a multiresolution analysis from Mallat's function $m_0(\xi) = \sum_{-\infty}^{\infty} \alpha_k \exp(ik\xi)$. The main result shows that the possibility of reconstructing the multiresolution analysis $(V_j)_{j\in\mathbb{Z}}$ from $m_0(\xi)$ is equivalent to the possibility of solving the functional equation $\varphi(x) = 2\sum_{-\infty}^{\infty} \alpha_k\varphi(2x+k)$, where $\varphi \in L^2 \cap L^1$ and $\int_{-\infty}^{\infty} \varphi(x)dx = 1$, by a fixed point method. This leads to studying the convergence in $L^2(\mathbb{R})$ of the sequence $f_j(x)$ defined *by $f_{j+1}(x) = 2\sum_{-\infty}^{\infty} \alpha_k f_j(2x+k)$ when f_0 satisfies certain conditions. The authors also give a proof of A. Cohen's theorem [Ondelettes et traitement numérique du signal, RMA 25, Masson, Paris (1993)].*

J.S.Joel (Kelly)
Keywords : wavelets; orthonormal bases; multiresolution analysis; Mallat's function; functional equation
Classification :
∗*42C40* *Wavelets*
46B15 *Summability and bases in normed spaces*

Zbl 0822.42019

Coifman, R.R.; Meyer, Y.; Wickerhauser, V.
Size properties of wavelet-packets. *(English)*
Ruskai, Mary Beth (ed.) et al., Wavelets and their applications. Boston, MA etc.: Jones and Bartlett Publishers. 453-470 (1992). ISBN 0-86720-225-4/hbk

This paper begins with a very clear description, in Sections 2 through 5, of how wavelet packets are derived from orthonormal bases. Sections 6 and 7 are devoted to a study of the growth properties of the L^∞ and L^p norms of wavelet packets, with the goal of understanding the frequency localization of basic wavelet packets, for which lower bounds are given by the asymptotic behavior of the L^∞ norms of wavelet packets. In the introduction, the authors observe that their results imply that basic wavelet packet functions are not as sharply localized in frequency as one might hope.
In Section 8, the authors investigate an extended notion of wavelet packets, subsets of which are indexed by a covering set (possibly infinite) of dyadic subintervals on an interval. Theorem 6 shows how to construct orthonormal bases of $L^2(\mathbb{R})$ using generalized wavelet packets. In the conclusion, the authors show that Theorem 6 does not completely characterize all possible wavelet-packet orthonormal bases, some of which may correspond to dyadic subintervals which do not cover all but an exceptional denumerable subset of an interval.

C.Pfeffer Johnston (Boca Raton)
Keywords : wavelet packets; frequency localization; subsets; dyadic subintervals
Classification :
∗*42C40* *Wavelets*

Zbl 0792.94004

Coifman, Ronald R.; Meyer, Yves; Wickerhauser, Victor
Wavelet analysis and signal processing. *(English)*
Ruskai, Mary Beth (ed.) et al., Wavelets and their applications. Boston, MA etc.: Jones and Bartlett Publishers. 153-178 (1992). ISBN 0-86720-225-4/hbk

The authors define the concept of "libraries", structures relating to different time scales at different times (or different spatial scales at different locations), with the aim of realizing the basis in which a given signal can be most efficiently superposed in terms of *oscillatory modes on different time scales. This also provides an efficient compression method. The authors discuss two principal libraries, namely trigonometric waveform libraries (which are localized sine transforms) and the wavelet packet library (which contains the wavelet basis, Walsh functions and smooth versions of Walsh functions called wavelet packets). They give a simple characterization of subsets forming orthonormal bases and some examples of specific computations using their process.*

J.S.Joel (Kelly)

Keywords : compression method; trigonometric waveform libraries; wavelet packet library

Classification :

∗ *94A12 Signal theory*

42C40 Wavelets

65T99 Numerical methods in Fourier analysis

Zbl 0788.42012

Coifman, R.R.; Meyer, Y.

Fourier analysis adapted to a partition by Whitney cubes. (Analyse de Fourier adaptée à une partition par des cubes de Whitney.) *(French)*

Colloq. Math. 63, No.1, 111-117 (1992). ISSN 0010-1354; ISSN 1730-6302

http://journals.impan.gov.pl/cm/

http://matwbn.icm.edu.pl/spis.php?wyd=8jez=pl

Given an open domain $\Omega \subset \mathbb{R}^n$, consider a partition of Ω by dyadic cubes Q_j, $j \in J$. These cubes are called Whitney cubes if there exists a constant $C > 0$ such that for all $j \in J$, a distance L_j between Q_j and the boundary $\partial\Omega$ of Ω is connected with the diameter ℓ_j of Q_j by the relation $C^{-1}\ell_j < L_j < C\ell_j$. For every Q_j define a "window" w_j adapted to Q_j as a function belonging to $C_0^\infty(\Omega)$, supported by delated cube $2Q_j$ and such that $|\partial^\alpha w_j(x)| \le C_\alpha \ell_j^{-|\alpha|}$.

The authors pose the following problem: how to choose w_j in such a way that a family $\ell_j^{-n/2} w_j(x)\exp(2\pi ik\cdot x/\ell)$ would constitute an orthonormal basis of $L_2(\Omega)$?

In the paper the answer is given for the case $n = 2$ with the exponent replaced by the function

$$\sin\big[\pi(k_1+\tfrac{1}{2})(x_1-a_j^{(1)})/\ell_j\big]\sin\big[\pi(k_2+\tfrac{1}{2})(x_2-a_j^{(2)})/\ell_j\big],$$

$(a_j^{(1)}, a_j^{(2)})$ being an apex of Q_j.

B.Rubin (Jerusalem)

Keywords : windows; wavelets; Whitney cubes; orthonormal basis

Classification :

∗ *42C15 Series and expansions in general function systems*

Zbl 0782.00087

Ruskai, Mary Beth (ed.); Beylkin, Gregory (ed.); Coifman, Ronald (ed.); Daubechies, Ingrid (ed.); Mallat, Stephane (ed.); Meyer, Yves (ed.); Raphael,

Louise (ed.)
Wavelets and their applications. *(English)*
Boston, MA etc.: Jones and Bartlett Publishers. xiii, 474 p. (1992). ISBN 0-86720-225-4/hbk

The articles of this volume will be reviewed individually.

Keywords : Wavelets
Classification :
∗ *00B25 Proceedings of conferences of miscellaneous specific interest*
46-06 Proceedings of conferences (functional analysis)
42Cxx Non-trigonometric Fourier analysis

Zbl 0776.42019

Meyer, Yves
Wavelets and operators. Translated by D. H. Salinger. *(English)*
Cambridge Studies in Advanced Mathematics. 37. Cambridge: Cambridge University Press. xv, 223 p. £27.95/hbk; £ 49.95/hbk (1992). ISBN 0-521-42000-8/hbk

See the review of the French original edition [Ondelettes et opérateurs. I: Ondelettes] (1990) in Zbl 0694.41037. For Parts II and III of this series see Zbl 0745.42011 and Zbl 0745.42012, respectively.

Keywords : BMO; functions of bounded mean oscillation; singular integral operators; Besov spaces; Sobolev spaces; multiresolution analysis; ondelettes; wavelets; Fourier transform; boundedness of a discrete Hilbert transform; inequality of Bernstein; oblique structure; frame; Hardy space
Classification :
∗ *42C40 Wavelets*
42B20 Singular integrals, several variables
42-02 Research monographs (Fourier analysis)

Zbl 0773.42019

Meyer, Yves
Wavelets and concurrent algorithms. (Ondelettes et algorithmes concurrents.) *(French)*
Actualités Scientifiques et Industrielles. 1435. Paris: Hermann, Éditeurs des Sciences et des Arts. 84 p. (1992). ISBN 2-7056-1435-5

This is a new book of Prof. Yves Meyer who is a well known expert in wavelet analysis and in its applications. The book is mainly devoted to new algorithms which work well in image- and signal-processing and cover the situations when usual algorithms based on wavelets are not suitable.
The main topics of the book are the following: Fourier transform and its discrete modifications, Malvar wavelets [cf. H. S. Malvar, IEEE Trans. Acoust. Speech Signal

Process. 38, 969-978 (1990)], filtering and sampling, multiresolution analysis, pyramid algorithms of Burt and Adelson, non-orthogonal and orthogonal wavelets.

B.Rubin (Jerusalem)

Keywords : pyramid algorithms of Burt and Adelson; algorithms; Fourier transform; Malvar wavelets; filtering; sampling; multiresolution analysis; pyramid algorithms; non-orthogonal and orthogonal wavelets

Classification :

∗ *42C40 Wavelets*

Zbl 0772.26009

Dobyinsky, Sylvia; Meyer, Yves

Lemme div-curl et renormalisation. (Div-curl lemma and renormalization). *(French)*

Sémin. Équ. Dériv. Partielles, Éc. Polytech., Cent. Math., Palaiseau 1991-1992, No.II, 4 p. (1992).

The authors present a new proof based on renormalization operators of P. L. Lion's result:

Let $E(x) = (E_1(x), \ldots, E_n(x))$, $B(x) = (B_1(x), \ldots, B_n(x))$, $E_j(x) \in L^p(\mathbb{R}^n)$, $1 \le j \le n$, $B_j(x) \in L^q(\mathbb{R}^n)$, $1 \le j \le n$, $1 < p < \infty$, $1/p + 1/q = 1$, *div* $E(x) = 0$, *rot* $B(x) = 0$. *Then the scalar product* $E(x) \cdot B(x)$ *belongs to the space* $\mathcal{H}^1(\mathbb{R}^n)$ *of Stein and Weiss.*

G.Bruckner (Berlin)

Keywords : div-curl lemma; renormalization operators

Classification :

∗ *26B35 Special properties of functions of several real variables*

35S05 General theory of pseudodifferential operators

Zbl 0742.00079

Meyer, Yves (ed.)

Wavelets and applications. Proceedings of the 2nd international conference, held in Marseille, France, May 1989. *(English)*

Recherches en Mathématiques Appliquées. 20. Paris etc., Berlin etc.: Masson, Springer-Verlag. 450 p. with 10 color plates (1992). ISBN 2-225-82550-5; ISBN 3-540-54516-6

The articles of this volume will be reviewed individually.

Keywords : Marseille (France); Wavelets; Proceedings; Conference

Classification :

∗ *00B25 Proceedings of conferences of miscellaneous specific interest*

Zbl 0763.35014

DiPerna, R.J.; Lions, P.L.; Meyer, Y.

L^p **regularity of velocity averages.** *(English)*

Ann. Inst. Henri Poincaré, Anal. Non Linéaire 8, No.3-4, 271-287 (1991). ISSN 0294-1449
numdam:AIHPC_1991__8_3-4_271_0
http://www.sciencedirect.com/science/journal/02941449

The paper is concerned with the regularity of velocity averages for solutions of transport equations:

$$(1) \qquad v.\nabla_x f = g \quad \text{for } x \in \mathbb{R}^N, \quad v \in \mathbb{R}^N, \quad \text{or}$$

$$(2) \qquad \partial f/\partial t + v.\nabla_x f = g \quad \text{for } x \in \mathbb{R}^N, \quad v \in \mathbb{R}^N, \quad t \in \mathbb{R}.$$

In the time-independent case (equation (1)), the authors prove that, if $f \in L^p(\mathbb{R}^N \times \mathbb{R}^N)$ *and* $g \in L^p(\mathbb{R}^N \times \mathbb{R}^N)$, $1 < p \leq 2$, *then for every* $\psi \in D(\mathbb{R}^N)$, *the velocity average* $\bar{f}(x) = \int_{\mathbb{R}^N} f(x,v)\psi(v)dv$ *belongs to the Besov space* $B_2^{s,p}(\mathbb{R}^N)$ *where* $s = 1/p'$.
In the time dependent case (equation (2)), they prove similar results when g admits the following decomposition: $g = (I - \Delta_x)^{\tau/2}(I - \Delta_v)^{m/2}G$, $\tau \in [0,1)$, $m \geq 0$, $G \in L^p(\mathbb{R}^N \times \mathbb{R}^N \times \mathbb{R})$.
Some applications to Vlasov-Maxwell systems and to other models are also given. These results extend many previous results [see for example F. Golse, P. L. Lions, B. Perthame, R. Sentis, J. Funct. Anal. 76, No. 1, 110-125 (1988; Zbl 0652.47031)].

J.-P.Raymond (Toulouse)
Keywords : Littlewood-Paley type decompositions; interpolating arguments; spectral decomposition; Sobolev; Besov spaces; $L(\sup p)$*-multipliers; transport equations; time-independent case; time dependent case; Vlasov-Maxwell systems*
Classification :
- ∗*35B65 Smoothness of solutions of PDE*
- *35F05 General theory of first order linear PDE*
- *35Q40 PDE from quantum mechanics*
- *42B25 Maximal functions*
- *42B30 Hp-spaces (Fourier analysis)*
- *46E35 Sobolev spaces and generalizations*
- *42B15 Multipliers, several variables*
- *82B40 Kinetic theory of gases*
- *42B20 Singular integrals, several variables*

Zbl 0757.42014

Meyer, Y.

Orthonormal wavelets. *(English)*

Frontiers in pure and applied mathematics, Coll. Pap. Ded. J.-L. Lions Occas. 60th Birthday, 235-245 (1991).

[For the entire collection see Zbl 0722.00015.]
This is a short summary of the development of orthonormal wavelets up to the date when the paper was written. For instance, the Stromberg wavelets are discussed, and

the notion of multiresolution analysis is studied. Wavelet decomposition is compared with quadrature mirror filtering, and a discussion on the applications of wavelets to science and technology is presented.

C.K.Chui (College Station)

Keywords : orthonormal wavelets; Stromberg wavelets; multiresolution analysis; decomposition; quadrature mirror filtering
Classification :
∗*42C40* *Wavelets*

Zbl 0753.42015

Meyer, Yves

Ondelettes sur l'intervalle. (Wavelets on the interval). *(French)*

Rev. Mat. Iberoam. 7, No.2, 115-133 (1991). ISSN 0213-2230
http://projecteuclid.org/rmi
http://www.uam.es/departamentos/ciencias/matematicas/ibero/irevista.htm

The results concerning the wavelets analysis on the whole real line became classical due to achievements of I. Daubechies, Y. Meyer, S. Mallat and of some other authors. In the present paper the author has solved the difficult problem, how to construct multiresolution analysis of L^2*-space and the orthonormal basis of wavelets* $\{\psi_I\}$, $I \in J$, *on the finite interval* $[0,1]$. *The results are used for the characterization of the Hölder space* $C^s[0,1]$ *and of the space BMO[0,1]. The Calderón-Zygmund operators on the interval [0,1] are considered.*

B.Rubin (Jerusalem)

Keywords : wavelets; multiresolution analysis; Hölder space; BMO; Calderón- Zygmund operators
Classification :
∗*42C40* *Wavelets*

Zbl 0748.42013

Meyer, Yves F.

Wavelets and applications. *(English)*

Proc. Int. Congr. Math., Kyoto/Japan 1990, Vol. II, 1619-1626 (1991). ISBN 4-431-70047-1/hbk

[For the entire collection see Zbl 0741.00020.]
This is a brief but very substantial survey of wavelet theory. After a short historical introduction the author gives a sketch of the following issues: the windowed Fourier transform, wavelets with constant shape, pyramidal algorithms, quadrature mirror filters, orthonormal wavelets, wavelet packets.

B.Rubin (Jerusalem)

Keywords : applications; survey; windowed Fourier transform; wavelets with constant shape; pyramidal algorithms; quadrature mirror filters; orthonormal wavelets; wavelet packets
Classification :
∗42C40 *Wavelets*
42-02 *Research monographs (Fourier analysis)*

Zbl 0748.42012

Coifman, Ronald R.; Meyer, Yves

Remarques sur l'analyse de Fourier à fenêtre. (Remarks on windowed Fourier analysis). *(French)*

C. R. Acad. Sci., Paris, Sér. I 312, No.3, 259-261 (1991). ISSN 0764-4442

Summary: Dans le contexte général de l'analyse de Fourier à fenêtre, nous construisons de nouvelles bases orthonormées que l'on peut adapter à n'importe quelle segmentation.

Keywords : windowed Fourier analysis; orthonormal bases
Classification :
∗42C15 *Series and expansions in general function systems*

Zbl 0745.42012

Meyer, Yves; Coifman, R.R.

