
Zbl pre05606332

Li, Yan Yan; Nirenberg, Louis
Partial results on extending the Hopf Lemma. (English)
Rend. Mat. Appl., VII. Ser. 29, No. 1, 97-115 (2009). ISSN 1120-7183
http://www.mat.uniroma1.it/ rendicon/rendiconti.html

Classification :
∗35B50 Maximum principles (PDE)
35J60 Nonlinear elliptic equations

Zbl pre05320547

Brézis, H.; Nirenberg, L.; Stampacchia, G.
A remark on Ky Fan's minimax principle. (English)
Boll. Unione Mat. Ital. (9) 1, No. 2, 257-264 (2008). ISSN 1972-6724
http://umi.dm.unibo.it/

Classification :
∗49K35 Minimax problems (necessity and sufficiency)
49J35 Minimax problems (existence)
49J45 Optimal control problems inv. semicontinuity and convergence

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Zbl 1126.00013

Berestycki, Henri (ed.); Bertsch, Michiel (ed.); Browder, Felix E. (ed.); Nirenberg, Louis (ed.); Peletier, Lambertus A. (ed.); Véron, Laurent (ed.)
Perspectives in nonlinear partial differential equations in honor of Haïm Brezis. Based on the conference celebration of Haïm Brezis' 60th birthday, June 21–25, 2004. (English)
Contemporary Mathematics 446. Providence, RI: American Mathematical Society (AMS). xxiii, 494 p. $ 121.00 (2007). ISBN 978-0-8218-4190-7/pbk

The articles of this volume will be reviewed individually.

Classification :
∗00B30 Festschriften
00B25 Proceedings of conferences of miscellaneous specific interest
35-06 Proceedings of conferences (partial differential equations)

Zbl 1149.53302

Li, YanYan; Nirenberg, Louis
A geometric problem and the Hopf lemma. II. (English)
Chin. Ann. Math., Ser. B 27, No. 2, 193-218 (2006). ISSN 0252-9599; ISSN 1860-6261

http://dx.doi.org/10.1007/s11401-006-0037-3
http://www.springerlink.com/openurl.asp?genre=journalissn=0252-9599

Summary: A classical result of *A. D. Aleksandrov* [Vl. Vestn. Leningr. Univ. 13, No. 19, 5–8 (1958; Zbl 0101.13902) states that a connected compact smooth n-dimensional manifold without boundary, embedded in $\mathbb{R}^{n+1}$, and such that its mean curvature is constant, is a sphere. Here we study the problem of symmetry of M in a hyperplane X_{n+1} =constant in case M satisfies: for any two points (X', X_{n+1}), $(X', \hat{X}_{n+1})$ on M, with $X_{n+1} > \hat{X}_{n+1}$, the mean curvature at the first is not greater than that at the second. Symmetry need not always hold, but in this paper, we establish it under some additional conditions. Some variations of the Hopf Lemma are also presented. Several open problems are described. Part I [J. Eur. Math. Soc. (JEMS) 8, No. 2, 317–339 (2006; Zbl 1113.53003)] dealt with corresponding one dimensional problems.

Classification :

∗53A07 Higher-dimension and -codimension surfaces in Euclidean n-space
35B50 Maximum principles (PDE)
35J60 Nonlinear elliptic equations
53A05 Surfaces in Euclidean space

Zbl 1113.53003

Li, YanYan; Nirenberg, Louis

A geometric problem and the Hopf lemma. I. (English)

J. Eur. Math. Soc. (JEMS) 8, No. 2, 317-339 (2006). ISSN 1435-9855; ISSN 1435-9863
http://dx.doi.org/10.4171/JEMS/55
http://www.ems-ph.org/journals/journal.php?jrn=jems

The authors prove the following result: Let M be a closed planar C^2-embedded curve such that M stays on one side of any tangent to M parallel to the y-axis. Under the assumption that for all points $(x, y_1), (x, y_2) \in M$ (points on M on a line parallel to the y-axis) with $y_1 \leq y_2$ the inequality $\kappa|_{(x,y_2)} \leq \kappa|_{(x,y_1)}$ holds, the curve must be symmetric w.r.t. a line parallel to the x-axis. (κ denotes the curvature of M.) Some additional results concerning equality of two functions or line-symmetry of their graphs are derived. The paper also outlines some analogous but open problems for higher dimensions. Moreover, counter-examples are given in case of omitting some of the assumptions in the 2-dimensional case.

