
Zbl pre05729450

Ngô, Báo Châu

The Fundamental Lemma for Lie algebras. (Le Lemme Fondamental pour les algèbres de Lie.) (French)

Publ. Math., Inst. Hautes Étud. Sci. 111, 1-271 (2010). ISSN 0073-8301; ISSN 1618-1913
http://dx.doi.org/10.1007/s10240-010-0026-7
http://link.springer.de/link/service/journals/10240/

Classification :

∗22E57
22Exx Lie groups
11Fxx Discontinuous groups and automorphic forms

Zbl 1182.22009

Ngô, Báo Châu

Report on the proof of some conjectures on orbital integrals in Langlands' program. (English)

Vietnam J. Math. 37, No. 2-3, 127-140 (2009). ISSN 0866-7179
http://www.math.ac.vn/publications/vjm/VJM_37/127.htm
http://www.math.ac.vn/publications/vjm/

The paper is a brief survey of the celebrated proof by the author of the so-called fundamental lemma of the Langlands program. In the author's words, he recalls "the basics about orbital integrals, the natural places in mathematics where we encounter them, the fundamental lemma and the transfer conjectures stated in a precise form only in certain cases". The author surveys various contributions to the proof of these conjectures, focuses on certain algebraic varieties that play a central role in the understanding of non-Archimedean orbital integrals.
The complete proof of the above results exists so far as a preprint of about 200 pages; see `http://arxiv.org/abs/0801.0446`. For the case of unitary groups see *G. Laumon* and the author [Ann. Math. (2) 168, No. 2, 477–573 (2008; Zbl 1179.22019)].

Anatoly N. Kochubei (Kyïv)

Keywords : Langlands program; fundamental lemma; orbital integral; transfer conjectures; Hitchin fibration

Classification :

∗22E57
14D24
22E35 Analysis on p-adic Lie groups
22E50 Repres. of Lie and linear algebraic groups over local fields

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Zbl 1179.22019

Laumon, Gérard; Ngô, Bao Châu

The fundamental lemma for unitary groups. (Le lemme fondamental pour les groupes unitaires.) (French)

Ann. Math. (2) 168, No. 2, 477-573 (2008). ISSN 0003-486X; ISSN 1939-0980
http://dx.doi.org/10.4007/annals.2008.168.477
http://annals.math.princeton.edu/annals/2008/168-2/p03.xhtml
http://annals.math.princeton.edu/annals/about/cover/cover.html
http://pjm.math.berkeley.edu/annals/about/journal/about.html
http://www.jstor.org/journals/0003486X.html

This article gives a proof of the fundamental lemma for unitary groups of rank n over a local field of equal characteristics $> n$. The study is based on the geometric interpretation of elliptic endoscopy given by Ngô, which uses the Hitchin fibration. In the article the Hitchin fibration is described explicitly for $G = \mathrm{U}(n)$, as well as its relation with the Hitchin fibration for an endoscopic group $H = \mathrm{U}(n_1) \times \mathrm{U}(n_2)$ for G, where $n = n_1 + n_2$. (For general reductive groups see *B. C. Ngô* [Invent. Math. 164, No. 2, 399–453 (2006; Zbl 1098.14023)].) From a result on the perverse cohomology of the Hitchin fibration a numerical identity is deduced, using a fixed-point theorem. Explicit computation of the factors in this global identity gives the fundamental lemma.

J. G. M. Mars (Utrecht)

Keywords : endoscopy; Hitchin fibration
Classification :
∗22E50 Repres. of Lie and linear algebraic groups over local fields
11F72 Spectral theory
22E35 Analysis on p-adic Lie groups
14H60 Vector bundles on curves
14F20 Grothendieck cohomology and topology

Zbl 1141.22005

Ngô, Bao Châu; Ngô, Tuân Dac

Counting of G-shtukas: the elliptic regular set. (Comptage de G-chtoucas: La partie régulière elliptique.) (French)

J. Inst. Math. Jussieu 7, No. 1, 181-203 (2008). ISSN 1474-7480; ISSN 1475-3030
http://dx.doi.org/10.1017/S147474800700014X
http://www.journals.cambridge.org/journal_JournaloftheInstituteofMathematicsofJussieu

A few years ago, the predicted Langlands correspondence for the group GL_n over a function field has finally been established, in full generality, by *L. Lafforgue* in his fundamental treatise [Invent. Math. 147, 1–241 (2002; Zbl 1038.11075)]. Lafforgue's approach was based on a refined analysis of the cohomology of the moduli space of

Drinfeld shtukas, including a crucial method of counting the points corresponding to GL_n-shtukas.