Wavelets and operators III: Multilinear operators. (Ondelettes et opérateurs III: Opérateurs multilinéaires.) *(French)*

Actualités Mathématiques. Paris: Hermann, Éditeurs des Sciences et des Arts. xii, p. 383-532 (1991). ISBN 2-7056-6127-1

The main contents of this volume are multilinear operators and some of their applications as a continuation of the previous two volumes [for Part I (1990) see Zbl 0694.41037 and Part II the preceding review].
Chapter XII. Generalized Hardy spaces.
Let Γ *be the graph in the complex plane of a uniformly Lipschitzian function* $a:\mathbb{R}\to\mathbb{R}$. *Let* $\Omega_1=\{(x,y): y>a(x)\}$ *and* $\Omega_2=\{(x,y): y<a(x)\}$. *A holomorphic function* $F(z)$ *in* Ω_1 *is said to belong to* $H^p(\Omega_1)$, $0<p<\infty$, *if*

$$\sup_{\tau>0}\Big(\int_{\Gamma+i\tau}|F(z)|^p ds\Big)^{1/p}=\|F\|_{H^p(\Omega_1)}<\infty.$$

Suppose $1<p<\infty$. *If* $F\in H^p(\Omega_1)$, *then as* $\tau\downarrow 0$ *the sequence of functions* $F_\tau(z)=F(z+i\tau)$ *converges in* $L^p(\Gamma)$ *to a function which is denoted by* $\theta(F)$ *and called the trace of* F *on* Γ. *The operator* $\theta: H^p(\Omega_1)\to L^p(\Gamma)$ *is an isomorphism between* $H^p(\Omega_1)$ *and a closed subspace of* $L^p(\Gamma)$ *which is again denoted by* $H^p(\Omega_1)$.
The object of this chapter is to prove the theorem of G. David which asserts that for a rectifiable Jordan curve Γ *a necessary and sufficient condition in order that* $L^p(\Gamma)$ *coincides with the direct sum of* $H^p(\Omega_1)$ *and* $H^p(\Omega_2)$ *is that* Γ *is regular in the sense*

of Ahlfors, i.e. there exists a constant $C > 1$ such that the measure of the set $\{z \in \mathbb{C} : |z - z_0| \leq R\} \cap \Gamma$ does not exceed CR for any $z_0 \in \Gamma$.
Chapter XIII. Multilinear operators.
Let $A = A(\mathbb{R}^n)$ be the Wiener algebra, which is the totality of Fourier transforms $\hat{f}$ of functions $f \in L^1(\mathbb{R}^n)$ with the norm $\|\hat{f}\|_A = \|f\|_{L^1(\mathbb{R}^n)}$. It is shown that the $k+1$-linear continuous operator $\pi : A^k \times L^2 \to L^2$ which commutes with translations has a symbol $\tau \in L^\infty(\mathbb{R}^{n(k+1)})$ in the sense that

$$\pi(a_1, \dots, a_k, f) = (2\pi)^{-n(k+1)} \iint e^{ix(\xi+\tilde{\eta})} \tau(\eta, \xi) \hat{a}(\eta) \hat{f}(\xi) d\eta d\xi, \tag{2.1}$$

where $a(x) = a_1(x_1) \dots a_k(x_k)$, $x_1 \in \mathbb{R}^n, \dots, x_k \in \mathbb{R}^n$, $\eta = (\eta_1, \dots, \eta_k)$, $\tilde{\eta} = \eta_1 + \dots + \eta_k$. Furthermore, if π commutes with dilations, then τ is homogeneous of degree 0. Conversely, if $\tau \in L^\infty(\mathbb{R}^{n(k+1)})$, then (2.1) defines a multilinear operator which commutes with translations, and we have

$$\|\pi(a_1, \dots, a_k, f)\|_2 \leq \|\tau\|_\infty \|a_1\|_A \cdots \|a_k\|_A \|f\|_2.$$

Conditions in order that the operator π is extended by continuity to an operator: $C_0^k \times L^2 \to L^2$ and to an operator: $(BMO)^k \times L^2 \to L^2$ are given in terms of the symbol τ, where C_0 and BMO are the set of continuous functions on $\mathbb{R}^n$ vanishing at ∞ and the set of functions of bounded mean oscillation on $\mathbb{R}^n$, respectively.
These results are applied to the study of the Taylor expansion of a holomorphic functional from the open ball of $L^\infty(\mathbb{R}^n)$ to $\mathcal{L}(L^2(\mathbb{R}^n), L^2(\mathbb{R}^n))$ and the Hilbert transform on a Lipschitz curve.
The remaining part of this chapter is devoted to Calderón's program on the commutator of the Hilbert transform and an operator of pointwise multiplication by a function of bounded mean oscillation, and the formalism of McIntosh.
Chapter XIV. Multilinear analysis of the square root of accretive differential operators.
Let $A(x) = (a_{i,j}(x))_{i,j=1}^n$ be an $n \times n$ matrix with elements belonging to $L^\infty(\mathbb{R}^n)$. Suppose that there exists a constant $\delta > 0$ such that

$$Re \sum_{j,k=1}^n a_{j,k}(x) \zeta_j \bar{\zeta}_k \geq \delta(|\zeta_1|^2 + \dots + |\zeta_n|^2).$$

Let T_B be the operator associated with the sesquilinear form

$$B(f,g) = \sum_{j,k=1}^n \int_{\mathbb{R}^n} a_{j,k}(x) \frac{\partial f}{\partial x_j} \frac{\partial \bar{g}}{\partial x_k} dx, \quad f, g \in H^1(\mathbb{R}^n).$$

Namely,

$$B(f,g) = (T_B(f), g), \quad f \in V = D(T_B), \quad g \in H^1.$$

The opertor T_B is maximal accretive, and formally

$$T_B(f) = -div(A(x) \nabla f).$$

This chapter is devoted to the proof of the following theorem which is related to Kato's conjecture that the domain of the square root $T_B^{1/2}$ of T_B coincides with $H^1(\mathbb{R}^n)$: For any $n \geq 1$ there exists a constant $\varepsilon(n) > 0$ such that if $\|I - A(x)\|_\infty < \varepsilon(n)$, the square *root of* T_B *is written as*

$$T_B^{1/2} = \sum_{j=1}^{n} R_j(A) D_j, \quad D_j = -i\partial/\partial x_j,$$

where $R_j(A)$ *is a holomorphic function of* $A(x)$ *in the open set* $\|I - A(x)\|_\infty < \varepsilon(n)$ *of* $(L^\infty(\mathbb{R}^n))^{n^2}$ *taking values in* $\mathcal{L}(L^2(\mathbb{R}^n),\ L^2(\mathbb{R}^n))$*.* $R_1(I), \dots, R_n(I)$ *are the usual Riesz transformations. The operators* R_j*,* $1 \le j \le n$*, can be extended to operators from* $L^p(\mathbb{R}^n)$ *to* $L^p(\mathbb{R}^n)$ *for* $2 < p < \infty$ *and from* $L^\infty(\mathbb{R}^n)$ *to* $BMO(\mathbb{R}^n)$*. Using this theorem Kato's conjecture is solved affirmatively if* $\|I - A(x)\|_\infty < \varepsilon(n)$*. The theorem is proved with the aid of the analysis in the formalism of McIntosh of the previous chapter.*
Chapter XV. Potential theory in Lipschitzian domains.
Let Ω *be a bounded Lipschitzian domain in* $\mathbb{R}^n$*. To each point* $x \in \partial\Omega$ *a cone* $\Gamma(x)$ *is attached so that it lies in* Ω *and* x *is its vertex. A harmonic function* u *in* Ω *is said to belong to* $\mathcal{H}^2(\Omega)$ *if the function* $u^*(x) = \sup_{y\in\Gamma(x)} |u(y)|$ *belongs to* $L^2(\partial\Omega, d\sigma)$*, where* $d\sigma$ *is the surface measure of* $\partial\Omega$*. Let* $\mathcal{K}$ *be the operator of double layer potential, and* K *be the operator defined by*

$$Kf(x) = \frac{1}{\omega_n} \lim_{\varepsilon\to 0} \int_{|y-x|\ge\varepsilon} (y-x)n(y)|y-x|^{-n-1} f(y)d\sigma(y),$$

where ω_n *is the surface of the unit sphere of* $\mathbb{R}^n$ *and* $n(y)$ *is the normal vector at* y *which exists almost everywhere on* $\partial\Omega$*.*
The following result on the Dirichlet problem is proved: The mapping

$$\frac{1}{2} + K : L^2(\partial\Omega, d\sigma) \to L^2(\partial\Omega, d\sigma)$$

is an isomorphism. If $g \in L^2(\partial\Omega, d\sigma)$*, the harmonic function* $u = \mathcal{K}(\frac{1}{2} + K)^{-1} g$ *is the unique solution of the following Dirichlet problem*

$$\Delta u = 0 \text{ in } \Omega, \quad u^* \in L^2(\partial\Omega, d\sigma), \quad g(x) = \lim_{g\in\Gamma(x), y\to x} u(y) \ d\sigma\text{-almost everywhere.}$$

The corresponding result for the Neumann problem is given with $K, \mathcal{K}, u^* \in L^2(\partial\Omega, d\sigma)$ *replaced by the adjoint operator of* K*, the operator of simple layer potential and* $(\nabla u)^* \in L^2(\partial\Omega, d\sigma)$*, respectively.*
Chapter XVI. Paradifferential operators.
This chapter is devoted to the study of the algebra $B^r (r > 0)$ *of paradifferential operators introduced by J. M. Bony which contains the class* $O_p S^0_{1,0}$ *of classical pseudo-differential operators and operators of paraproduct with functions of class* C^r*.*
After stating the definition of paradifferential operators the author describes symbolic calculus of these operators, and an application to nonlinear partial differential equations. This volume terminates with an approximation of the operator of paraproduct with a function in the class C^r*,* $0 < r < 1$*, by an operator associated with multiresolution analysis.*

H. Tanabe (Toyonaka)

Keywords : multilinear operators; generalized Hardy spaces; BMO; functions of bounded mean oscillation; Hilbert transform on a Lipschitz curve; commutator; square root of accretive differential operators; Lipschitzian domain; Neumann problem; paradifferential

operators; pseudo-differential operators; paraproduct; multiresolution analysis
Classification :
∗ *42B20 Singular integrals, several variables*
42C40 Wavelets
47G30 Pseudodifferential operators
47H06 Accretive operators, etc. (nonlinear)

Zbl 0737.46011

Bourdaud, Gérard; Meyer, Yves

Fonctions qui opèrent sur les espaces de Sobolev. (Functions that operate on Sobolev spaces). *(French)*

J. Funct. Anal. 97, No.2, 351-360 (1991). ISSN 0022-1236
http://dx.doi.org/10.1016/0022-1236(91)90006-Q
http://www.sciencedirect.com/science/journal/00221236

Les auteurs cherchent à étendre des résultats concernant les fonctions F de la variable réelle opèrant dans un ensemble.
Ils montrent que si p et $q \in [1,+\infty]$, $s \in]0,\frac{1}{p}[$, $F : t \in \mathbb{R} \to |t|$ opère dans l'espace de Besov $B_p^{s,q}(\mathbb{R})$ et que l'on a plus prècisément: $\exists c \| |f| \|_{B_p^{s,q}} \le c\|f\|_{B_p^{s,q}}$ (th. 2), et donnent un contre exemple lorsque $p,q \in [1,+\infty]$ et $s \ge 1+\frac{1}{p}$.

M.-T.Lacroix (Saone)

Keywords : functions that operate on Sobolev spaces; Besov space
Classification :
∗ *46E35 Sobolev spaces and generalizations*

Zbl 0745.42011

Meyer, Yves

Wavelets and operators II: Calderón-Zygmund operators. (Ondelettes et opérateurs II: Opérateurs de Calderón-Zygmund.) *(French)*

Actualités Mathématiques. Paris: Hermann, Éditeurs des Sciences et des Arts. xii, p. 217-381 (1990). ISBN 2-7056-6126-7

The main contents of this volume are the studies of Calderón-Zygmund operators with the aid of the theory of ondelettes (= wavelets). [For Part I (1990) see Zbl 0694.41037 and Part III (1991) the following review.]
The first chapter, Chapter VII, consists of the definition of Calderón- Zygmund operators and their fundamental properties. Let T be a linear continuous operator from $\mathcal{D}(\mathbb{R}^n)$ to $\mathcal{D}'(\mathbb{R}^n)$ with distribution kernel S. The restriction of S to $\Omega = \{(x,y) \in \mathbb{R}^n \times \mathbb{R}^n : x \ne y\}$ is denoted by K. It is said that T is an operator of Calderón-Zygmund if the following conditions are satisfied:
(1) there exists a constant C_0 such that $K(x,y)$ is a locally integrable function satisfying $|K(x,y)| \le C_0|x-y|^{-n}$,

(2) there exist an exponent $\gamma \in]0,1]$ and a constant C_1 such that if $(x,y) \in \Omega$ and $|x'-x| \le \frac{1}{2}|x-y|$, then

$$|K(x',y) - K(x,y)| \le C_1|x'-x|^\gamma |x-y|^{-n-\gamma},$$

(3) analogously if $|y'-y| \le \frac{1}{2}|x-y|$ then

$$|K(x,y') - K(x,y)| \le C_1|y'-y|^\gamma |x-y|^{-n-\gamma},$$

(4) T is extended to a linear continuous operator in $L^2(\mathbb{R}^n)$.
It is shown that Calderón-Zygmund operators are bounded mappings from $L^p(\mathbb{R}^n)$ to itself for any $p \in]1,\infty[$, from the Hardy space $H^1(\mathbb{R}^n)$ to $L^1(\mathbb{R}^n)$, and from $L^\infty(\mathbb{R}^n)$ to the space BMO$(\mathbb{R}^n)$ of functions of bounded mean oscillations. Since $1 \in L^\infty(\mathbb{R}^n)$, $T(1)$ and $T^*(1)$ are defined. It is shown that T is extended to a linear continuous operator in $H^1(\mathbb{R}^n)$ (resp. BMO$(\mathbb{R}^n)$) if and only if $T^*(1) = 0$ (resp. $T(1) = 0$). The author then gives piecewise estimates of Calderón-Zygmund operators and the reconstruction of these operators by their kernels which are extensions of the corresponding results for classical singular integral operators of Calderón-Zygmund type.
Chapter VIII is devoted to the proof of the $T(1)$ theorem of David and Journé and its applications to the study of algebras of Calderón- Zygmund operators. Let V be a topological vector space such that $\mathcal{D}(\mathbb{R}^n) \subset V \subset L^2(\mathbb{R}^n) \subset V' \subset \mathcal{D}'(\mathbb{R}^n)$, e.g. $V = C_0^1(\mathbb{R}^n)$. A linear continuous operator $T : V \to V'$ is said to be weakly continuous on $L^2(\mathbb{R}^n)$ if there exist a constant C and an integer q such that for any ball $B \subset \mathbb{R}^n$ and for any pair $f, g \in V$ with support contained in B

$$|\langle Tf, g\rangle| \le CN_q^B(f)N_q^B(g),$$

where $N_q^B(f) = R^{n/2}\sum_{|\alpha|\le q} R^{|\alpha|}\|\partial^\alpha f\|_\infty$ and R is the radius of B. A linear continuous operator $T : V \to V'$ is said to be associated with a singular integral if there exist an exponent $\gamma \in]0,1]$, two constants C_0, C_1 and a function $K : \Omega \to \mathbb{C}$ such that (1), (2) and (3) hold, and

$$Tf(x) = \int K(x,y)f(y)dy$$

for any $f \in V$ and x not belonging to the support of f. For such an operator $T(1)$ is defined modulo a constant. The following $T(1)$ theorem is due to David and Journé.
Let $T : V \to V'$ be a linear continuous operator associated with a singular integral. Then a necessary and sufficient condition in order that T is extended to a continuous operator in $L^2(\mathbb{R}^n)$ is that the following three conditions are satisfied:
(a) $T(1)$ belongs to BMO$(\mathbb{R}^n)$,
(b) ${}^tT(1)$ belongs to BMO$(\mathbb{R}^n)$,
(c) T is weakly continuous on $L^2(\mathbb{R}^n)$.
The necessity part is a direct consequence of the previous chapter. The sufficiency part is established as follows. First consider the case $T(1) =^t T(1) = 0$. In this case some estimates of $\tau(\lambda,\lambda') = \langle T(\psi_\lambda), \psi_{\lambda'}\rangle$ are obtained, where ψ_λ, $\lambda \in \Lambda$, are ondelettes of class C^1 and with compact support. Then, the desired result is proved with the aid of Schur's lemma on infinite matrices. In the general case two operators R and S of Calderón-Zygmund are constructed so that

$$R(1) = T(1),\ {}^tR(1) = 0,\ S(1) = 0,\ {}^tS(1) =^t T(1).$$

Then $N = T - R - S$ is an operator associated with a singular integral which satisfies the conditions of the special case. Hence, R, S, N are all continuous in $L^2(\mathbb{R}^n)$, and so is T.
Chapter IX is devoted to examples of Calderón-Zygmund operators. A necessary and sufficient condition in order that an operator $T : \mathcal{S}(\mathbb{R}^n) \to \mathcal{S}'(\mathbb{R}^n)$ belongs to some class of pseudodifferential operators is given in terms of the matrix with elements $\langle T(\psi_\lambda), \psi_{\lambda'}\rangle$, $\lambda \in \Lambda$, $\lambda' \in \Lambda$. It is shown that the commutators of operators of pointwise multiplication by Lipschitz continuous functions and pseudodifferential operators of order 1 are operators of Calderón-Zygmund. The L^2 continuity of the operator v.p.$(A(x) - A(y))^k/(x - y)^{k+1}$ is studied, where A is a Lipschitz continuous function.
Chapter X is concerned with conditions in order that operators of Calderón-Zygmund are continuous in the class of Hölder continuous functions, Besov spaces and Sobolev spaces.
The main topic of Chapter XI is the $T(b)$ theorem which is a generalization of the $T(1)$ theorem of Chapter VIII. Let $b \in L^\infty(\mathbb{R}^n)$ be a function satisfying $Reb(x) \geq 1$. Then, for a given multiresolution analysis V_j, $-\infty < j < \infty$, one can construct ondelettes $\tilde\psi_\lambda$, $\lambda \in \Lambda$, so that $\tilde\psi_\lambda \in V_{j+1}$ for $\lambda \in \Lambda_j$, and $\{\tilde\psi_\lambda\}$ are biorthogonal with respect to the bilinear form $B(f,g) = \int_{\mathbb{R}^n} f(x)g(x)b(x)dx$. Let $T : \mathcal{D}(\mathbb{R}^n) \to \mathcal{D}'(\mathbb{R}^n)$ be a linear continuous operator such that the restriction of its distribution kernel to Ω is $b(x)K(x,y)b(y)$, where $K(x,y)$ is a function satisfying (1), (2), (3). Then $T(1)$, ${}^tT(1)$ can be defined, and one says that $T(1) \in BMO_b$ if $\sum_{Q(\lambda)\subset Q} |\langle T(1), \tilde\psi_\lambda\rangle|^2 \leq C|Q|$. Then, the $T(b)$ theorem asserts that T is extended to a linear continuous operator in $L^2(\mathbb{R}^n)$ if and only if T is weakly continuous and $T(1) \in BMO_b$, ${}^tT(1) \in BMO_b$.