Anton Gfrerrer (Graz)

Keywords : curvature; line-symmetry of a curve; Hopf Lemma

Classification :

∗53A04 Curves in Euclidean space
53A07 Higher-dimension and -codimension surfaces in Euclidean n-space

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Zbl pre05068915

Li, YanYan; Nirenberg, Louis

Generalization of a well-known inequality. (English)

Cazenave, Thierry (ed.) et al., Contributions to nonlinear analysis. A tribute to D. G. de Figueiredo on the occasion of his 70th birthday. Basel: Birkhäuser. Progress in Nonlinear Differential Equations and their Applications 66, 365-370 (2006). ISBN 3-7643-7149-8/hbk

Classification :

∗35-99 Partial differential equations (PDE)

26D99 Inequalities involving real functions

Zbl 1113.49041

Ekeland, Ivar; Nirenberg, Louis

Regularity in an unusual variational problem. (English)

J. Math. Fluid Mech. 7, Suppl. 3, S332-S348 (2005). ISSN 1422-6928; ISSN 1422-6952

http://dx.doi.org/10.1007/s00021-005-0163-9

http://link.springer.de/link/service/journals/00021/toc.htm

Given a symmetric positive definite $(2^n-1)\times(2^n-1)$ matrix Q the authors study an optimization problem of the following form: Find a function $u:\mathbb{R}^n_+\to\mathbb{R}$ which minimizes the functional

$$J(u)=\int\int\cdots\int_{R^n_+}[(\partial_{x_1}\partial_{x_2}\dots\partial_{x_n}u)^2+(Q\overline{D}^n_{n-1}u,\overline{D}^n_{n-1}u)]\,dx_1\,dx_2\dots dx_n$$

subject to $u(0)=1$ where $\overline{D}^n_{n-1}u=\{D^\alpha u|\alpha\in A_n,\ \alpha\notin(1,\dots,1)\}$ and $A_n=\{\alpha=(\alpha_1,\dots,\alpha_n)\mid$ with $\alpha_i=0$ or 1 for all $i\}$. Let $H_n(u)=\sum_{\alpha\in A}\|D^\alpha u\|^2_{L^2}$ be a quadratic form and E_n be the Hilbert space consisted of functions u with $H_n(u)<\infty$. The problem then is to find

$$\inf J(u),\quad u\in E_n,\ u(0)=1. \tag{1}$$

The existence of a unique solution of (1) was proved a the previous paper by the authors. In this paper the authors prove that the solution of (1) is C^∞ up to and including the boundary.

Yana Belopolskaya (St. Petersburg)

Keywords : Regularity; variational problems; Malliavin calculus; mathematical finance

Classification :

∗49N60 Regularity of solutions in the calculus of variations

35B65 Smoothness of solutions of PDE

49K20 Optimal control problems with PDE (nec./ suff.)

91B28 Finance etc.

60H07 Stochastic calculus of variations and the Malliavin calculus

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Zbl 1093.01541

Nirenberg, L.

Memories of Guido Stampacchia. (English)

Giannessi, Franco (ed.) et al., Variational analysis and applications. Proceedings of the 38th conference of the School of Mathematics "G. Stampacchia" in memory of G. Stampacchia and J.-L. Lions, Erice, Italy, June 20–July 1, 2003. New York, NY: Springer. Nonconvex Optimization and its Applications 79, 79-80 (2005). ISBN 0-387-24209-0/hbk; ISBN 0-387-24276-7/ebook

Keywords : Memory

Classification :

∗01A70 Biographies, obituaries, personalia, bibliographies

Zbl 1065.01014

Nirenberg, Louis

Some recollections of working with François Trèves. (English)

Chanillo, Sagun (ed.) et al., Geometric analysis of PDE and several complex variables. Dedicated to François Trèves. Providence, RI: American Mathematical Society (AMS). Contemporary Mathematics 368, 371-373 (2005). ISBN 0-8218-3386-3/pbk

Summary: These recollections concern L. Nirenberg's and F. Trèves' work on local solvability of linear PDEs [Commun. Pure Appl. Math. 16, 331–351 (1963; Zbl 0117.06104), ibid 23, 1–38 (1970; Zbl 0191.39103), ibid. 23, 459–509 (1970; Zbl 0208.35902)].