In the paper under review, the authors develop an alternative method of counting the number of moduli points of G-shtukas for certain reductive groups G over a function field. Their approach is inspired by *R. Kottwitz's* method of counting certain points on Shimura varieties [J. Am. Math. Soc. 5, No. 2, 373–444 (1992; Zbl 0796.14014)] and appears to be somewhat simpler than L. Lafforgue's very general construction.

More precisely, working with stacks of G-shtukas with arbitrary modifications and their compactifications [cf. *T. Ngo Dac*, Ph. D. Thesis, Université de Paris-Sud (2004)], constructing certain invariants by generalizing the notion of Kottwitz triplets for Shimura varieties to this categorical framework and establishing a counting formula for the isogeny classes with respect to a given invariant, the authors finally derive a counting formula for the moduli of regular elliptic G-shtukas, which certainly can be generalized to arbitrary cases.

In fact, the authors' formula is a faithful analogue of R. Kottwitz's counting formula for points on Shimura varieties. This reveals both its apparent general significance and its ubiquity in a wider arithmetic context.

Werner Kleinert (Berlin)

Keywords : Drinfeld modules; Drinfeld shtukas; torsors; stacks; Hecke operators; trace formula; Shimura varieties; Langlands correspondence

Classification :

∗22E55 Repres. of Lie and linear algebraic groups over global fields
11F70 Representation-theoretic methods in automorphic theory
11G09 Drinfel'd modules, etc.
11G18 Arithmetic aspects of modular and Shimura varieties
14G35 Modular and Shimura varieties

Zbl 1147.22012

Ngô, Bao Châu

Drinfeld D-shtukas with symmetric modification and base change identity. (D-chtoucas de Drinfeld à modifications symétriques et identité de changement de base.) (French. English summary)

Ann. Sci. Éc. Norm. Supér. (4) 39, No. 2, 197-243 (2006). ISSN 0012-9593
http://dx.doi.org/10.1016/j.ansens.2005.12.005
numdam:ASENS_2006_4_39_2_197_0
http://smf.emath.fr/en/Publications/AnnalesENS/
http://www.sciencedirect.com/science/journal/00129593
http://www.numdam.org/numdam-bin/browse?j=ASENSsl=2

Summary: We give a new proof of the base change fundamental lemma for GL_n for certain Hecke functions by comparing the cohomology of two different moduli spaces of D-shtukas. The original proofs, due to Clozel and Labesse, make use of the trace formula and of the unit case due to Kottwitz.

Classification :

∗22E50 Repres. of Lie and linear algebraic groups over local fields
11F70 Representation-theoretic methods in automorphic theory

Zbl 1098.14023

Ngô, Bao Châu

Hitchin fibration and endoscopy. (Fibration de Hitchin et endoscopie.) (English)

Invent. Math. 164, No. 2, 399-453 (2006). ISSN 0020-9910; ISSN 1432-1297
http://dx.doi.org/10.1007/s00222-005-0483-7
http://link.springer.de/link/service/journals/00222/

The Hitchin fibration is a tool to investigate moduli-spaces of G-bundles (G a reductive group) on a curve. Its total space is fibred in two ways: Once over the moduli space of bundles and once over the space of conjugacy classes of sections of the adjoint bundle. For sufficiently generic conjugacy classes (called "elliptic") the paper translates geometric data into the language of orbital integrals and endoscopic groups. This allows to proof certain purity statements.
Some of the proofs appear a little bit hasty but this does not affect the correctness of the results.

Gerd Faltings (Bonn)

Keywords : Hitchin fibration; orbital integrals

Classification :

∗14H60 Vector bundles on curves
14D20 Algebraic moduli problems
11F72 Spectral theory
11R39 Langlands-Weil conjectures, nonabelian class field theory
22E55 Repres. of Lie and linear algebraic groups over global fields
14F30 p-adic cohomology

Zbl 1096.14029

Ngô, Bao-Châu

Hitchin fibration and the endoscopic structure of the trace formula. (Fibration de Hitchin et structure endoscopique de la formule des traces.) (French)

Sanz-Solé, Marta (ed.) et al., Proceedings of the international congress of mathematicians (ICM), Madrid, Spain, August 22–30, 2006. Volume II: Invited lectures. Zürich: European Mathematical Society (EMS). 1213-1225 (2006). ISBN 978-3-03719-022-7/hbk

Summary: The Hitchin fibration is a well suited tool to study the geometric side of the trace formula for Lie algebra from the point of view of moduli spaces of vector bundles over a curve. The endoscopy groups appear naturally when we decompose the cohomology of the Hitchin fibration by its natural symmetries. Following this dictionary,

we can formulate a global and geometric version of Langlands–Shelstad's fundamental lemma. This conjecture has been proved in the case of unitary groups in a joint work with *G. Laumon* [Le lemme fondamental pour les groupes unitaires, preprint, `arXiv:math.AG/0404454`].