H. Tanabe (Toyonaka)
Keywords : Calderón-Zygmund operators; ondelettes; wavelets; BMO; functions of bounded mean oscillations; singular integral operators; $T(1)$ theorem; pseudodifferential operators; commutators; Lipschitz continuous function; Hölder continuous functions; Besov spaces; Sobolev spaces; $T(b)$ theorem; multiresolution analysis
Classification :
- ∗ *42B20 Singular integrals, several variables*
- *42C40 Wavelets*
- *47G10 Integral operators*
- *47G30 Pseudodifferential operators*

Zbl 0735.42017

Meyer, Yves

Construction de bases orthonormées d'ondelettes. (Construction of othonormal bases of wavelets). *(French)*
Colloq. Math. 60/61, No.1, 141-149 (1990). ISSN 0010-1354; ISSN 1730-6302
http://journals.impan.gov.pl/cm/
http://matwbn.icm.edu.pl/spis.php?wyd=8jez=pl

Let $\mathcal{S}(\mathbb{R})$ be the space of Schwartz tempered functions. A multiresolution analysis is an increasing subsequence of closed subspaces V_j of $L^2(\mathbb{R})$, $j \in \mathbb{Z}$, such that

(a) $\bigcap_{j=-\infty}^{\infty} V_j = \{0\}$, $\bigcup_{j=-\infty}^{\infty} V_j$ *is dense in* $L^2(\mathbb{R})$,
(b) $\forall j \in \mathbb{Z}$, $\forall f \in L^2(\mathbb{R})$, $f(x) \in V_j \Leftrightarrow f(2x) \in V_{j+1}$,
(c) $\forall j \in \mathbb{Z}$, $\forall f \in L^2(\mathbb{R})$, $f(x) \in V_0 \Leftrightarrow f(x-k) \in V_0$,
(d) There is a function $g \in \mathcal{S}(\mathbb{R})$ *such that the sequence* $g(x-k)$, $k \in \mathbb{Z}$, *is a Riesz basis for* V_0.
It is possible to show that such a multiresolution analysis can be used to construct wavelets.
In this paper, the author shows that in $\mathbb{R}^2$, *not all wavelets can be constructed in this way. The problem is still open in* $\mathbb{R}$ *where the author proves some partial results.*

D.C.Struppa (Fairfax)

Keywords : orthonormal bases; space of Schwartz tempered functions; multiresolution analysis; Riesz basis; wavelets

Classification :

∗*42C10 Fourier series in special orthogonal functions*
42C40 Wavelets
46F05 Topological linear spaces of test functions and distributions

---

Zbl 0705.46015

Coifman, R.; Lions, P.L.; Meyer, Y.; Semmes, S.

Compacité par compensation et espaces de Hardy. (Compactness by compensation and Hardy spaces.). *(French)*

Sémin. Équations Dériv. Partielles 1989-1990, No.14, 8 p. (1990).

numdam:SEDP_1989-1990____A16_0

For the nonlinear expressions $\det(\nabla u)$, $\sum \frac{\partial u_i}{\partial x_j}\frac{\partial u_j}{\partial x_i}$ *and* $\sum u_i v_i$, *better regularity properties are obtained. They consist in substituting a generalized Hardy space for* L^1. *Applications (e.g. Navier-Stokes equations) and extensions are discussed.*

C.Marinov

Keywords : compactness by compensation; Hardy space; weak solutions; regularity properties; Navier-Stokes equations

Classification :

∗*46E35 Sobolev spaces and generalizations*
46J15 Banach algebras of differentiable functions
35Q30 Stokes and Navier-Stokes equations
76D05 Navier-Stokes equations (fluid dynamics)

Zbl 0694.41037

Meyer, Yves

Ondelettes et opérateurs I: Ondelettes. *(French)*

Actualités Mathématiques. Paris: Hermann, Éditeurs des Sciences et des Arts. xii, 215 p. sFr. 186.00 (1990). ISBN 2-7056-6125-0

This book is concerned with the basic theory of ondelettes (= wavelets) which is useful in locating discontinuities or singularities of a given function and important in mechanics and engineering. The theory is now being developped and the book is an excellent introduction to it.
In Chapter I a theorem on whether a function f whose Fourier transform has a support contained in [-T,T] can be decided by its sampling f(kδ*),* $k \in \mathbb{Z}$*, or not is given. Next, the boundedness of a discret Hilbert transform in* $\ell^p(\mathbb{Z})$ *is proved. The final part of this chapter is a brief history of the theory of ondelettes. The study by A. Grossmann and J. Morlet is described concerning the expression of a function in the space* $\mathbb{H}^2(\mathbb{R})$ *which is the set of all functions f such that* $f(x+iy)$ *is holomorphic in the demiplane* $y > 0$ *and* $\|f(\cdot+iy)\|_2$ *is bounded for* $y > 0$ *by an integral of* $\psi_{(a,b)}(t) = a^{-1/2}\psi((t-b)/a)$*,* $a > 0$*,* $b \in \mathbb{R}$*, where* $\psi(t) = (t+i)^{-2}$*.*
In Chapter II there is given the definition of a multi-resolution analysis of $L^2(\mathbb{R}^n)$ *which is an increasing sequence* V_j*,* $j \in \mathbb{Z}$*, of closed subspaces of* $L^2(\mathbb{R}^n)$ *satisfying (1)* $\cap_{j=-\infty}^{\infty} V_j = \{0\}$*,* $\cup_{j=-\infty}^{\infty} V_j$ *is dense in* $L^2(\mathbb{R}^n)$*, (2)* $\forall f \in L^2(\mathbb{R}^n)$*,* $\forall j \in \mathbb{Z}$ $f(x) \in V_j \Leftrightarrow f(2x) \in V_{j+1}$*, (3)* $\forall f \in L^2(\mathbb{R}^n)$*,* $\forall k \in \mathbb{Z}^n$ $f(x) \in V_0 \Leftrightarrow f(x-k) \in V_0$*, (4) There exists a function* $g \in V_0$ *such that g(x-k),* $k \in \mathbb{Z}^n$ *is a Riesz base of* V_0*. A multiresolution analysis is called r-regular if we can choose the function g(x) in (4) so that* $|g(x)| \leq C_m(1+|x|)^{-m}$ *for any multiindices* α *such that* $|\alpha| \leq r$ *and for any* $m \in N$*. Several examples are given. One of them is the space of splines of order r for* $n = 1$ *in which* V_0 *is the set of all functions of class* C^{r-1} *whose restrictions to* $[k, k+1[$*,* $k \in \mathbb{Z}$*, are polynomials of order* $\leq r$*. The function g in (4) is* χ **...**χ *(r-times) in this example, where* χ *is the characteristic function of the interval [0,1].*
It is shown that there exists a function ϕ *such that* ϕ *(x-k), k* $\mathbb{Z}^n$*, forms an orthonormal base of* V_0*. Let* E_j *be the orthogonal projection onto* V_j*.* E_j *has a kernel* $E_j(x,y) = 2^{nj}E(2^jx, 2^jy)$ *where* $E(x,y) = \sum \phi(x-k)\bar{\phi}(y-k)$*, and* $E_jf(x)$ *may be considered as a sampling of f at the lattice points* $\Gamma_j = 2^{-j}\mathbb{Z}^n$*. By definition* $E_jf \to f$ *in* $L^2(\mathbb{R}^n)$ *for* $f \in L^2(\mathbb{R}^n)$*. The same holds with* $L^2(\mathbb{R}^n)$ *replaced by the Sobolev space* $H^s(\mathbb{R}^n)$*. A multiresolution analysis is also considered in* $L^p(\mathbb{R}^n)$ *defining* $V_j(p)$ *appropriately starting from* V_j*. The inequality of Bernstein* $\|\partial^\alpha f\|_p \leq C2^{|\alpha|j}\|f\|_p$ *is established for* $f \in V_j(p)$*,* $|\alpha| \leq r$*. The following remarkable inequality is proved:* $\int_{-\infty}^{\infty} E(x,y)y^\alpha dy = x^\alpha$ *for* $|\alpha| \leq r$*. It is also shown that* E_j *is a pseudodifferential operator whose symbol has an explicite representation with the use of* ϕ*.*
Chapter III. A function ψ *(x) of a real variable is called an ondelette of class m if (a)* ψ *and its derivatives of order up to m belong to* $L^\infty(R)$*, (b) the functions in (a) are rapidly decreasing at infinity, (c) its moment of order up to m vanish, and (d)* $2^{j/2}\psi(2^jx-k)$*,* $j \in \mathbb{Z}$*,* $k \in \mathbb{Z}$*, is an orthonormal base of* $L^2(\mathbb{R})$*. By virtue of (d) it is possible to expand a function in* $L^2(\mathbb{R})$ *as a series of* $\psi_I = 2^{j/2}\psi(2^jx-k)$*,* $I = [k2^{-j}, (k+1)2^{-j}[$*. However, this expansion does not function for functions in* $L^1(\mathbb{R})$ *or* $L^\infty(\mathbb{R})$*. In order to avoid this difficulty another function* ϕ *which is called the father of ondelettes (*ψ *is called the mother of ondelettes) is introduced. The function* ϕ *has the properties (a),(b) above,* $\int_{-\infty}^{\infty}\phi(x)dx = 1$*, and* ϕ *(x-k),* $k \in \mathbb{Z}$*,* $\psi_I(x)$*,* $|I| \leq 1$*, form an orthogonal base of* $L^2(\mathbb{R})$*. The closed subspace spanned by* $2^{j/2}\phi(2^jx-k)$*,* $k \in \mathbb{Z}$*, is* V_j*, and that spanned by* $2^{j/2}\psi(2^jx-k)$*,* $k \in \mathbb{Z}$*, is the orthogonal complement of* V_j *in* V_{j+1}*. This situation is the same in multidimensional cases. In Chapter III a method of constructing* ψ *for a given* ϕ *is stated. Several examples are described in detail.*

Chapter IV. Let ψ be a function in $L^1(\mathbb{R})$ whose integral over $\mathbb{R}$ vanishes. A sufficient condition in order that $2^{j/2}\psi(2^j x-k)$, $j\in\mathbb{Z}$, $k\in\mathbb{Z}$, is an oblique structure (= a frame) of $L^2(\mathbb{R})$ is given. It is stated that for $\psi(x)=(x+i)^{-2}$ this set of functions constitute a frame of $\mathbb{H}^2(\mathbb{R})$.
In Chapter V the Hardy space $H^1(\mathbb{R}^n)$ is defined as the set of functions $f\in L^1(\mathbb{R}^n)$ such that the series of ondelettes $\sum(f,\psi_\lambda)\psi_\lambda$ is convergent unconditionally to f. Ondelettes used here are regular and have compact supports. The definition turns out to be independent of the choice of ondelettes. One of the main theorems of this chapter is that the space of functions of bounded mean oscillation is the dual space of $H^1(\mathbb{R}^n)$.
In Chapter VI criterions are stated whether a given function or distribution belongs to various function spaces such as L^p spaces, Sobolev spaces, Hardy spaces, the space of Hölder continuous functions, the algebra of Beurling, the algebra of Wiener, Besov spaces, etc.

H.Tanabe
Keywords : ondelettes; wavelets; Fourier transform; boundedness of a discret Hilbert transform; inequality of Bernstein; oblique structure; frame; Hardy space
Classification :
- ∗41A58 Series expansions
- 41-02 Research monographs (approximations and expansions)
- 42A38 Fourier type transforms, one variable

Zbl 0850.42008

Meyer, Y.
Orthogonal wavelets. *(English)*
Wavelets. Time-frequency methods and phase space, Proc. Int. Conf., Marseille/Fr. 1987, 21-37 (1989).

Classification :
- ∗42C40 Wavelets

Zbl 0731.42025

Meyer, Yves
Orthonormal wavelets. *(English)*
Mathematical physics, 9th Int. Congr., Swansea/UK 1988, 38-47 (1989).

[For the entire collection see Zbl 0721.00025.]
The paper under review is a short but informative survey on the topic of orthonormal wavelets and their applications to signal processing and, e.g., speech analysis.

D.C.Struppa (Fairfax)
Keywords : Haar systems; trigonometric systems; orthonormal bases; survey; orthonormal wavelets
Classification :
- ∗42C10 Fourier series in special orthogonal functions
- 42C05 General theory of orthogonal functions and polynomials

42C40 Wavelets

Zbl 0704.46009

Jaffard, Stéphane; Meyer, Yves

Bases d'ondelettes dans des ouverts de $\mathbb{R}^n$. (Wavelet bases in open sets of $\mathbb{R}^n$). *(French)*

J. Math. Pures Appl., IX. Sér. 68, No.1, 95-108 (1989). ISSN 0021-7824
http://www.sciencedirect.com/science/journal/00217824

Let m and n be integers and $\Omega \subset \mathbb{R}^n$ be a domain in $\mathbb{R}^n$ which satisfies certain condition of accessibility from exterior. The authors construct a system of (2m-2)-smooth wavelets $(\Psi_\lambda)_{\lambda\in\Lambda}$ [see the review of Y. Meyer, Wavelets and operators, Lond. Math. Soc. Lect. Note Ser. 137, 256-365 (1989; review above)] which is an unconditional basis of $L^2(\Omega)$ and for which we have: $f \in L^2(\Omega)$ belongs to $C_0^r(\Omega)$ (the space of all Hölder functions of order r, $0 < r < 2m-2$, with supports in Ω) if and only if the coefficients of the decomposition $f = \sum_{\lambda\in\Lambda} a(\lambda)\Psi_\lambda$ satisfy the condition $\sup_\lambda |a(\lambda)|/\tau(\lambda) < \infty$ for certain function $\tau : \Lambda \to \mathbb{R}^+$.

M.Ostrovskij

Keywords : Sobolev spaces; wavelets; unconditional basis; space of all Hölder functions
Classification :
∗*46B15 Summability and bases in normed spaces*
46E15 Banach spaces of functions defined by smoothness properties
42C40 Wavelets
46E35 Sobolev spaces and generalizations

Zbl 0704.46008

Meyer, Yves

Wavelets and operators. *(English)*

Analysis at Urbana. Vol. 1: Analysis in function spaces, Proc. Spec. Year Mod. Anal./Ill. 1986-87, Lond. Math. Soc. Lect. Note Ser. 137, 256-365 (1989).

[For the entire collection see Zbl 0667.00018.]
The paper is devoted to recently introduced systems of functions which are called wavelets. Let m be an integer and let Ψ be a function from $L^2(\mathbb{R})$ satisfying conditions (1)-(4).
(1) Ψ is m times continuously differentiable;
(2) Ψ and all its derivatives of order $k \le m$ have a rapid decay at infinity;
(3) all moments of Ψ of order $k \le m$ vanish;
(4) the collection $2^{j/2}\Psi(2^jx - k)$; $j \in Z$, $k \in Z$ is an orthonormal basis of $L^2(\mathbb{R})$.
Then the system $(2^{j/2}\Psi(2^jx - k))$ is called a system of wavelets. The term "wavelets" is preserved for different generalizations of such systems: many-dimensional analogues, systems with support contained in a fixed domain, systems for which the word "orthonormal" in (4) is replaced by the word "unconditional". Wavelets combine good properties of the Haar system with good properties of the trigonometrical system. This is why they

have many applications. Some of them are described in the reviewed paper. Here is its brief contents:
Construction of a concrete system of wavelets called Franklin wavelets; Their applications to the construction of a basis for the disc-algebra and an unconditional basis for holomorphic H^1-space. Characterization of functions of the Zygmund class. Characterization of Calderon-Zygmund operators.

M.Ostrovskij

Keywords : Franklin wavelets; disc-algebra; unconditional basis for holomorphic H^1-space; Characterization of functions of the Zygmund class; Characterization of Calderon-Zygmund operators

Classification :

∗*46B15* *Summability and bases in normed spaces*
42C40 *Wavelets*
47B38 *Operators on function spaces*

Zbl 0689.47011

Coifman, R.; David, G.; Meyer, Y.; Semmes, S.
ω-Calderón-Zygmund operators. *(English)*
Harmonic analysis and partial differential equations, Proc. Int. Conf., El Escorial/Spain 1987, Lect. Notes Math. 1384, 132-145 (1989).

[For the entire collection see Zbl 0669.00010.]
Authors' abstract: "We enlarge the class of singular integrals of Calderón-Zygmund type by generalizing the usual assumptions on the kernel. These weaker conditions on the kernel arise naturally in the study of operators depending (linearly or not) on a functional parameter. Examples include the Cauchy integral operator, viewed as operating on one of the arguments while the others are frozen."

I.Gottlieb

Keywords : singular integrals of Calderón-Zygmund type; Cauchy integral operator

Classification :

∗*47B38* *Operators on function spaces*
47B10 *Operators defined by summability properties*

Zbl 0684.46044

Coifman, Ronald R.; Lions, Pierre-Louis; Meyer, Yves; Semmes, Stephen
Compacité par compenstion et espaces de Hardy. (Compensated compactness and Hardy spaces). *(French)*
C. R. Acad. Sci., Paris, Sér. I 309, No.18, 945-949 (1989). ISSN 0764-4442

Summary: We prove that various nonlinear quantities occuring in the compensated compactness theory belong to the Hardy space $\mathcal{H}^1$. We also indicate some applications, variants and extensions of such results.

Keywords : compensated compactness; Hardy space

Classification :
∗ *46J15* *Banach algebras of differentiable functions*
46A50 *Compactness in topological linear spaces, etc.*

Zbl 0684.42018

Jaffard, S.; Meyer, Y.

Les ondelettes. (The wavelets). *(French)*

Harmonic analysis and partial differential equations, Proc. Int. Conf., El Escorial/Spain 1987, Lect. Notes Math. 1384, 182-192 (1989).

[For the entire collection see Zbl 0669.00010.]
The wavelet series are presented and compared to the Fourier series. The relation to the concept of orthonormal and unconditional bases in a Hilbert space is discussed. After defining the graduate analysis some examples are given and then the construction of wavelet basis is presented by means of two theorems of K. Gröchenig and I. Daubechies.