Classification :

∗01A70 Biographies, obituaries, personalia, bibliographies

35-03 Historical (partial differential equations)

Zbl 1062.49021

Li, YanYan; Nirenberg, Louis

The distance function to the boundary, Finsler geometry, and the singular set of viscosity solutions of some Hamilton-Jacobi equations. (English)

Commun. Pure Appl. Math. 58, No. 1, 85-146 (2005). ISSN 0010-3640
http://dx.doi.org/10.1002/cpa.20051
http://onlinelibrary.wiley.com/journal/10.1002/%28ISSN%291097-0312

Authors consider the following boundary value problem: $\{H(x,\nabla u)=1,\ x\in\Omega;\ u|_{\partial\Omega}=0\}$, where $H(x,p)\in C^\infty(\overline{\Omega}\times\mathbb{R}^n)$. The viscosity solution is the function $u(x)\equiv\inf_{y\in\Omega}L(x,y)$, $x\in\overline{\Omega}$, where $L(x,y)$ is a distance function defined with respect to a suitable Finsler metric. One has $u>0$ in Ω and $u\in W^{1,\infty}$. The set Σ of singular points of the distance function to the boundary is related to the singular points set of the viscosity solution. Authors' aim is to generalize previous results where Ω is an open set in $\mathbb{R}^n$, to a domain of an n-dimensional smooth manifold with complete smooth

Finsler metric and prove that the Hausdorff measure $H^{n-1}(\Sigma)$ of the singular set Σ is finite.
Remark: Viscosity solutions were introduced in the theory of PDE's as generalized solutions for boundary value problems where smooth solutions are absent. Let us emphasize that weak solutions can be directly recognized in the geometric theory of PDE's. For example, the following Hamilton-Jacobi equation: $(HJ) \subset J\mathcal{D}^1(W)$: $ax^2 + bu_x^2 = c$, where $\pi : W \equiv \mathbb{R}^2 \to \mathbb{R}$, $(x, u) \mapsto (x)$, besides the boundary condition: $u|_{\partial\Omega} = 0$, where $\Omega \equiv [-\sqrt{\frac{c}{a}}, \sqrt{\frac{c}{a}}] \subset \mathbb{R}$, has no smooth solutions, but we recognize solutions in the class of weak solutions. These are peaces of helicoidal-lines passing from the boundary $\partial\Omega \equiv \{-\sqrt{\frac{c}{a}}, \sqrt{\frac{c}{a}}\} \subset \mathbb{R}$, that are integral lines of the Cartan distribution of (HE), that is 1-dimensional. For example, if we denote the two points of the boundary $\partial\Omega \equiv \{A, B\}$, a weak solution is one having two branches: the first starting from A and arriving in the first point A'' that projects on A by means of the projection $\pi_1 : J\mathcal{D}^1(W) \to \mathbb{R}$. The second branche starts from B and arrives on the first point A' that projects on A via π_1. Therefore, the integral line $V \equiv (AA'') \cup (A'B)$ can be considered as a weak solution with boundary $\partial V = A \cup B$. Note that the *boundary of the weak solution* V is, par definition, the topological boundary minus the set of singular points with discontinuity: $\partial V = A \cup A'' \cup A' \cup B \setminus \Sigma_S(V) = A \cup B$. Here $\Sigma_S(V) \equiv A'' \cup A'$.

Agostino Prástaro (Roma)
Keywords : Hamilto-Jacobi equations; viscosity solutions
Classification :
∗49L25 Viscosity solutions
35D05 Existence of generalized solutions of PDE

Zbl pre05044219

Nirenberg, L.

The distance function to the boundary and singular set of viscosity solutions of Hamilton-Jacobi equation. (English)
Giannessi, Franco (ed.) et al., Variational analysis and applications. Proceedings of the 38th conference of the School of Mathematics "G. Stampacchia" in memory of G. Stampacchia and J.-L. Lions, Erice, Italy, June 20–July 1, 2003. New York, NY: Springer. Nonconvex Optimization and its Applications 79, 765-772 (2005). ISBN 0-387-24209-0/hbk; ISBN 0-387-24276-7/ebook

Classification :
∗35J30 Higher order elliptic equations, general
49J25 Optimal control problems with equations with ret. arguments (exist.)