Keywords : endoscopy groups, fundamental lemma, moduli spaces of vector bundles
Classification :
∗14H60 Vector bundles on curves
11F72 Spectral theory
22E35 Analysis on p-adic Lie groups

Zbl 1047.20037

Haines, Thomas J.; Ngô, Bao Châu

Alcoves associated to special fibers of local models. (English)

Am. J. Math. 124, No. 6, 1125-1152 (2002). ISSN 0002-9327; ISSN 1080-6377
http://dx.doi.org/10.1353/ajm.2002.0037
http://muse.jhu.edu/journals/american_journal_of_mathematics/toc/ajm124.6.html
http://muse.jhu.edu/journals/american_journal_of_mathematics

Let G be a classical group over the p-adic field $\mathbb{Q}_p$, and let μ be a dominant minuscule coweight of G. The special fiber $M_{\mu,\overline{\mathbb{F}}_p}$ of the Rapoport-Zink local model M_μ has a stratification indexed by a finite subset $\mathrm{Perm}(\mu)$ of the extended affine Weyl group $\widetilde{W}(G)$ for G. Let W_0 denote the finite Weyl group of G. For each translation λ in the W_0-orbit $W_0(\mu)$ of μ, the element t_λ in $\widetilde{W}(G)$ is contained in $\mathrm{Perm}(\mu)$. Let $\mathrm{Adm}(\mu)$ denote the subset of $\mathrm{Perm}(\mu)$ indexing those strata which lie in the closure of the stratum indexed by t_λ for some $\lambda \in W_0(\mu)$.
The paper under review is concerned with the equality $\mathrm{Adm}(\mu) = \mathrm{Perm}(\mu)$. The main result of the paper is to prove that if the root system for G has type A_{n-1}, then the equality $\mathrm{Adm}(\mu) = \mathrm{Perm}(\mu)$ holds for every dominant coweight μ. The authors also show that the equality $\mathrm{Adm}(\mu) = \mathrm{Perm}(\mu)$ holds if μ is a sum of minuscule coweights for the symplectic group $G = \mathrm{GSp}_{2n}$. On the other hand, the authors show that if the root system for G is irreducible, of rank $\geqslant 4$ and not of type A_{n-1}, then $\mathrm{Adm}(\mu) \neq \mathrm{Perm}(\mu)$ for every sufficiently regular dominant coweight μ.

Shi Jian-yi (Shanghai)

Keywords : alcoves; affine Weyl groups; root systems; minuscule coweights; classical groups; bad reduction of Shimura varieties
Classification :
∗20G05 Representation theory of linear algebraic groups
20F55 Coxeter groups
14G35 Modular and Shimura varieties
11G18 Arithmetic aspects of modular and Shimura varieties
22E35 Analysis on p-adic Lie groups

Zbl 1046.14023

Ngô, Bao Chau; Genestier, Alain

Alcoves and p-rank of abelian varieties. (Alcôves et p-rang des variétés abéliennes.) (French)

Ann. Inst. Fourier 52, No. 6, 1665-1680 (2002). ISSN 0373-0956
numdam:AIF_2002__52_6_1665_0
http://aif.cedram.org/
http://annalif.ujf-grenoble.fr/aif-bin/feuilleter

The moduli space of principally polarized abelian varieties with level structure has a stratification induced by the action of the Iwahori subgroup and indexed by elements of a subset of a Weyl group. This paper shows this stratification is finer than that defined by the p-rank of the abelian variety.

Ki Hang Kim (Montgomery)

Keywords : p-rank of abelian variety; Iwahori subgroup; bad reduction; moduli space; principally polarized abelian varieties; stratification
Classification :
∗14K10 Algebraic moduli, classification (abelian varieties)

Zbl 1009.11042

Haines, T.; Ngô, Bao Châu

Nearby cycles for local models of some Shimura varieties. (English)

Compos. Math. 133, No.2, 117-150 (2002). ISSN 0010-437X; ISSN 1570-5846
http://dx.doi.org/10.1023/A:1019666710051
http://www.journals.cambridge.org/journal_CompositioMathematica

The authors solved a Kottwitz's conjecture, which is a formula for the semisimple trace for the Frobenius acting on the nearby cycles of a local model at p of a Shimura variety with prescribed level structure of Iwahori type at p. Such a computation is important for determining the local factor at p of the semisimple Hasse-Weil zeta-function of the Shimura variety. It is a surprising and interesting fact that some ideas used by the authors come from the geometric Langlands program, especially from Beilinson and Gaitsgory.