L.Gores

Keywords : unconditional bases; wavelet series; examples

Classification :
∗ *42C40* *Wavelets*
41A15 *Spline approximation*

Zbl 0674.42011

Meyer, Yves

Ondelettes, filtres miroirs en quadrature et traitement numérique de l'image. (Wavelets, quadratic mirror filters and numerical treatment of the image). *(French)*

Gaz. Math., Soc. Math. Fr. 40, 31-42 (1989). ISSN 0224-8999
http://smf.emath.fr/en/Publications/Gazette/

L'auteur donne un résumé de sa théorie des ondelettes. Il s'agit là d'un couple de fonctions réelles ψ *(x) et* ϕ *(x) appartenant à la classe* $S(\mathbb{R})$ *de Schwartz telles que la collection* $2^{j/2}\psi(2^jx-k)$, $j,k\in\mathbb{Z}$, *ou encore* ϕ *(x-k),* $2^{j/2}\psi(2^jx-k)$, $j\in\mathbb{N}$, $k\in\mathbb{Z}$, *soit une base orthonormée de* $L^2(\mathbb{R})$. *De même pour les produits et les espaces* $S(\mathbb{R}^2)$, $L^2(\mathbb{R}^2)$. *De façon plus général, ce sont des bases inconditionnelles de tous les espaces fonctionnels classiques, à l'exception de* L^1 *et* L^∞ *et des espaces qui s'en déduisent. Une ondelette* ψ *est de régularité* $r\in\mathbb{N}$, *si* $(1+x^2)^m\psi(x)$ *appartient à l'espace de Sobolev* $H^r(\mathbb{R})$ *pour tout* $m\in\mathbb{N}$. *Plusieurs constructions de bases orthonormées bien connues, celles de Haar, Faber et Franklin sont à l'origine ou parentes des ondelettes, comme les travaux, récents de Ciesielski, Jaffard et Strömberg l'ont montré. En outre, la fonction* ψ *associée au système de Franklin est explicite, ainsi que les coefficients de la décomposition d'un* $f\in L^2$. *Ensuite, l'auteur décrit un algorithme basé sur une analyse multirésolution de* $L^2(\mathbb{R}^n)$ *c'est à dire une suite croissante de sous-espaces fermés* V_j *ayant certaines propriétés menant aussi bien au système de Franklin qu'aux ondelettes de Strömberg. Il* *existe un lien étroit entre les analyses multirésolution r-régulières et les filtres miroirs en quadrature. Ceux-ci constituent un algorithme de traitement numérique d'un signal échantillonné et décrit par une suite $f(k) \in \ell^2$, algorithme visant la réduction du bruit et l'optimisation du code. Une autre application concerne le codage et la transmission d'images en commençant par un modèle simplifié qui sera ensuite complété. Pour ce faire, on décompose l'image en utilisant les bases de fonctions similaires aux ondelettes.*

W.Luther

Keywords : wavelet; orthogonal bases; Sobolev space; Franklin system
Classification :
∗ 42C40 Wavelets
33C45 Orthogonal polynomials and functions of hypergeometric type

---

Zbl 0719.42028

Meyer, Yves

Constructions de bases orthonormées d'ondelettes. (Construction of orthonormal bases of wavelets). *(French)*

Rev. Mat. Iberoam. 4, No.1, 31-39 (1988). ISSN 0213-2230
http://projecteuclid.org/rmi
http://www.uam.es/departamentos/ciencias/matematicas/ibero/irevista.htm

A function w(x), $x \in \mathbb{R}$, is said to be a wavelet of order $m \geq 1$ if it belongs to the Sobolev space $H^m(\mathbb{R})$, if it has rapid decay together with all derivatives of order m and if $2^{j/2}w(2^jx-k)$, $j \in \mathbb{Z}$, $k \in \mathbb{Z}$, constitute an orthonormal basis for $L^2(\mathbb{R})$.
The author shows that the Strömberg wavelet basis of order m [J.- O. Strömberg, Harmonic analysis, Conf. in Honor A. Zygmund, Chicago 1981, Vol. 2, 475-494 (1983; Zbl 0521.46011)] can be obtained by using the method of constructing a wavelet basis suggested by S. Mallat and the author [the author, in Analysis at Urbana, Vol. I (Urbana, IL, 1986- 1987), 256-365, Cambridge Univ. Press, Cambridge (1989)].

Keywords : orthonormal basis; Strömberg wavelet basis
Classification :
∗ 42C40 Wavelets
42C10 Fourier series in special orthogonal functions

Zbl 0693.35166

Bourdaud, Gérard; Meyer, Yves

Inégalités L^2 précisées pour la classe $S^0_{0,0}$. (Precise L^2-inequalities for the class $S^0_{0,0}$). *(French)*

Bull. Soc. Math. Fr. 116, No.4, 401-412 (1988). ISSN 0037-9484
numdam:BSMF_1988__116_4_401_0
http://smf.emath.fr/en/Publications/Bulletin/

The authors consider the Beurling algebra $A_\Omega(\mathbb{R}^n \times \mathbb{R}^n)$, where $\Omega(x,\xi) = \omega_1(x)\omega_2(\xi)$

and ω_1, ω_2 are two Beurling's weights in $\mathbb{R}^n$. They assume that the function σ (x,ξ) is a multiplier in the algebra $A_\Omega(\mathbb{R}^n \times \mathbb{R}^n)$, i.e. there exists a constant $C > 0$ and such that $\|\sigma\phi\|_{A_\Omega} \leq C\|\phi\|_{A_\Omega}$, $\forall\phi \in C_0^\infty$ $(\mathbb{R}^n \times \mathbb{R}^n)$. Under this condition it is proved in Theorem 3 that the pseudodifferential operator σ (x,$\mathcal{D}$) with the symbol σ (x,ξ) is bounded in $L_2(\mathbb{R}^n)$. As a corollary the famous Calderon- Vaillancourt theorem on the L_2-boundedness of the operators belonging to the class $S^0_{0,0}$ is shown in Theorem 2 with a minimal Hölder continuity assumption on the symbol σ. When proving this result Plancherel equality is used as well as the "presque orthogonalité lemme" of Coifman-Meyer.

P.Popivanov

Keywords : Calderon-Vaillancourt; L_2-boundedness; symbol

Classification :

∗ *35S05 General theory of pseudodifferential operators*

Zbl 0714.42022

Meyer, Yves

Ondelettes, fonctions splines et analyses graduées. (Wavelets, spline functions and graduated analyses). *(French)*

Rend. Semin. Mat., Torino 45, No.1, 1-42 (1987). ISSN 0373-1243

http://www.emis.de/journals/RSMT/

http://seminariomatematico.dm.unito.it/rendiconti/

From the computational point of view, expansions $\sum_{\lambda\in\Lambda} < f|\psi_\lambda > \psi_\lambda$ of elements $f \in L^2(\mathbb{R}^n)$ with respect to orthonormal bases $(\psi_\lambda)_{\lambda\in\Lambda}$ are most convenient to handle. In image processing, however, the nonorthogonal Gabor functions, i.e., the system of coherent states based on the central projection slice of the three-dimensional Heisenberg nilpotent Lie group [the reviewer, Result. Math. 16, No.3/4, 345-382 (1989; Zbl 0686.68007)] provide an excellent fit to the two-dimensional receptive field profiles [J. P. Jones and L. A. Palmer, J. Neurophysiol. 58, No.6, 1233-1258 (1987)]. For the purpose of data compression, there is a tendency to replace the Gabor functions by wavelets. Instead of using the affine coherent states approach to wavelets [cf. A. Grossmann, J. Morlet and T. Paul, Ann. Inst. Henri Poincaré, Phys. Théor. 45, 293-309 (1986; Zbl 0601.22001)], the author of the paper under review describes a new approach to construct orthonormal bases $(\psi_\lambda)_{\lambda\in\Lambda}$ of wavelets. His method is based on an increasing sequence $(V_j)_{j\in\mathbb{Z}}$ of closed subspaces of the standard complex Hilbert space $L^2(\mathbb{R}^n)$. The starting point of the geometrically scaled tensor product construction is formed by suitable "mother" functions $\psi^{(0)} = \phi \in V_0$ and $\psi^{(1)} = \psi \in W_0$, where $V_0 \oplus W_0 = V_1$. The index set is given by the lattice $\Lambda = \mathbb{Z} \times \mathbb{Z}^n \times A$, where A denotes the set of binary n-tuples. The operators canonically associated with the $\mathbb{Z}$- graduation $(V_j)_{j\in\mathbb{Z}}$ of $L^2(\mathbb{R}^n)$ include the martingale transformations.

Keywords : orthonormal bases of wavelets; artificial neural network; neurocomputer architecture; expansions; nonorthogonal Gabor functions; martingale transformations

Classification :

∗ *42C10 Fourier series in special orthogonal functions*

42C40 Wavelets

Zbl 0657.42028

Lemarié, P.G.; Meyer, Y.

Ondelettes et bases hilbertiennes. (Wavelets and Hilbert bases). *(French)*

Rev. Mat. Iberoam. 2, No.1-2, 1-18 (1987). ISSN 0213-2230
http://projecteuclid.org/rmi
http://www.uam.es/departamentos/ciencias/matematicas/ibero/irevista.htm

The authors consider some special systems of functions called "wavelets" and demonstrate that they are complete systems in different functional spaces.

A. Venkov

Keywords : unconditional basis; Hilbert space; wavelets

Classification :

∗ *42C30 Completeness of sets of functions of non-trigonom. Fourier analysis*

46B15 Summability and bases in normed spaces

Zbl 0648.42011

Meyer, Yves

Principe d'incertitude, bases hilbertiennes et algèbres d'opérateurs. (The uncertainty principle, Hilbert base and operator algebras). *(French)*

Sémin. Bourbaki, 38ème année, Vol. 1985/86, Exp. Astérisque 145/146, 209-223 (1987).
numdam:SB_1985-1986__28__209_0

[For the entire collection see Zbl 0609.00002.]

Using the representation of functions in wavelet series, the author presents a discrete version of Littlewood-Paley-Stein theory characterising the classical functional spaces by the modules of the wavelet coefficients. A discrete version of the uncertainty principle is established and a Hilbert type base compatible with this principle is obtained. The relation to periodic representations as well as with certain operator algebras is investigated.

L. Goras

Keywords : Littlewood-Paley-Stein theory; wavelet coefficients; uncertainty principle; operator algebras

Classification :

∗ *42B99 Fourier analysis in several variables*

46E20 Hilbert spaces of functions defined by smoothness properties

42C15 Series and expansions in general function systems

Zbl 0646.42015

Meyer, Y.

Les ondelettes. (The wavelets). *(French)*

Contributions to nonlinear partial differential equations, Vol. II, Proc. 2nd Franco-Span. Colloq., Paris 1985, Pitman Res. Notes Math. Ser. 155, 158-171 (1987).

[For the entire collection see Zbl 0614.00011.]

The paper under review is a survey devoted to the theory of wavelets in different Banach function spaces developed by the author and his collaborators. In the case of $L^2(R)$ the wavelets are defined in the following way: $f_{j,k} = a^j f(b^j x+k)$, where $f \in L^2(R)$; $a, b > 0$; $j, k = 0, \pm 1, \pm 2, \ldots$ When the choice of f, a, b is good then the corresponding wavelets have many useful properties, e.g., they are orthonormal or unconditional basis in L 2(R). Several families of wavelets and their applications are described. Several unsolved problems are proposed.

I.V.Oskovskij

Keywords : orthonormal basis; wavelets; Banach function spaces; applications

Classification :

∗*42C40 Wavelets*

46E30 Spaces of measurable functions

46B15 Summability and bases in normed spaces

41A50 Best approximation

Zbl 0642.46028

Meyer, Y.

Ondelettes et fonctions splines. (Wavelets and spline functions). *(French)*

Sémin., Équations Dériv. Partielles 1986-1987, Exp. No.6, 18 p. (1987).

numdam:SEDP_1986-1987____A5_0

An orthonormal basis of wavelets ("ondelettes") is defined to be an orthonormal basis of $L^2(\mathbb{R}^d)$ of the form $2^{dj/2}\psi_m(2^j x-\gamma)$, $x \in \mathbb{R}^d$, $j \in \mathbb{Z}$, $1 \le m < 2^d (m \in \mathbb{N})$, $\gamma \in \Gamma$ where Γ is a lattice in $\mathbb{R}^d$ (i.e. Γ is a discrete subgroup of $\mathbb{R}^d$ with compact quotient $\mathbb{R}^d/\Gamma$) and the $\psi_m's$ belong to the class of functions $\psi : \mathbb{R}^d \to \mathbb{C}$ such that, for some $C > 0$,

$$|\psi(x)| \le C(1+|x|)^{-d-1}, |\partial/\partial x_j \psi(x)| \le C(1+|x|)^{-d-1}, 1 \le j \le d, \quad x \in \mathbb{R}^d$$

and $\int_{\mathbb{R}^d} \psi(x)dx = 0$. The author points out the usefulness of these special orthonormal bases and outlines a way of obtaining many of them (perhaps all -this remains an open question) via what he calls an "analyse graduée dyadique" of $L^2(\mathbb{R}^d)$; this latter is an increasing family of closed subspaces of $L^2(\mathbb{R}^d)$, V_j, $j \in \mathbb{Z}$, satisfying certain requirements (some reminiscent of those in time series analysis). The author points out the fascinating relationship between "analyse graduée", martingale theory and classical methods of numerical analysis (like those involving splines and finite elements). The existence of orthonormal bases of wavelets is not at all obvious; on the other hand, they *seem to be a powerful tool in many different problems. Articles referred to in the paper and others to come should be of interest for further understanding.*

S.D.Chatterji

Keywords : graded dyadic analysis; orthonormal basis of wavelets; ondelettes; martingale theory

Classification :

- ∗*46E30* *Spaces of measurable functions*
- *46B15* *Summability and bases in normed spaces*
- *42B20* *Singular integrals, several variables*

Zbl 0623.47052

Coifman, R.R.; Meyer, Yves

Nonlinear harmonic analysis, operator theory and P.D.E. *(English)*

Beijing lectures in harmonic analysis, Ann. Math. Stud. 112, 3-45 (1986).

[For the entire collection see Zbl 0595.00015.]

The authors describe results involving the study of nonlinear analytic dependance of same functionals arising in P.D.E. or operator theory connected with functional calculus.

D.Robert

Keywords : nonlinear analytic dependance; functional calculus

Classification :

- ∗*47F05* *Partial differential operators*
- *47A60* *Functional calculus of operators*

Zbl 0608.46014

Daubechies, Ingrid; Grossmann, A.; Meyer, Y.

Painless nonorthogonal expansions. *(English)*

J. Math. Phys. 27, 1271-1283 (1986). ISSN 0022-2488

http://dx.doi.org/10.1063/1.527388

http://jmp.aip.org/

In a Hilbert space $\mathcal{H}$, discrete families of vectors $\{h_j\}$ with the property that $f = \sum_j < h_j|f > h_j$ for every f in $\mathcal{H}$ are considered. This expansion formula is obviously true if the family is an orthonormal basis of $\mathcal{H}$, but also can hold in situations where the h_j are not mutually orthogonal and are "overcomplete". The two classes of examples studied here are (i) appropriate sets of Weyl-Heisenberg coherent states, based on certain (non-Gaussian) fiducial vectors, and (ii) analogous families of affine coherent states. It is believed, that such "quasiorthogonal expansions" will be a useful tool in many areas of theoretical physics and applied mathematics.

Keywords : orthonormal basis; overcomplete; Weyl-Heisenberg coherent states; affine coherent states; quasiorthogonal expansions

Classification :
 ∗ 46C99 *Inner product spaces, Hilbert spaces*
 46B15 *Summability and bases in normed spaces*

Zbl 0604.46024

Meyer, Y.

De la recherche petrolière à la géometrie des espaces de Banach en passant par les paraproduite. (From soil investigation to geometry of Banach spaces via paraproducts). *(French)*

Sémin., Équations Dériv. Partielles 1985-1986, Exposé No.1, 11 p. (1986).
numdam:SEDP_1985-1986____A1_0

A la suite des travaux de P. Goupillaud sur la vibro-sismique, J. Morlet (O.R.I.C., Elf-Aquitaine) a étudié systématiquement les algorithmes de traitement numérique des signaux sismiques. En particulier, en partant des "ondelettes de Gabor" ou "analyse de Fourier en fenêtre glissante", il a été conduit à des ondelletes à localisation arbitraire et à largeur arbitraire. Les travaux de J. Morlet on reçu une mise en forme mathématique grâce à la collaboration entre A. Grossmann (mécanique quantique) et J. Morlet. Le texte qui est analysé, décrit une petite amélioratir des idées de Grossmann et Morlet: la possibilité de construire une base orthonormale d'ondelettes. Cette base orthonormale permet alors de donner des démonstrations simplifiées de certains théorèmes classiques en géométrie des espaces de Banach (p.ex. le theóréme de Maurey). L'utilisation de cette base orthonormée d'ondelettes est adaptée au para-produit de J. M. Bony. Mais il ne semble pas, à l'heure actuelle, que les "ondelettes orthonormées" soient utiles à la recherche pétrolière (bien qu'elles aient été motivées par les travaux de J. Morlet).

Keywords : soil investigation; paraproduct; orthonormal small waves
Classification :
 ∗ 46B20 *Geometry and structure of normed spaces*
 46B15 *Summability and bases in normed spaces*
 46N99 *Appl. of functional analysis*

Zbl 0603.42018

Coifman, Ronald R.; Meyer, Yves F.

A simple proof of a theorem by G. David and J.-L. Journé on singular integral operators. *(English)*

Probability theory and harmonic analysis, Pap. Mini-Conf., Cleveland/Ohio 1983, Pure Appl. Math., Marcel Dekker 98, 61-65 (1986).

[For the entire collection see Zbl 0577.00016.]
Recently, G. David and J.-L. Journé [Ann. Math., II. Ser. 120, 371-379 (1985; Zbl 0567.47025)] have obtained a remarkably simple criterion on guaranteeing boundedness in L^2 (or L^p) of a class of singular integral operators whose singularity is of Calderon-Zygmund type. In particular, their results yield estimates for operators like the so-called

Calderon-nth commutator:

$$\mathcal{C}_n(a,f)=\int[\frac{A(x)-A(y)}{x-y}]^n\frac{f(y)}{x-y}dy.$$

The present authors give an elementary proof of their theorem and describe the "principal" part of such operators and its probabilistic version. For simplicity, they restrict their attention to $\mathbb{R}^1$.

R.Srivastava

Keywords : singular integral operators; Calderon-Zygmund type; Calderon-nth commutator

Classification :

∗42B20 *Singular integrals, several variables*

Zbl 0641.47039

Kenig, C.; Meyer, Y.

Kato's square roots of accretive operators and Cauchy kernels on Lipschitz curves are the same. *(English)*

Recent progress in Fourier analysis, Proc. Semin., El Escorial/Spain 1983, North-Holland Math. Stud. 111, 123-143 (1985).

[For the entire collection see Zbl 0581.00009.]