Zbl 1169.01321

Nirenberg, Louis

About Olga Arsen'evna Oleinik. (English)
J. Math. Sci., New York 120, No. 3, 1241-1241 (2004). ISSN 1072-3374; ISSN 1573-8795
http://dx.doi.org/10.1023/B:JOTH.0000016045.00993.ef
http://www.springerlink.com/content/106477/

Classification :
∗01A70 Biographies, obituaries, personalia, bibliographies

Zbl 1168.01327

Friedlander, Susan; Lax, Peter; Morawetz, Cathleen; Nirenberg, Louis; Seregin, Gregory; Ural'tseva, Nina; Vishik, Mark

Olga Alexandrovna Ladyzhenskaya (1922–2004). (English)

Notices Am. Math. Soc. 51, No. 11, 1320-1330 (2004). ISSN 0002-9920; ISSN 1088-9477

http://www.ams.org/notices/200411/200411-toc.html
http://www.ams.org/notices

Summary: The authors recall the life and mathematical legacy of the influential Russian mathematician.

Keywords : Obituary
Classification :
∗01A70 Biographies, obituaries, personalia, bibliographies

Zbl 1159.01337

Kohn, Joseph J.; Griffiths, Phillip A.; Goldschmidt, Hubert; Bombieri, Enrico; Cenkl, Bohous; Garabedian, Paul; Nirenberg, Louis

Donald C. Spencer (1912–2001). (English)

Notices Am. Math. Soc. 51, No. 1, 17-29 (2004). ISSN 0002-9920; ISSN 1088-9477

http://www.ams.org/notices/200401/200401-toc.html
http://www.ams.org/notices

Summary: The authors recall the mathematical legacy and life of the influential American mathematician Donald C. Spencer.

Keywords : Obituary
Classification :
∗01A70 Biographies, obituaries, personalia, bibliographies

Zbl 1125.35340

Nirenberg, L.

Comment on: "Estimates for elliptic systems for composite material". (English)

Chuong, N. M. (ed.) et al., Abstract and applied analysis. Proceedings of the international conference, Hanoi, Vietnam, August 13–17, 2002. River Edge, NJ: World Scientific. 249-255 (2004). ISBN 981-238-944-X/hbk

This short paper contains the notes from a lecture given by Louis Nirenberg concerning his paper with YanYan Li [Commun. Pure Appl. Math. 56, No. 7, 892–925 (2003; Zbl 1125.35339)].

Classification :
- ∗35J55 Systems of elliptic equations, boundary value problems
- 35B45 A priori estimates
- 35D10 Regularity of generalized solutions of PDE
- 74G99 Equilibrium (steady-state) problems
- 74E30 Composite and mixture properties

Zbl 1058.00011

Chuong, N. M. (ed.); Nirenberg, L. (ed.); Tutschke, W. (ed.)

Abstract and applied analysis. Proceedings of the international conference, Hanoi, Vietnam, August 13–17, 2002. (English)

River Edge, NJ: World Scientific. x, 567 p. £ 72.00 (2004). ISBN 981-238-944-X/hbk

The articles of this volume will be reviewed individually.

Classification :
- ∗00B25 Proceedings of conferences of miscellaneous specific interest

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Zbl 1125.35339

Li, YanYan; Nirenberg, Louis

Estimates for elliptic systems from composite material. (English)

Commun. Pure Appl. Math. 56, No. 7, 892-925 (2003). ISSN 0010-3640
http://dx.doi.org/10.1002/cpa.10079
http://onlinelibrary.wiley.com/journal/10.1002/%28ISSN%291097-0312

Suppose that D is a bounded domain in $\mathbb{R}^n$ that contains subdomains D_m, $m=1,\dots,L$, with $D=\bigcup D_m$. Let $\sum_{\alpha,\beta=1}^n\sum_{j=1}^N\partial_\alpha A_{i,j}^{\alpha,\beta}\partial_\beta u_j=b_i$ with $i=1,\dots,N$ be a (weakly) elliptic system of equations where $A_{i,j}^{\alpha,\beta}$ are Hölder continuous in D_m but not necessarily continuous on ∂D_m. Such problems appear naturally in elasticity theory of composite material. The version of the elliptic condition used here indeed does allow for these systems. The estimates that Li and Nirenberg seek to establish answer the following question: does an interior Hölder type bound exist for ∇u in terms of u and b? Assuming that the boundaries ∂D_m are $C^{1,\gamma}$ their main result gives the affirmative answer.
Set $D_\varepsilon=\{x\in D;\operatorname{dist}(x,\partial D)>\varepsilon\}$. Then there exist C and $\gamma^*>0$ such that for all $\gamma'\in(0,\gamma^*)$ and $b_i:=h_i+\sum_{\beta=1}^n\partial_\beta g_i^\beta$ any weak solution u satisfies

$$\sum_{m=1}^L\|u\|_{C^{1,\gamma'}(\overline{D}_m\cap D_\varepsilon)}\le C\left(\|u\|_{L^2(D)}+\|h\|_{L^\infty(D)}+\sum_{m=1}^L\|g\|_{C^{\gamma'}(\overline{D}_m)}\right).$$