Marc Reversat (Toulouse)

Keywords : nearby cycles; Shimura varieties; Hasse-Weil zeta-function
Classification :
∗11G18 Arithmetic aspects of modular and Shimura varieties
14G35 Modular and Shimura varieties
20C08 Hecke algebras and their representations
14M15 Grassmannians, Schubert varieties

Zbl 1041.14002

Ngô, Bao Châu; Polo, P.

Affine Demazure resolutions and the geometric Casselman-Shalika formula. (Résolutions de Demazure affines et formule de Casselman-Shalika géométrique.) (French)

J. Algebr. Geom. 10, No. 3, 515-547 (2001). ISSN 1056-3911
http://www.ams.org/distribution/jag/

Summary: We prove a conjecture of *E. Frenkel*, *D. Gaitsgory*, *D. Kazhdan* and *K. Vilonen* [J. Am. Math. Soc. 11, 451–484 (1988; Zbl 1068.11501)] which is supported by a paper by *W. Casselman* and *J. Shalika* [Compos. Math. 41, 207–231 (1980; Zbl 0472.22005)] related to Fourier coefficients of spherical perverse sheaves on the affine Grassmannian associated to a split reductive group.

Keywords : characteristic p; Demazure resolution; spherical perverse sheaves; Grassmannian
Classification :
∗14F10 Special sheaves
14G15 Finite ground fields
14M15 Grassmannians, Schubert varieties

Zbl 0976.11052

Ngô, Bao Châu (Ngô, Bao Châu)

Proof of a conjecture of Frenkel-Gaitsgory-Kazhdan-Vilonen for general linear groups. (Preuve d'une conjecture de Frenkel-Gaitsgory-Kazhdan-Vilonen pour les groupes linéaires généraux.) (French)

Isr. J. Math. 120, Pt. A, 259-270 (2000). ISSN 0021-2172; ISSN 1565-8511
http://www.springerlink.com/content/0021-2172/
http://www.ma.huji.ac.il/ ijmath/

The conjecture in the title is a conjecture concerning intersection cohomology on Grassmannians [cf. *E. Frenkel, D. Gaitsgory, D. Kazhdan* and *K. Vilonen*, J. Am. Math. Soc. 11, 451–484 (1998; Zbl 1068.11501)]. It is related to the geometric interpretation of Langlands' correspondence in characteristic different from zero. The conjecture is stated in [loc. cit.] for any split reductive algebraic group and proved in the present paper for GL(n). The proof consists of a study of a resolution of the scheme of lattices.

J.G.M.Mars (Utrecht)

Keywords : intersection cohomology; Grassmannian; split reductive algebraic group
Classification :
∗11R39 Langlands-Weil conjectures, nonabelian class field theory
22E55 Repres. of Lie and linear algebraic groups over global fields

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Zbl 1047.11517

Ngô, Báo Châu (Báo Châu Ngô)
The fundamental lemma of Jacquet and Ye in positive characteristic. (Le lemme fondamental de Jacquet et Ye en caractéristique positive.) (French)
Duke Math. J. 96, No. 3, 473-520 (1999). ISSN 0012-7094
http://dx.doi.org/10.1215/S0012-7094-99-09615-1
http://www.dukemathjournal.org
http://projecteuclid.org/handle/euclid.dmj