Kato's definition of the square root $\sqrt{T}$ *of an m-accretive operator* T *is slightly generalized and used in the proof of the following theorem: Let* A *and* B *be pointwise multiplication operators by* $a\in L^\infty$ *and* $b\in L^\infty$*, respectively, let* $D=-id/dx$ *and let* $T=BDAD$*. Then* $(1+\epsilon T)^{-1}$ *is bounded on L 2 for every* $\epsilon\geq 0$*, the norms of these operators are bounded independently of* ϵ *and the domain of* $\sqrt{T}$ *is the Sobolev space H 1. Moreover,* $\sqrt{T}=J(A,B)D$*, where J(A,B):L 2* $\rightarrow$ *L 2 is an isomorphism.*

The second main result is concerned with the Cauchy operator C_Φ *whose kernel is* $(i\pi)^{-1}PV[z(y)-z(x)]^{-1}$*, where* $z(x)=x+i\Phi(x)$ *and* Φ *denotes a Lipschitz function. It is then proved that* $C_\Phi=J(A,A)$*, where J(A,A) is defined as in the preceding theorem with* $a(x)=[1+i\Phi'(x)]^{-1}$*. This result shows that the boundedness of the Cauchy operator is a special case of a more general theorem concerning square roots of second-order differential operators.*

Keywords : square root of an m-accretive operator; pointwise multiplication operators; Cauchy operator; Lipschitz function; square roots of second- order differential operators

Classification :

∗47B44 *Accretive operators, etc. (linear)*

47E05 *Ordinary differential operators*

47A60 *Functional calculus of operators*

47Gxx *Integral operators and their generalizations*

Zbl 0628.46035

Meyer, Y.

Minimalité de certains espaces fonctionnels et applications à la théorie des

opérateurs. (Minimality of certain functional spaces and applications to operator theory). *(French)*

Sémin., Équations Dériv. Partielles 1984/1985, Exp. No.3, 12 p. (1985).
numdam:SEDP_1984-1985____A3_0

This is a fairly informal exposé of a number of results centering around work attributed to a variety of individuals, principally G. Weiss and R. Coifman; occasional indications of proof are given.
A charming introductory section explains that infinite sums of the form $\sum \lambda_k g_k$, *where the* g_k *are gaussians on* $\mathbb{R}^n$ *and the* λ_k *are real constants satisfying* $\sum |\lambda_k| < \infty$, *form a linear subspace B of* $C_0(\mathbb{R}^n)$ *such that (i)* $\mathcal{D}(\mathbb{R}^n) \subset B \subset \mathcal{D}'(\mathbb{R}^n)$ *with continuous injections (*$\mathcal{D}$ *is the space of rest functions,* $\mathcal{D}'$ *the space of distributions), and (ii) the changes of variables* $x \to tx$ $(t > 0)$ *and* $x \to x - x_0$ $(x_0 \in \mathbb{R}^n)$ *induce isometries of B; further, B is minimal with these properties. B is then identified as the homogeneous Besov space* $B_1^{n,1}(\mathbb{R}^n)$, *which is defined and characterized. It turns out that the gaussian* $\exp(-|x|^2)$ *and its images under the transformations in (ii) above can be replaced by any (not identically zero) radial rapidly decreasing function* $\theta \in \mathcal{S}(\mathbb{R}^n)$ *and its images.*
The second section treats $B_1^{s,1}(\mathbb{R}^n)$ *for* $0 \le s < n$*; here (i) is replaced by* $\mathcal{S}(\mathbb{R}^n) \subset E \subset \mathcal{S}'(\mathbb{R}^n)$ *(*$\mathcal{S}$ *is the space of rapidly decreasing functions,* $\mathcal{S}'$ *the space of tempered distributions) if* $s < 0$, *with* $\mathcal{S}$ *replaced by its subspace* $\mathcal{S}_0$ *of functions with integral 0 if* $s = 0$, *and the change of variables in (ii) acquires a fudge factor:* $t^{n-s} f(tx)$ *has the same norm in E as does f(x). In each case, again, E can be obtained as a set of sums* $\sum \lambda_k \chi_k$ *where the* χ_k *are transformed versions of a single function* χ, *and* $\sum |\lambda_k| < \infty$.
The third and last section deals with conditions which force linear operators to be continuous on $L^2(\mathbb{R}^n)$. *The connection with the earlier sections and Besov spaces is clearest in Theorem 4, which we now describe.*
Suppose $\psi \in \mathcal{D}_0(\mathbb{R}^n)$ *(test functions with integral 0) is a (not identically zero) radial test function of integral 0, and* $\mathcal{A}$ *is a family of continuous linear transformations* $T : \mathcal{D} \to \mathcal{D}$ *closed under taking adjoints and under conjugation by the unitary maps* $f(x) \to t^{-n/2} f((x - x_0)/t)$ $(t > 0, x_0 \in \mathbb{R}^n)$. *If the norms in* $B_1^{0,1}$ *of the function* $T(\psi)$ *are bounded as T runs through* $\mathcal{A}$, *then all the operators in* $\mathcal{A}$ *are continuous on* L^2.
As the author says rather poetically, this makes it possible "de réaliser le rêve de tout analyste: démontrer la continuitié d'un opérateur en se limitant au calcul de l'action de cet opérateur sur une seule fonction," though at the price of having to compute the norm of T(ψ*).*

S.J.Sidney

Keywords : space of rest functions; space of distributions; homogeneous Besov space; space of rapidly decreasing functions; space of tempered distributions; change of variables; fudge factor; conditions which force linear operators to be continuous
Classification :
∗46F05 *Topological linear spaces of test functions and distributions*
46E35 *Sobolev spaces and generalizations*
47B38 *Operators on function spaces*

Zbl 0616.42008

Meyer, Y

Continuité sur les espaces de Hölder et de Sobolev des opérateurs définis par des intégrales singulières. (Continuity on the Hölder and Sobolev spaces defined by singular integrals). *(French)*

Recent progress in Fourier analysis, Proc. Semin., El Escorial/Spain 1983, North-Holland Math. Stud. 111, 145-172 (1985).

[For the entire collection see Zbl 0581.00009.]

The author studies the continuity of Calderón-Zygmund operators on Hölder and Sobolev spaces. Let T be a continuous operator $T : D(\mathbb{R}^n) \to D'(\mathbb{R}^n)$, let $0 < \epsilon \leq 1$, and let K(x,y) be the standard kernel associated to T. Then one says that $T \in_\epsilon$ if the following conditions are satisfied: (i) $|K(x, y)| \leq c|x - y|^{-n}$ if $x \neq y$, (ii) $|K(x', y) - K(x, y)| \leq c|x - x'|^\epsilon |x - y|^{-n-\epsilon}$ if $|x' - x| \leq \times - y|$ and $x \neq y$. If $\epsilon > 1$ then write $\epsilon = m + \eta$, where $m \in \mathbb{N}$, $0 < \eta \leq 1$, and replace conditions (i) and (ii) by (i)' $|\partial^\alpha/\partial^\alpha x^\alpha K(x, y)| \leq c|x - y|^{-n-|\alpha|}$, if $0 \leq |\alpha| \leq m$,

$$(ii)' \quad |\partial^\alpha/\partial x^\alpha K(x, y) - \partial^\alpha/\partial x^\alpha K(x', y)| \leq c|x - x'|^\eta |x - y|^{-n-\epsilon}.$$

Moreover, T is "of order zero in the weak sense" if $R_u \delta_t T \delta_t^{-1} R_u^{-1} : D(\mathbb{R}^n) \to D'(\mathbb{R}^n)$ is bounded for all $u \in \mathbb{R}^n$, $t \in \mathbb{R}_+$, where $(R_u f)(x) = f(x - u)$ and $(\delta_t f)(x) = f(x/t)$. Here we provide a sample of the results obtained. Theorem A (i). Let $0 < s < \epsilon < 1$, $T \in L_\epsilon$. Then, $T : \Lambda^s(\mathbb{R}^n) \to \Lambda^s(\mathbb{R}^n)$ continuously if and only if T is of order zero in the weak sense and $T(1) = 0$. (ii) Let $0 < s < \epsilon$, $m_0 = [s]$, $T \in L_\epsilon$. Then, $T : \Lambda^s(\mathbb{R}^n) \to \Lambda^s(\mathbb{R}^n)$ continuously if and only if T is of order zero in the weak sense and $T(x^\alpha) = 0 \forall \alpha \in \mathbb{N}^n$, $|\alpha| \leq m_0$. Similar results obtained for the continuity of C-Z operators on Sobolev spaces. Theorem B. Suppose that $0 < s < \epsilon < 1$, and $T \in L_\epsilon$. Then, if T is of order zero in the weak sense and $T(1) = 0$ then T can be extended to a bounded operator on $B^s(\mathbb{R}^n)$.

M.Milman

Keywords : Hölder space; Calderón-Zygmund operators; Sobolev spaces

Classification :

∗*42A50* *Singular integrals, one variable*

Zbl 0594.47025

Meyer, Yves

Le lemme de Cotlar et Stein et la continuité L^2 des opérateurs définis par des intégrales singulières. (The lemma of Cotlar and Stein and the L^2 continuity of operators defined by singular integrals). *(French)*

Colloq. Honneur L. Schwartz, Éc. Polytech. 1983, Vol. 1, Astérisque 131, 115-125 (1985).

[For the entire collection see Zbl 0566.00010.]

In the early sixties Alberto Calderón was interested in proving L^2 estimates for certain operators T which are not given by convolutions but satisfy the following condition:

there exists a function K(x,y), $x \in \mathbb{R}^n$, $y \in \mathbb{R}^n$ ($x \neq y$) with the following estimates $|K(x,y)| \leq C|x-y|^{-n}$, $|(\partial/\partial x_j)K(x,y)| \leq C|x-y|^{-n-1}$, $|(\partial/\partial y_j)K(x,y)| \leq C|x-y|^{-n-1}$ *(1*$\leq j \leq n$*) and such that*

$$Tf(x) = \int_{\mathbb{R}^n} K(x,y)f(y)dy$$

when x does not belong to the support of the function f.
The real variable methods of Calderón and Zygmund apply to those operators as long as they are bounded on $L^2(\mathbb{R}^n)$ *but it took about 20 years to find the necessary and sufficient condition yielding the* L^2*-estimate. This condition has been found by G. David and J. L. Journé [Ann. Math. II. Ser. 120, 371-397 (1984; Zbl 0567.47025)]. For example the David-Journé criterion gives immediately all the estimates which have been obtained before 1981. The paper is a survey without proofs but they are given in full details in the David-Journé paper in the Annals.*

Keywords : L^2*-estimate; David-Journé criterion*
Classification :
∗*47B38 Operators on function spaces*
47Gxx Integral operators and their generalizations
45E10 Integral equations of the convolution type

Zbl 0591.47041

Meyer, Yves

Real analysis and operator theory. *(English)*

Pseudodifferential operators and applications, Proc. Symp., Notre Dame/Indiana 1984, Proc. Symp. Pure Math. 43, 219-235 (1985).

[For the entire collection see Zbl 0562.00004.]
In his Helsinki adress, A. P. Calderón proposed a systematic study of the third generation of singular integral operators and explained the role played by some operators of this class in complex analysis and in elliptic P.D.E.'s in Lipschitz domains. An operator of the third generation is given as a weakly defined linear operator T mapping test functions into distributions whose distributional kernel satisfies some smoothness and size estimates away from the diagonal. Those estimates are reminiscent of the ones in the case of the Riesz transforms: off the diagonal the distributional kernel coincides with an ordinary function K(x,y) with the following estimates
(1) $|K(x,y)| \leq C|x-y|^{-n}$
(2) $|K(x',y) - K(x,y)| \leq C|x'-x|^{\delta}|x-y|^{-n-\delta}$ ($0 < \delta \leq 1$ *is a fixed exponent and (2) should hold for* $|x'-x| \leq 1/2|x-y|$)
(3) $|K(x,y') - K(x,y)| \leq C|y'-y|^{\delta}|x-y|^{-n-\delta}$ *for* $|y'-y| \leq 1/2|x-y|$.
Major break-throughs have been made by G. David and J.-L. Journé [Ann. Math., II. Ser. 120, 371-397 (1984; Zbl 0567.47025)] who completely solved the fundamental problem of giving a necessary and sufficient condition for such an operator T to be bounded on $L^2(\mathbb{R}^n; dx)$*. Soon afterwards P. G. Lemarié [Ann. Inst. Fourier 35, No.4, 175-187 (1985; Zbl 0555.47032)] could give a sufficient condition for the continuity of T on homogeneous Besov spaces* $B^s_{p,q}$ *when* $0 < s < \delta$, $1 \leq p \leq +\infty$, $1 \leq q \leq +\infty$.

Keywords : singular integral operators; Lipschitz domains; distributional kernel; Riesz transforms; homogeneous Besov spaces
Classification :
∗*47Gxx Integral operators and their generalizations*
42B20 Singular integrals, several variables
46J15 Banach algebras of differentiable functions
47B38 Operators on function spaces
46E35 Sobolev spaces and generalizations
47L10 Algebras of operators on Banach spaces, etc.
22E30 Analysis on real and complex Lie groups

Zbl 0587.35004

Coifman, R.R.; Meyer, Yves

Nonlinear harmonic analysis and analytic dependence. *(English)*

Pseudodifferential operators and applications, Proc. Symp., Notre Dame/Indiana 1984, Proc. Symp. Pure Math. 43, 71-78 (1985).

[For the entire collection see Zbl 0562.00004.]
This is a written version of a lecture describing recent results, due mainly to the authors, concerning what used to be called the operational calculus. The functionals studied are expanded in a vectorial Taylor series, or represented through an integral formula, thus bringing in holomorphism. Techniques for analysing such expressions are shown on several significant examples.

E.J.Akutowicz

Keywords : differential operators; integral representations; Fourier transform; operational calculus; vectorial Taylor series
Classification :
∗*35A20 Analytic methods (PDE)*
47A60 Functional calculus of operators
47A67 Representation theory of linear operators
47A05 General theory of linear operators

Zbl 0584.47030

McIntosh, Alan; Meyer, Yves

Algèbres d'opérateurs définis par des intégrales singulières. (Algebras of singular integral operators). *(French)*

C. R. Acad. Sci., Paris, Sér. I 301, 395-397 (1985). ISSN 0764-4442

For each function $b \in L^\infty(\mathbb{R}^n)$ such that Re b(x)≥ 1 (a.e.), an operator algebra A_b is constructed. A characterization of the distributional kernels of operators in A_b is given leading to new criteria for L^2-boundedness of singular integral operators.

Keywords : algebras of singular integral operators; distributional kernels; L^2-boundedness of singular integral operators

Classification :
 ∗ *47B38 Operators on function spaces*
 47Gxx Integral operators and their generalizations
 45E10 Integral equations of the convolution type
 47B35 Toeplitz operators, etc.

Zbl 0573.42010

Meyer, Yves

Les nouveaux opérateurs de Calderón-Zygmund. *(French)*

Colloq. Honneur L. Schwartz, Éc. Polytech. 1983, Vol. 1, Astérisque 131, 237-254 (1985).

[For the entire collection see Zbl 0566.00010.]
The author provides a new proof, using a generalized calculus of singular integrals, of the following remarkable result of G. David and J. L. Jourńe: Theorem. Let T be a linear operator given by an integration kernel K(x,y) on $\mathbb{R}^n \times \mathbb{R}^n$. Suppose that K satisfies certain hypotheses regarding smoothness (in the finite part of space) and order of vanishing (near infinity). [These are called the F_ϵ-hypotheses.] If T maps the constant function 1 to BMO and if the adjoint of T has the same property then T maps $L^2(\mathbb{R}^n)$ to $L^2(\mathbb{R}^n)$. This condition is also necessary.

S.Krantz

Keywords : Lipschitz domains; Hilbert space

Classification :
 ∗ *42B10 Fourier type transforms, several variables*
 42B20 Singular integrals, several variables
 42B30 Hp-spaces (Fourier analysis)
 44A05 General integral transforms

Zbl 0569.42016

Coifman, R.R.; Meyer, Y.; Stein, E.M.

Some new function spaces and their applications to harmonic analysis. *(English)*

J. Funct. Anal. 62, 304-335 (1985). ISSN 0022-1236
http://dx.doi.org/10.1016/0022-1236(85)90007-2
http://www.sciencedirect.com/science/journal/00221236

This paper is devoted to the definition of a new family of function spaces and to the investigation of their fundamental properties. These spaces, called "tent spaces" are of functions on $X\times\mathbb{R}_+$ where X is a Euclidean space and the spaces are so defined that the functions have "good" boundary values on the boundary X of this space. Such boundary values play a central role in harmonic analysis and the theory developed in this paper systemises a great deal of the earlier work. It is so rich in material that it is hardly possible in a short review to summarize the results in detail. To show the range of these *methods the authors give a number of applications at the close of this paper, to maximal functions, to the Hilbert transform and to the theory of Hardy spaces.*

S.J.Patterson

Keywords : tent spaces; maximal functions; Hilbert transform
Classification :
$*$ 42B25 *Maximal functions*
31B25 *Boundary behavior of harmonic functions (higher-dim.)*

Zbl 0552.42002

Meyer, Yves; Taibleson, Mitchell H.; Weiss, Guido

Some functional analytic properties of the spaces B_q generated by blocks. *(English)*

Indiana Univ. Math. J. 34, 493-515 (1985). ISSN 0022-2518
http://dx.doi.org/10.1512/iumj.1985.34.34028
http://www.iumj.indiana.edu/IUMJ/issues.php

In [Conference on harmonic analysis in honor of Antonio Zygmund (Univ. Chicago), Vol. I, 95-113 (1983; Zbl 0532.42001)] two of the authors (Taibleson and Weiss) introduced certain function spaces B_q, $1 < q \leq \infty$, generated by functions b satisfying: The domain of b is $T = (-,]$, Supp $b \subset I \subset T$ (where I is an interval) and $(*)$ $(\int_T |b(x)|^q(dx/|I|))^{1/q} \leq 1/|I|$ (when $q = \infty$, the left side of (*) is $\|b\|_\infty$. Such functions b are called q-blocks. The space B_q consists of all those functions f having the form $f = \sum m_j b_j$, where each b_j is a q-block and the numerical sequence $m = \{m_j\}$ satisfies $M(m) \equiv \sum |m_j|(1 + \log^+(\sum |m_k|)/|m_j|) < \infty$. Letting $M_q(f)$ denote the infimum of all such expressions, over all representations of f, it is clear that $M_q(f + g) \leq M_q(f) + M_q(g)$. Consequently, $d_q(f, g) \equiv M_q(f - g)$ is a metric on B_q. The same topology is induced by the quasi-norm $N_q(f) \equiv \inf\{N(m) : f = \sum m_j b_j\}$. N_q enjoys the advantage, over M_q, of being positive homogeneous; however, instead of Minkowski's inequality, it satisfies $N_q(f+g) \leq (1+\log 2)(N_q(f)+N_q(g)$. (The constant $1 + \log 2$ is best possible.)
Some of the properties of the spaces B_q are: (i) The Fourier series of an $f \in B_q$ converges a.e. to f; (ii) If $q_1 > q_2$ then $B_{q_1} \subsetneqq B_{q_2}$: Moreover $J \subset B_\infty$, where J is the space of measurable functions having infinite entropy. (Introduced by R. Fefferman); (iii) The spaces B_q are "close" to $L^1(T)$ and are closely related to the phenomenon of a.e. convergence of Fourier series; (iv) The spaces (B_q, d_q) are complete metric spaces. As topological vector spaces they are not locally convex. Properties (i) and (ii) are established in the paper of Taibleson and Weiss referred to above, where one can also find justifications for the assertions in (iii) and (iv). In an accompanying paper by F. Soria one can find further justifications; in particular, he gives a "distribution function characterization" of B_∞ and uses it to study other properties of these spaces. Higher dimensional extensions of all this theory can be found in [S. Lu, M. H. Taibleson and G. Weiss, Lect. Notes Math. 908, 311-318 (1982; Zbl 0514.42009); S. Lu, ibid., 319-325 (1982; Zbl 0513.42016)].
The topics studied in this paper are the duals of the spaces B_q, the associated convolu-

tion structures, their weak envelopes and the examination of certain operators whose properties shed some light on the theory of interpolation involving these spaces. At the end of the paper it is shown that in the definition of N_q (or M_q) one must take the infimum over all representations (*), involving infinite sums, even when f equals a finite linear combination of q-blocks. This fact, as we shall see, is a reflection of the difference between the Lebesgue and Riemann integral; moreover, this phenomenon is also true for the definition of the atomic H^p-space norms for $0 < p \leq 1$.*

Keywords : block spaces

Classification :

∗*42A20 Convergence trigonometric series*

46A04 Locally convex Frechet spaces, etc.