The constant C that is obtained does not depend on the distance between subdomains D_m and hence allows even some “touching” of subdomains. Although related results are available in the literature, the present combination of “system” and “composite material" is new and makes the long and hard analysis in this paper necessary. The authors mention a preceding result for the scalar equation which is due to Li and Vogelius. The present paper also pays tribute to results of Chipot, Kinderlehrer, and Vergara-Caffarelli, and of Avellaneda and Lin.

Guido Sweers (MR1990481)

Classification :

∗35J55 Systems of elliptic equations, boundary value problems

35D10 Regularity of generalized solutions of PDE

Zbl 1082.58501

Ekeland, Ivar; Nirenberg, Louis

A convex Darboux theorem. (English)

Methods Appl. Anal. 9, No. 3, 329-344 (2002). ISSN 1073-2772

http://www.intlpress.com/MAA/

http://projecteuclid.org/maa

Summary: We give necessary and sufficient conditions for a smooth, generic, differential one-form ω on $\mathbb{R}^n$ to decompose into a sum $\omega = a^1 du_1 + \cdots + a^k du_k$, where the functions a^ℓ are positive and the u_ℓ convex (or quasi-convex) near the origin.

Classification :

∗58A15 Exterior differential systems (Cartan theory)

91B16 Utility theory

Zbl 0992.47023

Nirenberg, Louis (Artino, Ralph A.)

Topics in nonlinear functional analysis. Notes by Ralph A. Artino. Revised reprint of the 1974 original. (English)

Courant Lecture Notes in Mathematics. 6. Providence, RI: American Mathematical Society (AMS). New York, NY: Courant Institute of Mathematical Sciences, xii, 145 p. $ 24.00 (2001). ISBN 0-8218-2819-3/pbk

This is the second edition of the well-known and remarkable book on Nonlinear Analysis. The text is presented unchanged from the first edition except the proof of Proposition 1.7.2. Although the first edition of the book was printed in 1974 one can say that it continues to be actual also this time. Here we recall the contents of the book:
Chapter 1 (Topological Approach: Finite Dimensions) in which the Brouwer-Hopf degree theory is presented in detail on the base of Sard's lemma; in the chapter one can find also some information about homotopic theory of continuous mappings between finite-dimensional spaces of different dimensions. Chapter 2 (Topological Degree in Banach Spaces) deals with Leray-Schauder degree theory with some elements of Calculus in Banach Spaces. Chapter 3 (Bifurcation Theory) is devoted to Morse lemma, Krasnosel'ski and Rabinowitz' bifurcation theorems and some their modifications. Chapter 4 (Further Topological Methods) is devoted to some generalizations of Leray-Schauder theory and

the theory of framed cobordisms; here the known lectures by J. Ize about application of cohomotopy groups in Nonlinear Analysis are presented. Chapter 5 (Monotone Operators and the Min-Max Theorem) presents also a lecture by N. Bitzenhofer in which it was proved that a monotone set-valued operator in Banach space is in fact single-valued at most point. The last Chapter 6 (Generalized Implicit Functions Theorem) deals with an elegant account to the theorem of Kolmogorov-Arnold-Mozer.
The book is useful for all specialists in Nonlinear Analysis, first for young mathematicians that, due to this book, can become acquainted with a series of fundamental and brilliant ideas and methods of Nonlinear Analysis.