In the trace formula approach to the lifting problems of automorphic representations of reductive groups, the matching of local orbital integrals has been reduced to that for the unit element of the Hecke algebra related to relevant groups, which is usually called the fundamental lemma. In general, the fundamental lemma is one of the most difficult open problems in the trace formula approach to the theory of automorphic representations.
The fundamental lemma of Jacquet and Ye is the one in the relative trace formula approach to the quadratic base change problem for GL(n) [*H. M. Jacquet* and *Y. Ye*, Bull. Soc. Math. Fr. 120, 263–295 (1992; Zbl 0785.11032)]. An equivalent formulation of the fundamental lemma of Jacquet and Ye is an identity of two distributions, called Jacquet and Ye's conjecture. One of the distributions is of generalized Kloosterman integral type and is defined over GL(n, E), and the other distribution is of relative Kloosterman integral type and is defined over GL(n, F), where E/F is a quadratic extension. Jacquet and Ye verified their conjecture for GL(2) and GL(3) over a local non-Archimedean field of characteristic zero.
The aim of the paper under review is to prove the above conjecture of Jacquet and Ye for GL(n) over local fields of positive characteristic. The main idea is to interpret the function-field analogies of the distributions of Kloosterman type defined by Jacquet and Ye as the trace of the Frobenius endomorphism over the l-adic cohomology of a certain algebraic variety, in the sense of Grothendieck. The conjectural identity of Jacquet and Ye in the function-field case becomes a certain quasi-invariant property of the trace with respect to a simply defined involution. It is a beautiful geometric argument.
The original conjecture of Jacquet and Ye over local fields of characteristic zero is still open for GL(n), $n \geq 4$.

Dihua Jiang (Minneapolis)
Classification :
∗11F70 Representation-theoretic methods in automorphic theory
14F20 Grothendieck cohomology and topology

Zbl 1002.11046

Ngô, Báo Châu (Báo Châu Ngô)
Perverse sheaves, base change homomorphism, and fundamental lemma of Jacquet and Ye. (Faisceaux pervers, homomorphisme de changement de base et lemme fondamental de Jacquet et Ye.) (French)
Ann. Sci. Éc. Norm. Supér. (4) 32, No. 5, 619-679 (1999). ISSN 0012-9593

http://dx.doi.org/10.1016/S0012-9593(01)80002-1
numdam:ASENS_1999_4_32_5_619_0
http://smf.emath.fr/en/Publications/AnnalesENS/
http://www.sciencedirect.com/science/journal/00129593
http://www.numdam.org/numdam-bin/browse?j=ASENSsl=2

Summary: We give a geometric interpretation of the base change homomorphism between the Hecke algebra of GL(n) for an unramified extension of local fields of positive characteristic. For this, we use some results of *V. Ginzburg* [Perverse sheaves on a loop group and Langlands duality. Preprint alg-geom, 9511007 (1995)], *I. Mirkovic* and *K. Vilonen* [Perverse sheaves on loop Grassmannians and Langlands duality. Preprint alg-geom, 9703010 (1997), see also Math. Res. Lett. 7, 13–24 (2000; Zbl 0987.14015)] related to the geometric Satake isomorphism. We give a new proof of these results in the positive characteristic case.

By using that geometric interpretation of the base change homomorphism, we prove the fundamental lemma of *H. Jacquet* and *Y. Ye* [C. R. Acad. Sci., Paris, Sér. I 311, 671–676 (1990; Zbl 0715.11026)] for an arbitrary Hecke function in the case of equal characteristic.

Keywords : base change homomorphism; Hecke algebra; Satake isomorphism; fundamental lemma

Classification :

∗11F70 Representation-theoretic methods in automorphic theory
22E50 Repres. of Lie and linear algebraic groups over local fields
14F43 Other algebro-geometric (co)homologies

Zbl 0979.11031

Ngô, Bao Châu (Ngô, Bao Châu)

The fundamental lemma of Jacquet and Ye in equal characteristics. (Le lemme fondamental de Jacquet et Ye en caractéristiques égales.) (French. Abridged English version)

C. R. Acad. Sci., Paris, Sér. I, Math. 325, No. 3, 307-312 (1997). ISSN 0764-4442
http://dx.doi.org/10.1016/S0764-4442(97)83961-4
http://www.sciencedirect.com/science/journal/07644442

Summary: We prove in the case of equal characteristics a fundamental lemma conjectured by *H. M. Jacquet* and *Y. Ye* [C. R. Acad. Sci., Paris, Sér. I, Math. 311, No. 11, 671–676 (1990; Zbl 0715.11026); Bull. Soc. Math. Fr. 120, 263–295 (1992; Zbl 0785.11032)] for GL(r). We use the étale cohomology interpretation of exponential sums, perverse sheaves and the Fourier-Deligne transformation.

For a proof for the unit element see the author, Duke Math. J. 96, 473–520 (1999; Zbl 1047.11517). See also the author's recent paper in Isr. J. Math. 120, Pt. A, 259–270 (2000; Zbl 0976.11052).

Keywords : equal characteristics; fundamental lemma; etale cohomology; exponential sums; perverse sheaves; Fourier-Deligne transformation
Classification :
∗11F70 Representation-theoretic methods in automorphic theory
22E50 Repres. of Lie and linear algebraic groups over local fields