28D20 Entropy and other measure-theoretic invariants

Zbl 0579.47052

Meyer, Yves

Intégrales singulières, opérateurs multilinéaires, analyse complexe et équations aux derivées partielles. (Singular integrals, multilinear operators, complex analysis and partial differential equations). *(French)*

Proc. Int. Congr. Math., Warszawa 1983, Vol. 2, 1001-1010 (1984).

[For the entire collection see Zbl 0553.00001.]

Dans cet exposé l'auteur présente des résultats de continuité L^2 pour des opérateurs linéaires $T : \mathcal{D}(\mathbb{R}^n) \to \mathcal{D}'(\mathbb{R}^n)$ dont le noyau-distribution vérifie certaines estimations (dites de Calderon-Zygmund). Il expose en particulier une condition nécessaire et suffisante de continuité L^2 pour ces opérateurs ainsi que des applications à l'analyse complexe et aux équations aux dérivées partielles (conjecture de T. Kato sur le domaine de $\sqrt{A}$ où: $A = -\sum_{i,j}(\partial/\partial x_j)(a_{ji}(x)\partial/\partial x_i)$).

D.Robert

Keywords : singular integrals; multilinear operators; complex analysis; partial differential equations; distribution kernel; Calderon-Zygmund estimate

Classification :

∗*47Gxx Integral operators and their generalizations*

47F05 Partial differential operators

46F10 Operations with distributions (generalized functions)

Zbl 0547.47032

Meyer, Y.

Continuité sur les espaces de Hölder et de Sobolev des opérateurs definis par des intégrales singulières. *(French)*

Sémin. Goulaouic-Meyer-Schwartz 1983-1984, Équat. dériv. part., Exposé No.1, 11 p. (1984).

numdam:SEDP_1983-1984____A1_0

Une nouvelle approche à la theorie des opérateurs a été définie par R. R. Coifman et Y. Meyer et étudiée par G. David et J. L. Iowiné. On part d'un opérateur (linéaire) défini au sens faible, c'est à dire d'un opérateur linéaire continu $T:\mathcal{D}(\mathbb{R}^n)\to\mathcal{D}'(\mathbb{R}^n)$. On suppose, en outre, que le noyau- distribution de T, une fois restreint à l'ouvert $x\neq y$ de $\mathbb{R}^n\times\mathbb{R}^n$, est une fonction K(x,y) vérifiant $|K(x,y)|\le C|x-y|^{-n}$ et $|K(x',y)-K(x,y)|\le C|x'-x|^{\alpha}|x-y|^{-n-\alpha}(0<\alpha\le 1,\ x\neq y,\ |x'-x|\le(1/2)|x-y|)$. Alors on donne une condition nécessaire et suffisante pour que T se prolonge en un opérateur linéaire continu sur l'espace de Sobolev homogène B^s, $0<s<\alpha$ ou l'espace de Hölder homogène C^s, $0<s<\alpha$.

Keywords : singular integral operator; kernel distribution; homogeneous Sobolev space; homogeneous Hölder space

Classification :

- ∗ *47Gxx Integral operators and their generalizations*
- *46E35 Sobolev spaces and generalizations*
- *46F10 Operations with distributions (generalized functions)*
- *47B38 Operators on function spaces*
- *45E05 Integral equations with kernels of Cauchy type*

Zbl 0529.00023

(Goulaouic, Ch.; Meyer, Y.)

Seminaire Goulaouic-Meyer-Schwartz: Equations aux dérivées partielles, 1983-1984. *(French)*

Palaiseau: Ecole Polytechnique, Centre de Mathematiques. 330 p. (1984).

Keywords : Equations aux derivees partielles; Seminaire

Classification :

- ∗ *00Bxx Conference proceedings and collections of papers*
- *35-06 Proceedings of conferences (partial differential equations)*

Zbl 0585.47010

Coifman, R.R.; Meyer, Y.

L'analyse harmonique non linéaire. (Non linear harmonic analysis). *(French)*

Topics in modern harmonic analysis, Proc. Semin., Torino and Milano 1982, Vol. II, 707-721 (1983).

[For the entire collection see Zbl 0527.00011.]

The authors introduce the new concept of non-linear harmonic analysis for the class of functions $T:B\to\mathcal{L}(L^2(\mathbb{R}^n),L^2(\mathbb{R}^n))$ satisfying the following conditions.

i) T is holomorphic in B,

ii) T commutes with the translations,

iii) T commutes with dilatations,

where B is the unit ball in $L^\infty(\mathbb{R}^n)$.

After explaining these conditions precisely, the author describes the different methods he uses for the non-linear harmonic analysis. He establishes some important properties

on the following topics.
1. The Cauchy operator c_Γ is an analytic function of Γ on the variety of specially defined curves of Lavrentiev.
2. The conformal representation of functions analytic in the curve of Lavrentiev.
3. Opertors defined by singular integrals.
4. Potential Theory.
5. Multilinear Operators of McIntosh.

D.Somasundaram

Keywords : non-linear harmonic analysis; curves of Lavrentiev; conformal representation
Classification :
∗*47A56 Functions whose values are linear operators*
43A99 Miscellaneous topics in harmonic analysis
47Gxx Integral operators and their generalizations

Zbl 0557.31004

Meyer, Y.

Théorie du potentiel dans les domaines lipschitziens (d'après G. C. Verchota). *(French)*

Sémin. Goulaouic-Schwartz 1979-1980, Équat. dériv. part., Exposé No.5, 16 p. (1983).
numdam:SEDP_1982-1983____A5_0

L'article donne un compte rendu des résultats de G. C. Verchota [Layer potentials and boundary value problems for Laplace equation in Lipschitz domains, Univ. Minnesota (1982)]. La solution du problème de Dirichlet et de celui de Neumann, pour un domaine Lipschitzien, borné ou non borné, est donnée par des potentiels de double couche. Les résultats sont apparentés à ceux de B. E. J. Dahlberg [Arch. Ration. Mech. Anal. 65, 275-288 (1977; Zbl 0406.28009)] et de D. S. Jerison et C. E. Kenig [Ann. Math., II. Ser. 113, 367-382 (1981; Zbl 0453.35036)]; Bull. Am. Math. Soc., New Ser. 4, 203-207 (1981; Zbl 0471.35026)].

E.M.J.Bertin

Keywords : Dirichlet problem; Neumann problem; Lipschitz domain; non; tangential limit; double layer potential
Classification :
∗*31B20 Boundary value and inverse problems (higher-dim. potential theory)*
31B15 Potentials, etc. (higher-dimensional)

Zbl 0532.42001

Beckner, William (ed.); Calderón, Alberto P. (ed.); Fefferman, Robert (ed.); Jones, Peter W. (ed.) *(Stein, E.M.; Beckner, W.; Janson, S.; Jerison, D.; Halász, G.; Montgomery, H.L.; Pychorides, S.K.; Taibleson, M.H.; Weiss, G.; Zafran, M.; Gatto, A.E.; Gutierrez, C.E.; Wheeden, R.L.; Hunt, R.A.; Kurtz, D.S.; Neugebauer, C.J.; Gundy, R.R.; Jodeit, M.; Ruiz, A.; Arocena, R.; Cotlar, Mischa; Sadosky, C.;*

Burkholder, D.L.; Cordoba, A.; Coifman, R.R.; McIntosh, A.; Meyer, Y.; Fefferman, R.; Phong, D.H.; Gasper, G.)

Conference on harmonic analysis in honor of Antonio Zygmund. (Papers presented at the Chicago Conference on Harmonic Analysis, March 23-28, 1981. Vol. 1). *(English)*

The Wadsworth Mathematics Series. Belmont, California: Wadsworth International Group, a Division of Wadsworth, Inc. XV, XII, 837 p. £ 107.95 (1983).

From the book's preface: "Between March 23 and 28, 1981, some two hundred mathematicians gathered at the University of Chicago to honor Professor Antoni Zygmund on the occasion of his eightieth birthday. It was a public tribute to the undisputed highest authority in Fourier series and the foremost analyst in the world.... These proceedings reflect the impact of Professor Zygmund's work and teaching. Most of the contributions to the conference are on topics he has either initiated or permanently shaped."
The proceedings of the conference have been edited in two volumes. The volume under review is divided into four parts: "Introductory paper", "Trigonometric series", "Fourier analysis on $\mathbb{R}^n$ *and real analysis", and "Singular integrals and pseudodifferential operators".*
Together with the second volume (divided also into four parts: "Hardy spaces", "Differentiation theory", "Partial differential equations", and "Other topics related to harmonic analysis") the proceedings give state of the art account of many areas of current interest in classical analysis. Moreover, these accounts are provided by the foremost experts in each subject.
The first part consists of an expository paper by E. M. Stein ("The development of square functions in the work of A. Zygmund") offering his personal perspective on a subject of fundamental importance in harmonic analysis.
The second part starts with an article by W. Beckner, S. Janson and D. Jerison ("Convolution inequalities on the circle"). They show that the classical convolution theorem of Young; to wit $\|f*g\|_q \leq \|f\|_p\|h\|_r$, $\frac{1}{q}=\frac{1}{r}+\frac{1}{q}-1$; *can be improved (for* $h\geq 0$) *to read* $\|f*h\|_{p+\delta}\leq\|f\|_p$, *for some* δ *depending on p,r and* $\|h\|_r$. *G. Gasper's article ("A convolution structure and positivity of a generalized translation operator for continuous Q Jacobi polynomials") contains a new proof of the Jacobi polynomial characterization and contains a list of open problems. G. Halász and H. L. Montgomery ("Bernstein's inequality for finite intervals") obtain the following extension of Bernstein's inequality. Let* $F(x)=\sum_{n=1}^N a_n e^{it_n x}$, $-T\leq t_n\leq T$, $n=1,...,N$. *Then,* $\|F'\|_{L^\infty[0,1]}\leq\Delta\|F\|_{L^\infty[0,1]}$ *with* $\Delta=C(N^2+T\log 6N$. *J. P. Kahane's article ("Slow points of Gaussian processes") studies the Gaussian series* $F(t)=\sum_{n=1}^\infty a_n\xi_n e^{2\pi int}$, $0\leq a_n\leq n^{-frac12-\alpha}$, $0<\alpha<1$, $n=1,...,$ *where the* ξ_n *are independent, normalized Gaussian random variables. It is shown that a.s.* $F(t+h)-F(t)=0(|h|^\alpha)$, $(h\to 0)$ *for a random set of t's, of Hausdorff dimension 1. S. K. Pychorides ("Notes on trigonometric polynomials") discusses the recent solutions to some well known conjectures on trigonometric polynomials. In particular, the article contains a discussion of the McGehee-Pigno-Smith-Konjagin solution of Littlewood's conjecture. M. H. Taibleson and G. Weiss ("Certain function spaces connected with almost everywhere convergence of Fourier series") study almost everywhere convergence of Fourier series on certain function spaces. In particular, they show that if a function*

has finite entropy then its Fourier series converges a.s. M. Zafran ("Exponential estimates in multiplier algebras") obtains the following theorem: Let $1 \leq p < \infty$, $p \neq 2$, $n \in \mathbb{N}$, *and let* $G = \mathbb{T}^n$, $\mathbb{R}^n$ *or* $\mathbb{Z}^n$ *then there exist* $C > 0$, $\beta > 1$, *such that* $\forall J \in \mathbb{Z}$, $\sup\{\|e^{ij\mu}\|_{M_p(G)} : \quad \mu \in M(G)\} \geq c\beta^{|\gamma|}$.
The third part of the book collects a number of articles on weighted norm inequalities: A. E. Gatto, C. E. Gutierrez and R. L. Wheeden ("On weighted fractional integrals"), R. A. Hunt and D. S. Kurtz ("The Hardy- Littlewood maximal function on L(p,1)") and R. A. Hunt, D. S. Kurtz and C. J. Neugebauer ("A note on the equivalence of A_p *and Sawyer's condition for equal weights"). R. R. Gundy ("The density of the area integral") studies a characterization of* H^p *spaces in terms of the density of the area integral. Let* $A(u)$ *be the Lusin area integral of a harmonic function u in* $\mathbb{R}^2_+$, *then write* $A^2(u)(x_0) = \int D(u,x_0)(r)dr$, *and let* $D(u)(x_0) = ess_r \sup D(u,x_0)(r)$. *The functional D is the maximal density of* A^2. *It is shown that* $\|A(u)\|_p \cong \|D(u)\|_p \cong \|N(u)\|_p, \quad 0 < p < \infty$, *(here N is the maximal nontangential maximal function). The article by M. Jodeit jun., ("On the decompostion of* $L^1_1(\mathbb{R})$ *functions into humps") gives an interesting decomposition of functions in the Sobolev space* $L^1_1(\mathbb{R})$ *that is useful in a number of problems: fractional integration, duality, multipliers and composition operators. A. Ruiz ("On the restriction of Fourier transform to curves") and T. Walsh ("Minimal smoothness for a bound on the Fourier transform of a surface measure") study restriction of the Fourier transforms to various curves and surfaces.*
The last part of the book contains two articles considering weighted norm inequalities from a different perspective. R. Arocena and M. Cotlar ("A generalized Herglotz-Bochner theorem and L^2 *weighted inequalities with finite measures") apply their generalized Herglotz-Bochner theorem to obtain weighted norm inequalities for the Hilbert transform. M. Cotlar and C. Sadosky ("On some* L^p *versions of the Helson-Szegö theorem") adapt the methods of their* L^2 *theory to deal with weighted* L^p *inequalities. Closely related to the weighted norm inequalities are the vector valued norm inequalities. These are considered in the articles by D. L. Burkholder ("A geometric condition that implies the existence of certain singular integrals of Banach space valued functions") and A. Cordoba ("Vector valued inequalities for multipliers"). Burkholder discusses a geometrical characterization of the Banach spaces with the UMD property (unconditionality property for martingale differences). He also shows that UMD implies the boundedness of the Hilbert transform on vector valued* L^p *spaces. (The converse is also valid and was proved afterwards by J. Bourgain.) The duality between vector valued inequalities and weighted norm inequalities is made evident in Cordoba's approach to some vector valued estimates for multiplier operators. R. R. Coifman, A. McIntosh and Y. Meyer ("Estimations* L^2 *pour les noyaux singuliers") discuss their solution to a problem of A. P. Calderón on commutators of singular integrals. R. Fefferman ("Some topics in Calderón-Zygmund theory") presents results on singular integrals invariant under two parameter family of dilations. D. H. Phong and E. M. Stein ("Singular integrals with kernels of mixed homogeneities") discuss the* $\bar{\partial}$*- Neumann problem on strongly pseudoconvex domains.*

M.Milman

Keywords : Conference; Harmonic analysis; Chicago
Classification :
∗*41-06 Proceedings of conferences (approximations and expansions)*

00Bxx Conference proceedings and collections of papers

Zbl 0527.47034

Meyer, Y.

Intégrales singulieres, opérateurs multilineaires et équations aux dérivées partielles. *(French)*

Sémin. Goulaouic-Meyer-Schwartz 1982-1983, Équat. dériv. part., Exposé No.19, 21 p. (1983).

numdam:SEDP_1982-1983____A19_0

Keywords : multilinear operators; partial differential equations with bounded and measurable coefficients; Sobolev space; convolution operator; distribution kernel

Classification :

∗*47F05 Partial differential operators*

47A30 Operator norms and inequalities

47B38 Operators on function spaces

47Gxx Integral operators and their generalizations

Zbl 0523.42016

Coifman, R.R.; Meyer, Y.; Stein, E.M.

Un nouvel espace fonctionnel adapte à l'étude des opérateurs definis par des intégrales singulieres. *(French)*

Harmonic analysis, Proc. Conf., Cortona/Italy 1982, Lect. Notes Math. 992, 1-15 (1983).

Keywords : kernel; McIntosh's formula; atomic decomposition; commutator

Classification :

∗*42B20 Singular integrals, several variables*

44A15 Special transforms

Zbl 0518.42024

Coifman, R.R.; David, G.; Meyer, Y.

La solution des conjectures de Calderon. *(French)*

Adv. Math. 48, 144-148 (1983). ISSN 0001-8708

http://dx.doi.org/10.1016/0001-8708(83)90084-1

http://www.sciencedirect.com/science/journal/00018708

Keywords : Calderon-Zygmund operator; singular integrals; Lipschitz mappings

Classification :

∗*42B20 Singular integrals, several variables*

42A50 Singular integrals, one variable

Zbl 0516.35083

Bourdaud, Gerard; Meyer, Yves
Inégalités L^2 precisees pour la classe $S^0_{0,0}$. *(French)*
Publ. Math. Orsay 83-02, 47-58 (1983).

Keywords : bounded pseudodifferential operator; Beurling's algebras; Beurling's weights; multipliers of Sobolev spaces
Classification :
∗ *35S05 General theory of pseudodifferential operators*
35B45 A priori estimates
46E35 Sobolev spaces and generalizations

Zbl 0509.46023

Meyer, Yves
Sur un problème de Michael Herman. *(French)*
Harmonic analysis, Conf. in Honor A. Zygmund, Chicago 1981, Vol. 2, 726-731 (1983).