Peter Zabreiko (Minsk)

Keywords : topological degree in Banach space; calculus in Banach spaces; Brouwer-Hopf degree; Sard's lemma; homotopic theory of continuous mappings; Leray-Schauder degree theory; Morse lemma; Krasnosel'ski and Rabinowitz' bifurcation theorems; framed cobordisms; cohomotopy groups; monotone set-valued operator
Classification :
∗47H05 Monotone operators (with respect to duality)
47H11 Degree theory
47-02 Research monographs (operator theory)
46G05 Derivatives, etc. (functional analysis)
46J15 Banach algebras of differentiable functions

Zbl 1009.58004

Li, Yanyan; Nirenberg, Louis

A variational result in a domain with boundary. (English)

Methods Appl. Anal. 7, No.3, 489-494 (2000). ISSN 1073-2772
http://www.intlpress.com/MAA/
http://projecteuclid.org/maa

The authors prove: Let F be a real C^2 function in the closure $\overline{\Omega}$ of a smooth bounded domain in $\mathbb{R}^n$. Assume that

$$\varphi := F|_{\partial\Omega} : \partial\Omega \to \mathbb{R}$$

has only two critical values, max and min. Denote by m the set where φ takes its minimum. Assume: (i) m is contractible to a point in $\overline{\Omega}$; (ii) in some α-neighborhood on $\partial\Omega$ of m, m is not contractible to a point. Then F has a critical point in Ω.
The result is extended to a domain Ω in Hilbert space, assuming uniform continuity in some β-neighborhood of $\partial\Omega$ of the Fréchet derivative F'.

Dian K.Palagachev (Bari)

Keywords : critical point of function; variational methods
Classification :
∗58E05 Abstract critical point theory
35A15 Variational methods (PDE)

Zbl 0980.58005

Nirenberg, L.

Variational methods in nonlinear problems. (English)

Bitar, Khalil (ed.) et al., Proceedings of the international conference on the mathematical sciences after the year 2000, Beirut, Lebanon, January 11-15, 1999. Singapore: World Scientific. 116-122 (2000). ISBN 981-02-4223-9/hbk

This excellent expository survey presents some of the main min-max methods developed in the last few decades. The results recalled in the paper are very useful because many nonlinear problems for partial differential equations arise as Euler equations for some appropriate problems in the Calculus of Variations. There are presented several celebrated results, such as Ekeland's Variational Principle, the Mountain Pass Lemma of Ambrosetti and Rabinowitz, the Linking theorem, and it is also discussed the role of the Palais-Smale compactness condition for finding critical points of energy functionals. The author concludes with an attempt to use the Mountain Pass lemma to solve the Jacobian conjecture, a long-standing problem in algebra.

Vicenţiu D.Rădulescu (Craiova)

Keywords : variational principle; critical point theory; Palais-Smale condition; minimization problem

Classification :

∗58E05 Abstract critical point theory
34C25 Periodic solutions of ODE
37J45 Periodic, homoclinic and heteroclinic orbits, etc.
47J30 Variational methods

Zbl 0977.01034

Mather, John N.; McKean, Henry P.; Nirenberg, Louis; Rabinowitz, Paul H.

Jürgen K. Moser (1928–1999). (English)

Notices Am. Math. Soc. 47, No.11, 1392-1405 (2000). ISSN 0002-9920; ISSN 1088-9477
http://www.ams.org/notices

Keywords : Obituary

Classification :

∗01A70 Biographies, obituaries, personalia, bibliographies

Zbl 0956.46024

Brézis, Haïm; Li, Yanyan; Mironescu, Petru; Nirenberg, Louis

Degree and Sobolev spaces. (English)

Topol. Methods Nonlinear Anal. 13, No.2, 181-190 (1999). ISSN 1230-3429
http://www-users.mat.uni.torun.pl/ tmna/

Authors' abstract: Let u belong (for example) to $W^{1,n+1}(S^n \times \Lambda, S^n)_{\lambda\in\Lambda}$ where Λ is a connected open set in $\mathbb{R}^k$. For a.e. the map $x \mapsto u(x,\lambda)$ is continuous from S^n into S^n and therefore its (Brouwer) degree is well defined. We prove that this degree is independent of λ a.e. in Λ. This result is extended to a more general setting, as well to fractional Sobolev spaces $W^{s,p}$ with $sp \geq n+1$.

Josef Wloka (Kiel)

Keywords : Brouwer degree; fractional Sobolev spaces

Classification :

∗46E35 Sobolev spaces and generalizations

47H11 Degree theory

Zbl 0944.35025

Nirenberg, Louis

Estimates for elliptic equations in unbounded domains and applications to symmetry and monotonicity. (English)

Christ, Michael (ed.) et al., Harmonic analysis and partial differential equations. Essays in honor of Alberto P. Calderón's 75th birthday. Proceedings of a conference, University of Chicago, IL, USA, February 1996. Chicago, IL: The University of Chicago Press. Chicago Lectures in Mathematics. 263-274 (1999). ISBN 0-226-10456-7/hbk

This is a brilliant essay on qualitative properties (such as symmetry and monotonicity) of solutions of nonlinear elliptic equations, discussed on the model problem

$$\Delta u + f(u) = 0 \quad \text{in } \Omega,\ u > 0, \qquad u = 0 \quad \text{on } \partial\Omega$$

with a domain $\Omega \subset \mathbb{R}^n$ and Lipschitz continuous f.