Keywords : BMO; Sobolev space; Markov constant
Classification :
∗ *46E35 Sobolev spaces and generalizations*
46J15 Banach algebras of differentiable functions
39B52 Functional equations for functions with more general domains

---

Zbl 0497.35088

Coifman, Ronald R.; Deng, D.G.; Meyer, Yves
Domaine de la racine carrée de certains opérateurs différentiels accrétifs. *(French)*
Ann. Inst. Fourier 33, No.2, 123-134 (1983). ISSN 0373-0956
numdam:AIF_1983__33_2_123_0
http://aif.cedram.org/
http://annalif.ujf-grenoble.fr/aif-bin/feuilleter

Keywords : Carleson measure; bounded measurable coefficients; quadratic root operator
Classification :
∗ *35S05 General theory of pseudodifferential operators*
47B44 Accretive operators, etc. (linear)

Zbl 0548.42006

Coifman, Ronald; McIntosh, Alan; Meyer, Yves
The Hilbert transform on Lipschitz curves. *(English)*
Partial differential equations, Miniconf. Canberra 1981, Proc. Cent. Math. Anal. Aust. Natl. Univ. 1, 26-69 (1982).

[For the entire collection see Zbl 0535.00012.]
On prouve d'abord une conjecture de A.-P. Calderon [Proc. Natl. Acad. Sci. USA 74, 1324-1327 (1977; Zbl 0373.44003)]. Soit $\gamma:\mathbb{R}\to\mathbb{C}$, $\gamma(n)=x+ih(x)$, *h réelle et absolument continue,* $h'\in L^\infty(\mathbb{R})$. *Alors l'opérateur* H_γ *est bien défini par*

$$H_\gamma u(x)=(i/\pi)\quad v.p.\quad \int_{-\infty}^{\infty}\sqrt{\gamma'(x)\gamma'(y)}((\gamma(x)-\gamma(y))^{-1}u(y)dy,\quad u\in L\quad 2(\mathbb{R})$$

et borné de $L^2(\mathbb{R})$ *à* $L^2(\mathbb{R})$; *sa norme ne dépend que de* $\|h'\|_\infty$. *On considère ensuite* $f\in L^\infty(\mathbb{R})$, $Ref\geq\rho>0$ *et l'opérateur M défini dans* $L^2(\mathbb{R})$ *par* $Mu=-(d/dx)(fdu/dx)$, *de domaine* $D(M)=\{u\in H^1(\mathbb{R})fdu/dx\in H^1(\mathbb{R})\}$ *(*H^1 *l'espace de Sobolev bien connu). Alors* $D(M^{\frac12})=H^1(\mathbb{R})$ *et*

$$(1/6)\rho\|f\|_\infty^{-6}\|du/dx\|_2\leq\|M^{\frac12}u\|\leq 6\rho\|f\|_\infty^6\|du/dx\|_2,\quad u\in H^1(\mathbb{R}).$$

En outre, si $f=f_z$ *dépend analytiquement d'un paramètre* $z\in\Omega=\mathring{\Omega}\subset\mathbb{C}$ *et* $M=M_z$ *l'opérateur correspondant, alors pour tout* $u\in H^1(\mathbb{R})$, M_zu *dépend analytiquement de z.*

V.Iftimie

Keywords : Hilbert transform; Calderon conjecture; maximal accretive operator in $L_2(\mathbb{R})$*; domain of the square root*
Classification :
∗ *42A50 Singular integrals, one variable*
42B10 Fourier type transforms, several variables

Zbl 0505.35080

Coifman, R.; Meyer, Y.

Analyse harmonique non linéaire. *(French)*

Journ. Équ. Dériv. Partielles, Saint-Jean-De-Monts 1982, Exp. No.8, 7 p. (1982).
numdam:JEDP_1982____A8_0

Keywords : nonlinear harmonic analysis; maximal accretive operator; conjecture of Kato
Classification :
∗ *35R20 Partial operator-differential equations*
35S99 Pseudodifferential operators
47H06 Accretive operators, etc. (nonlinear)
43A99 Miscellaneous topics in harmonic analysis

Zbl 0497.42012

Coifman, R.R.; McIntosh, A.; Meyer, Y.

L'intégrale de Cauchy définit un opératuer borne sur L^2**pour les courbes lipschitziennes.** *(French)*

Ann. Math. (2) 116, 361-387 (1982). ISSN 0003-486X

Keywords : Cauchy integral; Lipschitz constants

Classification :
∗ *42B30 Hp-spaces (Fourier analysis)*
30D55 H (sup p)-classes

Zbl 0495.35011

Meyer, Y.

Nouvelles estimations pour les solutions d'équations aux dérivées partielles non linéaires. *(French)*

Sémin. Goulaouic-Meyer-Schwartz 1981-1982, Équat. dériv. part., Exposé No.6, 12 p. (1982).
numdam:SEDP_1981-1982____A5_0

Keywords : Besov spaces; optimal estimates; paradifferential operators of Bony; linearisation theorem; microlocal regularity
Classification :
∗ *35B45 A priori estimates*
35D10 Regularity of generalized solutions of PDE
35G20 General theory of nonlinear higher-order PDE

Zbl 0494.43006

Gatesoupe, Michel; Meyer, Yves

Quelques propriétés de l'algèbre de Fourier du groupe des entiers dyadiques. *(French)*

Ark. Mat. 20, 147-156 (1982). ISSN 0004-2080; ISSN 1871-2487
http://dx.doi.org/10.1007/BF02390506
http://www.arkivformatematik.org/
http://www.springerlink.com/content/0004-2080

Keywords : dyadic integers; Fourier algebra; Lusin's problem; regularity
Classification :
∗ *43A75 Analysis on specific compact groups*

Zbl 0512.42020

Coifman, R.R.; McIntosh, A.; Meyer, Y.

Estimations L^2pour les noyaux singuliers. *(French)*

Journ. Équ. Dériv. Partielles, Saint-Jean-de-Monts 1981, Exp. No.18, 7 p. (1981).
numdam:JEDP_1981____A18_0

Keywords : singular kernel; Calderon-Zygmund-kernel; square root of accretive operators
Classification :
∗ *42B20 Singular integrals, several variables*
45P05 Integral operators

47Gxx Integral operators and their generalizations

Zbl 0483.35084

Meyer, Yves

Les opérateurs pseudo-différentiels classique et leurs conjugues par changement de variable. *(French)*

Sémin. Goulaouic-Meyer-Schwartz 1980-1981, Équat. dériv. part., Exposé No.10, 16 p. (1981).

numdam:SEDP_1980-1981____A12_0

Keywords : classical pseudodifferential operator; regularity

Classification :

∗ *35S05 General theory of pseudodifferential operators*

Zbl 0474.46028

Meyer, Yves

Multiplication of distributions. *(English)*

Adv. Math., Suppl. Stud. 7B, 603-615 (1981).

Keywords : multiplication of distributions; paraproduct

Classification :

∗ *46F10 Operations with distributions (generalized functions)*

Zbl 0473.35021

Meyer, Yves

Remarques sur un théorème de J. M. Bony. *(French)*

Rend. Circ. Mat. Palermo, II. Ser. 1981, Suppl. 1, 1-20 (1981). ISSN 0009-725X

Keywords : microlocal regularity; nonlinear partial differential equation; Littlewood-Paley decomposition; paradifferential operator

Classification :

∗ *35D10 Regularity of generalized solutions of PDE*

35S05 General theory of pseudodifferential operators

Zbl 0462.35012

Meyer, Yves

Régularité des solutions des équations aux dérivées partielles non linéaires (d'apres J.-M. Bony). *(French)*

Semin. Bourbaki, 32e annee, Vol. 1979/80, Exp. 16. 560, Lect. Notes Math. 842, 293-302 (1981).

numdam:SB_1979-1980__22__293_0

Keywords : quasilinear equations; pseudodifferential operators; characteristic direction

Classification :
∗ 35D10 *Regularity of generalized solutions of PDE*
35S05 *General theory of pseudodifferential operators*

Zbl 0541.42008

Coifman, R.R.; Meyer, Y.
Une généralisation du théorème de Calderón sur l'intégrale de Cauchy. *(French)*
Fourier analysis, Proc. Semin., El Escorial 1979, 87-116 (1980).

[For the entire collection see Zbl 0451.00015.]
Let $\mathcal{G}$ be the group of increasing homeomorphisms h of $\mathbb{R}$ that preserve the class of sets of measure zero and operate on BMO ($f\mathbb{O}h \in BMO$ for $f \in BMO$). The map $h \mapsto \log h'$ (derivative in the sense of distributions) takes $\mathcal{G}$ onto an open set U in $BMO_{\mathbb{R}}$.
Let H denote the Hilbert transform and, for $h \in \mathcal{G}$, V_h the operator $f \mapsto f\mathbb{O}h$ on BMO.
Theorem: The map Λ : log h'$\mapsto V_h$ from U to $\mathcal{L}(BMO)$ is real analytic.
This theorem is applied to give a direct sum decomposition of BMO(Γ) (Γ a plane curve subject to a so called chord-arc condition) and to prove boundedness of singular Calderón integral operators.

W.Kugler
Keywords : increasing homeomorphisms; sets of measure zero; BMO; Hilbert transform; real analytic; direct sum decomposition; plane curve; chord-arc condition; boundedness; singular Calderón integral operators
Classification :
∗ 42A50 *Singular integrals, one variable*

Zbl 0492.42023

Meyer, Yves
Les nouvelles intégrales singulieres de Calderon. *(French)*
Seminaire Bourbaki, Vol. 1978/79, Lect. Notes Math. 770, 57-65 (1980).
numdam:SB_1978-1979__21__57_0

Keywords : singular integrals on Lipschitz curves
Classification :
∗ 42B20 *Singular integrals, several variables*

Zbl 0427.42006

Coifman, R.R.; Meyer, Y.
Fourier analysis of multilinear convolutions, Calderon's theorem, and analysis on Lipschitz curves. *(English)*
Euclidean harmonic analysis, Proc. Semin., Univ. Maryland 1979, Lect. Notes Math. 779, 104-122 (1980).

Keywords : multilinear convolutions; first commutator; Fourier transform for Lipschitz curves
Classification :
∗42B10 Fourier type transforms, several variables
47B20 Subnormal operators, etc.
47B15 Hermitian and normal operators
42A85 Convolution (Fourier analysis)

Zbl 0452.42008

Meyer, Yves
Produits de Riesz généralises. *(French)*
Publ. Math. Orsay 79.07, 38-48 (1979).

Keywords : Riesz product; Ap-class of weights; bounded mean oscillation
Classification :
∗42A55 Lacunary series
42A32 Trigonometric series of special types
42A16 Fourier coefficients, etc.
42B25 Maximal functions

Zbl 0435.30029

Coifman, Ronald; Meyer, Yves
Le théorème de Calderon par les "Méthodes de variable réelle". *(French)*
Publ. Math. Orsay 79.07, 49-55 (1979).

Keywords : Cauchy integral; Hilbert transform; Riemann's mapping function; bounded mean oscillation
Classification :
∗30D55 H (sup p)-classes
30C35 General theory of conformal mappings
42A45 Multipliers, one variable
30E20 Integration (one complex variable)

Zbl 0433.35073

Meyer, Yves
Sharp estimates for pseudodifferential operators. *(English)*
Harmonic analysis in Euclidean spaces, Part 2, Williamstown/ Massachusetts 1978, Proc. Symp. Pure Math., Vol. 35, 105-113 (1979).

Keywords : sharp estimates; Calderon-Zygmund approach; pseudo differential operators; Littlewood-Paley-theory; commutators
Classification :
∗35S05 General theory of pseudodifferential operators
47F05 Partial differential operators
47Gxx Integral operators and their generalizations

58J40 Pseudodifferential and Fourier integral operators on manifolds

---

Zbl 0427.42007

Coifman, Ronald; Meyer, Yves
Le théorème de Calderon par les "méthodes de variable réelle". *(French)*
C. R. Acad. Sci., Paris, Sér. A 289, 425-428 (1979).

Keywords : Cauchy integral; Riemann mapping theorem
Classification :
- ∗ *42B30 Hp-spaces (Fourier analysis)*
- *42B20 Singular integrals, several variables*
- *30C35 General theory of conformal mappings*

Zbl 0422.43006

Hartman, S.; Meyer, Y.
Interpolation harmonique sur les compacts. *(French)*
Colloq. Math. 40, 265-276 (1979). ISSN 0010-1354; ISSN 1730-6302
http://journals.impan.gov.pl/cm/
http://matwbn.icm.edu.pl/spis.php?wyd=8jez=pl

Keywords : Fourier algebra; trigonometric polynomial; Siden set; Helsen set
Classification :
- ∗ *43A46 Special sets in abstract harmonic analysis*
- *42B10 Fourier type transforms, several variables*

Zbl 0483.35082

Coifman, Ronald R.; Meyer, Yves
Au dela des opérateurs pseudo-différentiels. *(French)*
Asterisque 57, 185 p. (1978).

Keywords : Calderon-Vaillancourt theorem; decomposition of operators; commutators; boundedness of pseudodifferential operators; Calderon-Zygmund operators; minimal conditions of regularity; Calderon-Zygmund singular integrals
Classification :
- ∗ *35S05 General theory of pseudodifferential operators*
- *35B35 Stability of solutions of PDE*
- *35-02 Research monographs (partial differential equations)*

Zbl 0415.42012

Coifman, R.; Meyer, Y.
Commutators of singular integrals. *(English)*
Fourier analysis and approximation theory, Vol. I, Budapest 1976, Colloq. Math. Soc. Janos Bolyai 19, 265-273 (1978).

Keywords : kernels of Calderon-Zygmund type
Classification :
∗ *42B20 Singular integrals, several variables*

Zbl 0406.35061

Meyer, Yves
Inégalités L^2 à poids pour les opérateurs différentiels. *(French)*
Publ. Math. Orsay No. 7812, 54-62 (1978).

Keywords : Spectral Method; Complex Method; Real Method; Weighted L_2-Estimate; Differential Operator
Classification :
∗ *35R20 Partial operator-differential equations*
35R45 Partial differential inequalities

Zbl 0401.47026

Meyer, Yves
Estimations L^2 pour les opérateurs pseudo-différentiels. *(French)*
Publ. Math. Orsay No.78-12, 47-53 (1978).

Keywords : L2-Boundedness; Pseudodifferential Operators
Classification :
∗ *47Gxx Integral operators and their generalizations*
35S05 General theory of pseudodifferential operators

Zbl 0384.35053

Meyer, Y.
Nouvelles estimations pour les opérateurs pseudo-différentiels. *(French)*
Sémin. Goulaouic-Schwartz 1977-1978, Équat. dériv. part. Anal. fonct., Exposé No.10, 6 p. (1978).
numdam:SEDP_1977-1978____A11_0

Classification :
∗ *35S05 General theory of pseudodifferential operators*

Zbl 0368.47031

Coifman, Ronald R.; Meyer, Yves
Commutateurs d'intégrales singulières et opérateurs multilinéaires. *(French)*
Ann. Inst. Fourier 28, No.3, 177-202 (1978). ISSN 0373-0956
numdam:AIF_1978__28_3_177_0
http://aif.cedram.org/
http://annalif.ujf-grenoble.fr/aif-bin/feuilleter

Classification :
∗ *47Gxx Integral operators and their generalizations*
47B47 Derivations and linear operators defined by algebraic conditions

---

Zbl 0424.15003

Lesieur, Leonce; Meyer, Yves; Joulain, Claude; Lefebvre, Jean
Algèbre linéaire, géométrie. Mathématiques supérieures, 1er cycle, 1re annee. *(French)*
Serie "Mathematiques". Paris: Librairie Armand Colin. IV, 348 p. (1977).

Keywords : linear algebra; geometry
Classification :
∗ *15-01 Textbooks (linear algebra)*
51-01 Textbooks (geometry)

Zbl 0369.43009

Meyer, Yves
Quelques problèmes sur les fonctions presque-périodiques. *(French)*
Publ. math. d'Orsay, No.77-77, Semin. d'Anal. harmon. 1976-1977, 21-27 (1977).

Classification :
∗ *43A60 Almost periodic functions on groups, etc.*
42A75 Periodic functions and generalizations

Zbl 0366.47023

Meyer, Yves; Coifman, R.
Opérateurs pseudo-différentiels et théorème de Calderon. *(French)*
Publ. math. d'Orsay, No.77-77, Semin. d'Anal. harmon. 1976-1977, 28-40 (1977).

Classification :
∗ *47Gxx Integral operators and their generalizations*
35S05 General theory of pseudodifferential operators

Zbl 0339.42019

Meyer, Yves
Harmonic analysis of mean-periodic functions. *(English)*
Stud. harmon. Anal., Proc. Conf. Chicago 1974, MAA Stud. Math. 13, 151-160 (1976).

Classification :
∗ *42A85 Convolution (Fourier analysis)*
42A38 Fourier type transforms, one variable
30E10 Approximation in the complex domain
43A45 Spectral synthesis on groups, etc.

Zbl 0324.42018

Meyer, Yves
Comportement asymptotique des solutions de certaines équations de convolution. *(French)*
J. Math. pur. appl., IX. Sér. 55, 69-97 (1976). ISSN 0021-7824
http://www.sciencedirect.com/science/journal/00217824

Classification :
∗ *42A85 Convolution (Fourier analysis)*
46E30 Spaces of measurable functions
46J15 Banach algebras of differentiable functions

Zbl 0318.42028

Meyer, Yves
Remarques sur un théorème de J. Delsarte. *(French)*
Ann. Inst. Fourier 26, No.2, 133-152 (1976). ISSN 0373-0956
numdam:AIF_1976__26_2_133_0
http://aif.cedram.org/
http://annalif.ujf-grenoble.fr/aif-bin/feuilleter

Classification :
∗ *42A75 Periodic functions and generalizations*

Zbl 0431.42010

Coifman, R.; Meyer, Y.
La décomposition de l'opérateur de Calderon. *(French)*
Publ. Math. Orsay No.75-44, 62-65 (1975).

Keywords : Hilbert transform; Calderon operator
Classification :
∗ *42B20 Singular integrals, several variables*
44A15 Special transforms

Zbl 0409.42016

Coifman, R.; Meyer, Y.
Le théorème des commutateurs de Calderon. *(French)*
Publ. Math. Orsay No.75-44, 37-46 (1975).

Keywords : Double Commutator
Classification :
∗ *42B20 Singular integrals, several variables*

Zbl 0371.13001

Lesieur, Leonce; Meyer, Yves; Joulain, Claude; Lefebvre, Jean
Algèbre générale. Mathématiques supérieures, 1er cycle, 1re annee. *(French)*
Serie "Mathematiques". Paris: Librairie Armand Colin. VIII, 243 p. F 65.00 (1975).

Classification :
∗ *13-01 Textbooks (commutative rings and algebras)*
12-01 Textbooks (field theory)
26C15 Rational functions (real variables)
12D05 Factorization of real or complex polynomials
13F20 Polynomial rings

Zbl 0334.42025

Gramain, F.; Meyer, Y.
Quelques fonctions moyenne-périodiques bornees. *(French)*
Colloq. Math. 33, 133-137 (1975). ISSN 0010-1354; ISSN 1730-6302
http://journals.impan.gov.pl/cm/
http://matwbn.icm.edu.pl/spis.php?wyd=8jez=pl

Classification :
∗ *42A75 Periodic functions and generalizations*
43A45 Spectral synthesis on groups, etc.
11R04 Algebraic numbers
11R06 Special algebraic numbers

Zbl 0324.44005

Coifman, R.R.; Meyer, Yves
On commutators of singular integrals and bilinear singular integrals. *(English)*
Trans. Am. Math. Soc. 212, 315-331 (1975). ISSN 0002-9947; ISSN 1088-6850
http://dx.doi.org/10.2307/1998628
http://www.ams.org/tran/

Classification :
∗ *42B25 Maximal functions*

Zbl 0287.43009

Meyer, Yves
Les nombres de Pisot et la synthèse spectrale. *(French)*
Bull. Soc. Math. Fr., Suppl., Mém. 37 (Journées arithmétiques, Grenoble 1973), 117-120 (1974). ISSN 0037-9484
numdam:MSMF_1974__37__117_0

Classification :
∗ *43A45 Spectral synthesis on groups, etc.*
11K55 Metric theory of other number-theoretic algorithms and expansions
11R06 Special algebraic numbers

Zbl 0287.42003

Meyer, Yves
Théorie L^p des sommes trigonométriques apériodiques. *(French)*
Ann. Inst. Fourier 24, No.4, 189-211 (1974). ISSN 0373-0956
numdam:AIF_1974__24_4_189_0
http://aif.cedram.org/
http://annalif.ujf-grenoble.fr/aif-bin/feuilleter

Classification :
∗ *42A05 Trigonometric polynomials*

Zbl 0258.42028

Gramain, Francois; Meyer, Yves
Ensembles de frequences et fonctions presque-périodiques. *(French)*
Colloq. Math. 30, 269-275 (1974). ISSN 0010-1354; ISSN 1730-6302
http://journals.impan.gov.pl/cm/
http://matwbn.icm.edu.pl/spis.php?wyd=8jez=pl

Classification :
∗ *42A75 Periodic functions and generalizations*

Zbl 0338.35058

Meyer, Yves
Nombres premiers et vibrations. *(French)*
Semin. Delange-Pisot-Poitou, 13e annee 1971/72, Theorie des Nombres, Fasc. 1, 2, Expose 15, 5 p. (1973).
numdam:SDPP_1971-1972__13_2_A3_0

Classification :
∗ *35L05 Wave equation*
11N05 Distribution of primes

Zbl 0328.43009

Meyer, Yves
Adeles et séries trigonométriques speciales. *(French)*
Semin. Delange-Pisot-Poitou, 13e annee 1971/72, Theorie des Nombres, Fasc. 1, 2, Expose 11, 16p. (1973).

numdam:SDPP_1971-1972__13_1_A10_0

Classification :

- ∗*43A60 Almost periodic functions on groups, etc.*
- *11K55 Metric theory of other number-theoretic algorithms and expansions*
- *11L03 Trigonometric and exponential sums, general*
- *11R06 Special algebraic numbers*
- *42A75 Periodic functions and generalizations*
- *43A05 Measures on groups, etc.*
- *43A40 Character groups and dual objects*
- *11R56 Adele rings and groups*
- *43A25 Fourier type transforms on locally compact abelian groups*
- *42A05 Trigonometric polynomials*
- *43A50 Convergence of Fourier series and of inverse transforms*

Zbl 0318.42002

Meyer, Yves

Trois problèmes sur les sommes trigonométriques. *(French)*

Astérisque 1, 1-86 (1973). ISSN 0303-1179

http://smf.emath.fr/en/Publications/Asterisque/index.html

Classification :

- ∗*42A05 Trigonometric polynomials*
- *42-00 Reference works (Fourier analysis)*
- *42A10 Trigonometric approximation*
- *43A45 Spectral synthesis on groups, etc.*

Zbl 0259.42003

Meyer, Yves

Sur un problème de M. Mandelbrojt. *(French)*

Analyse harmon. Domaine compl., Actes Table ronde internat. Centre Nat. Rech. Sci., Montpellier 1972, Lect. Notes Math. 336, 161-167 (1973).

Classification :

- ∗*42A05 Trigonometric polynomials*
- *42-00 Reference works (Fourier analysis)*
- *43-00 Reference works (abstract harmonic analysis)*

Zbl 0259.42002

Meyer, Yves

Théorie L^p des sommes trigonométriques aperiodiques. *(French)*

C. R. Acad. Sci., Paris, Sér. A 277, 15-17 (1973).

Classification :

∗ *42A05 Trigonometric polynomials*
42B05 Fourier series and coefficients, several variables
35L05 Wave equation

Zbl 0248.10029

Meyer, Y.

Adeles et séries trigonométriques speciales. *(French)*

Ann. Math. (2) 97, 171-186 (1973). ISSN 0003-486X

Classification :

∗ *11L03 Trigonometric and exponential sums, general*
43-XX Abstract harmonic analysis
43A05 Measures on groups, etc.
11J99 Diophantine approximation
11K55 Metric theory of other number-theoretic algorithms and expansions
11R56 Adele rings and groups

Zbl 0267.43001

Meyer, Yves

Algebraic numbers and harmonic analysis. *(English)*

North-Holland Mathematical Library. Vol. 2. Amsterdam-London: North- Holland Publishing Co mpany. X, 274 p. Hfl. 52.50; £ 16.50 (1972).

Classification :

∗ *43-02 Research monographs (abstract harmonic analysis)*
42-02 Research monographs (Fourier analysis)
43-XX Abstract harmonic analysis
11D99 Diophantine equations
11E95 p-adic theory of forms
42A16 Fourier coefficients, etc.

Zbl 0246.43010

Meyer, Y.

Étude asymptotique des vibrations des sphères. *(French)*

Sémin. Goulaouic-Schwartz 1971-1972, Équat. dériv. part. anal. fonct., Exposé No.XXVIII, 9 p. (1972).

numdam:SEDP_1971-1972____A28_0

Classification :

∗ *43A60 Almost periodic functions on groups, etc.*
35B05 General behavior of solutions of PDE
35B15 Almost periodic solutions of PDE
42A16 Fourier coefficients, etc.

42A75 Periodic functions and generalizations
43A85 Analysis on homogeneous spaces

Zbl 0239.42031

Meyer, Yves
Series trigonométriques speciales et corps quadratiques. *(French)*
Stud. Math. 44, 321-333 (1972). ISSN 0039-3223; ISSN 1730-6337
http://journals.impan.gov.pl/sm/
http://matwbn.icm.edu.pl/spis.php?wyd=2

Classification :
∗ *42A75 Periodic functions and generalizations*
43A60 Almost periodic functions on groups, etc.
11-XX Number theory

Zbl 0234.43003

Meyer, Yves
Recent advances in spectral synthesis. *(English)*
Conf. harmonic Analysis, College Park, Maryland 1971, Lect. Notes Math. 266, 239-253 (1972).

Classification :
∗ *43A45 Spectral synthesis on groups, etc.*

Zbl 0239.42032

Meyer, Yves
Special trigonometric series. *(English)*
Journees arithmetiques Francaises, Univ. Provence 1971, 5p. (1971).

Classification :
∗ *42A75 Periodic functions and generalizations*
11-XX Number theory
43A60 Almost periodic functions on groups, etc.

Zbl 0227.10042

Meyer, Yves
Nombres transcendants et répartition modulo 1. (Transcendent numbers and distribution mod 1.). *(French)*
Bull. Soc. Math. Fr., Suppl., Mém. 25, 143-149 (1971).
numdam:MSMF_1971__25__143_0

Classification :
∗ *11K31 Special sequences*

11K36 Well-distributed sequences and other variations

Zbl 0226.43006

Lindahl, L.-{Å}. (ed.); Poulsen, F. (ed.) *(Körner, Thomas W.; Tewari, U.; Hedberg, T.; Björk, J.-E.; Varopoulos, N.Th.; Meyer, Y.)*

Thin sets in harmonic analysis. Seminars held at Institute Mittag-Leffler 1969/70. *(English)*

Lecture Notes in Pure and Applied Mathematics. 2. New York: Marcel Dekker, Inc. IX, 185 p. £ 9.50 (1971).

Keywords : Harmonic analysis; Thin sets; Seminar

Classification :

∗*43A46 Special sets in abstract harmonic analysis*

43-06 Proceedings of conferences (abstract harmonic analysis)

00B25 Proceedings of conferences of miscellaneous specific interest

---

Zbl 0225.43010

Meyer, Yves

Nombres de Pisot et analyse harmonique. (Pisot numbers in harmonic analysis). *(French)*

Actes Congr. internat. Math. 1970, 2, 663-665 (1971).

Classification :

∗*43A45 Spectral synthesis on groups, etc.*

11K70 Harmonic analysis and almost periodicity

11R06 Special algebraic numbers

Zbl 0213.32802

Meyer, Y.

Nombres algébriques, nombres transcendants et équirépartition modulo 1. *(French)*

Acta Arith. 16, 347-350 (1970). ISSN 0065-1036; ISSN 1730-6264

http://journals.impan.gov.pl/aa/

http://matwbn.icm.edu.pl/spis.php?wyd=6

Classification :

∗*11J81 Transcendence (general theory)*

11K31 Special sequences

Zbl 0199.20301

Meyer, Y.

Les nombres de Pisot et l'analyse harmonique. *(French)*

Stud. Math. 34, 127-147 (1970). ISSN 0039-3223; ISSN 1730-6337

http://journals.impan.gov.pl/sm/
http://matwbn.icm.edu.pl/spis.php?wyd=2

Keywords : functional analysis

Zbl 0198.47702

Meyer, Yves
Les nombres de Pisot et la synthese harmonique. *(French)*
Ann. Sci. Éc. Norm. Supér. (4) 3, 235-246 (1970). ISSN 0012-9593
numdam:ASENS_1970_4_3_2_235_0
http://smf.emath.fr/en/Publications/AnnalesENS/
http://www.sciencedirect.com/science/journal/00129593
http://www.numdam.org/numdam-bin/browse?j=ASENSsl=2

Keywords : functional analysis

Zbl 0197.32702

Meyer, Yves
Nombres algébriques et analyse harmonique. *(French)*
Ann. Sci. Éc. Norm. Supér. (4) 3, 75-110 (1970). ISSN 0012-9593
numdam:ASENS_1970_4_3_1_75_0
http://smf.emath.fr/en/Publications/AnnalesENS/
http://www.sciencedirect.com/science/journal/00129593
http://www.numdam.org/numdam-bin/browse?j=ASENSsl=2

Keywords : number theory

Zbl 0189.14301

Meyer, Y.
Nombres de Pisot, nombres de Salem et analyse harmonique. Cours Peccot donne au College de France en avril-mai 1969. *(French)*
Lecture Notes in Mathematics. 117. Berlin-Heidelberg-New York: Springer-Verlag. 63 p. (1970).

Keywords : functional analysis

Zbl 0218.12001

Meyer, Y.
Caractérisation des nombres de Pisot-Vijayaraghavan. *(French)*
Semin. Delange-Pisot-Poitou 9 (1967/68), Theorie Nombres, No.8, 4p. (1969).
numdam:SDPP_1967-1968__9_1_A8_0

Classification :
∗ *11R06 Special algebraic numbers*

Zbl 0208.15403

Meyer, Y.
Problèmes de l'unicité, de la synthèse et des isomorphismes en analyse harmonique. *(French)*
Sémin. Bourbaki 1967/68, No.341, 9 p. (1969).
numdam:SB_1966-1968__10__463_0

Classification :
∗ *43A20 L1 algebras on groups, etc.*

Zbl 0207.35401

Meyer, Y.
Nombres de Pisot, nombres de Salem et répartition modulo 1. *(French)*
Sémin. Delange-Pisot-Poitou 10 (1968/69), Théorie Nombres, No.2, 6 p. (1969).
numdam:SDPP_1968-1969__10_1_A2_0
numdam:SDPP_1976-1977__18_2_A13_0

Classification :
∗ *11R06 Special algebraic numbers*
11K06 General theory of distribution modulo 1

Zbl 0181.06703

Bonami, A.; Meyer, Y.
Propriétés de convergence de certaines séries trigonométriques. *(French)*
C. R. Acad. Sci., Paris, Sér. A 269, 68-70 (1969).

Keywords : approximation and series expansion

Zbl 0179.46501

Meyer, Yves; Schreiber, Jean-Pierre
Quelques fonctions moyennes-périodiques non bornées. *(French)*
Ann. Inst. Fourier 19, No.1, 231-236 (1969). ISSN 0373-0956
numdam:AIF_1969__19_1_231_0
http://aif.cedram.org/
http://annalif.ujf-grenoble.fr/aif-bin/feuilleter

Keywords : functional analysis

Zbl 0179.46402

Meyer, Yves
Algèbres de restriction non isomorphes. *(French)*
Ann. Inst. Fourier 19, No.1, 117-124 (1969). ISSN 0373-0956
numdam:AIF_1969__19_1_117_0
http://aif.cedram.org/
http://annalif.ujf-grenoble.fr/aif-bin/feuilleter

Keywords : functional analysis

Zbl 0177.18601

Meyer, Y.
Les problèmes de l'unicité et de la synthese harmonique et la répartition modulo 1. *(French)*
C. R. Acad. Sci., Paris, Sér. A 269, 320-322 (1969).

Keywords : functional analysis

Zbl 0165.36403

Meyer, Y.
Nombres algébriques et répartition modulo I. *(French)*
C. R. Acad. Sci., Paris, Sér. A 268, 25-27 (1969).

Keywords : number theory

Zbl 0175.44501

Meyer, Yves
Isomorphismes entre certaines algèbres de restrictions. *(French)*
Ann. Inst. Fourier 18, No.2, 73-86 (1968). ISSN 0373-0956
numdam:AIF_1968__18_2_73_0
http://aif.cedram.org/
http://annalif.ujf-grenoble.fr/aif-bin/feuilleter

Keywords : functional analysis

Zbl 0169.18001

Meyer, Y.
Endomorphismes des ideaux fermes de $L^1(G)$, classes de Hardy et séries de Fourier lacunaires. *(French)*
Ann. Sci. Éc. Norm. Supér. (4) 1, No. 4, 499-580 (1968). ISSN 0012-9593
numdam:ASENS_1968_4_1_4_499_0
http://smf.emath.fr/en/Publications/AnnalesENS/

http://www.sciencedirect.com/science/journal/00129593
http://www.numdam.org/numdam-bin/browse?j=ASENSsl=2

Keywords : functional analysis

Zbl 0159.42501

Meyer, Y.
Spectres des mesures et mesures absolutement continues. *(French)*
Stud. Math. 30, 87-99 (1968). ISSN 0039-3223; ISSN 1730-6337
http://journals.impan.gov.pl/sm/
http://matwbn.icm.edu.pl/spis.php?wyd=2

Keywords : functional analysis

Zbl 0159.42402

Meyer, Y.
Nombres de Pisot et analyse harmonique. *(French)*
C. R. Acad. Sci., Paris, Sér. A 267, 196-198 (1968).

Keywords : functional analysis

Zbl 0159.42401

Meyer, Y.
Problème de l'unicité et de la synthese en analyse harmonique. *(French)*
C. R. Acad. Sci., Paris, Sér. A 266, 275-276 (1968).

Keywords : functional analysis

Zbl 0159.42304

Meyer, Y.
Une caractérisation des nombres de Pisot. *(French)*
C. R. Acad. Sci., Paris, Sér. A 266, 63-64 (1968).

Keywords : functional analysis

Zbl 0166.11603

Meyer, Y.
Ideaux fermes de L^1 dans lesquels une suite approche I'identite. *(French)*
Math. Scand. 19, 219-222 (1967). ISSN 0025-5521
http://www.mscand.dk/
http://www.digizeitschriften.de/resolveppn/PPN35397434X

Keywords : functional analysis

Zbl 0154.15203

Meyer, Y.
Isomorphismes entre certaines algèbres de restrictions. *(French)*
C. R. Acad. Sci., Paris, Sér. A 265, 18-20 (1967).

Keywords : functional analysis

Zbl 0166.11602

Meyer, Y.
Multiplicateurs des coefficients de Fourier des fonctions intégrables analytiques. *(French)*
C. R. Acad. Sci., Paris, Sér. A 263, 385-387 (1966).

Keywords : functional analysis

Zbl 0166.11601

Meyer, Y.
Prolongement des multiplicateurs d'ideaux fermes de $L^1(R^n)$. *(French)*
C. R. Acad. Sci., Paris, Sér. A 262, 744-745 (1966).

Keywords : functional analysis

Zbl 0161.11202

Meyer, Y.
Le spectre de Wiener. *(French)*
Stud. Math. 27, 189-201 (1966). ISSN 0039-3223; ISSN 1730-6337
http://journals.impan.gov.pl/sm/
http://matwbn.icm.edu.pl/spis.php?wyd=2

Keywords : functional analysis

Zbl 0161.11201

Meyer, Y.; Rosenthal, H.P.
Convexite et ensemble de Ditkin forts. *(French)*
C. R. Acad. Sci., Paris, Sér. A 262, 1404-1406 (1966).

Keywords : functional analysis

Zbl 0134.28401

Meyer, Y.
Complement à un théorème de Paley. *(French)*

C. R. Acad. Sci., Paris, Sér. A 262, 281-282 (1966).

Keywords : approximation and series expansion