Dian K.Palagachev (Bari)

Keywords : nonlinear elliptic equations; maximum principles; symmetry; monotonicity

Classification :

∗35J65 (Nonlinear) BVP for (non)linear elliptic equations

35B05 General behavior of solutions of PDE

35B50 Maximum principles (PDE)

35-06 Proceedings of conferences (partial differential equations)

Zbl 0933.35083

Li, Yanyan; Nirenberg, Louis

The Dirichlet problem for singularly perturbed elliptic equations. (English)

Commun. Pure Appl. Math. 51, No.11-12, 1445-1490 (1998). ISSN 0010-3640

http://dx.doi.org/10.1002/(SICI)1097-0312(199811/12)51:11/12¡1445::AID-CPA9¿3.0.CO;2-Z

http://onlinelibrary.wiley.com/journal/10.1002/%28ISSN%291097-0312

This remarkable paper is devoted to the Dirichlet problem for a singularly perturbed elliptic equation

$$-\varepsilon^2\Delta\widetilde{u}+\widetilde{u}=\widetilde{u}^q,\ \widetilde{u}>0,$$

in a bounded domain $\Omega\subset\mathbb{R}^n$, $\widetilde{u}|_{\partial\Omega}=0$, where $1<q<\infty$ if $n\in\{1,2\}$ and $1<q<(n+2)/(n-2)$ if $n\geq 3$, $\varepsilon>0$ is a small real parameter. The authors present two main results concerning the existence of a family of solutions $\widetilde{u}_\varepsilon$ of the problem under consideration. The first result is the following. Given the inequality $\max_{Q\in\partial V} d(Q,\partial\Omega)<\max_{Q\in\partial\overline{V}} d(Q,\partial\Omega)$, where $d(Q,\partial\Omega)\equiv\text{dist}(Q,\partial\Omega)$, V is an open set and $\overline{V}\subset\Omega$. Then there exists $\overline{\varepsilon}>0$ and $\widetilde{u}_\varepsilon$ for $0<\varepsilon<\overline{\varepsilon}$ such that $\widetilde{u}_\varepsilon$ has a unique local maximum point $\widetilde{Q}_\varepsilon\in V$, $d(\widetilde{Q}_\varepsilon,\partial\Omega)\to\max_{Q\in\partial\overline{V}} d(Q,\partial\Omega)$ as $\varepsilon\to 0$ and $\widetilde{Q}_\varepsilon$ is the unique critical point of $\widetilde{u}_\varepsilon$ provided that $n\in\{1,2\}$ or Ω is convex. The second result consists in the following statement. If V is open in Ω, $\overline{V}\subset\Omega$, $\partial V\subset\mathcal{O}$ ($\mathcal{O}\subset\Omega$) and the Brouwer degree $\deg(\nabla d(Q,\partial\Omega),V,0)\neq 0$, then there exists $\overline{\varepsilon}>0$ and $\widetilde{u}_\varepsilon$ for $0<\varepsilon<\overline{\varepsilon}$ such that $\widetilde{u}_\varepsilon$ has a unique local maximum point $\widetilde{Q}_\varepsilon\in V$, $d(\widetilde{Q}_\varepsilon,S)\to 0$ ($S=\Omega\setminus\mathcal{O}$) as $\varepsilon\to 0$ and also $\widetilde{Q}_\varepsilon$ is the unique critical point of $\widetilde{u}_\varepsilon$ provided that $n\in\{1,2\}$ or Ω is convex.

Dimitar Kolev (Sofia)

Keywords : Dirichlet problem; singularly perturbed elliptic equations; Brouwer degree; local maximum point; unique critical point

Classification :

∗35J70 Elliptic equations of degenerate type
35B25 Singular perturbations (PDE)
35B38 Critical points

Zbl 0928.00066

Nirenberg, Louis (ed.)

Special issue dedicated to the memory of Fritz John. Part 2. (English)

Commun. Pure Appl. Math. 51, No.11-12, 1247-1492 (1998). ISSN 0010-3640

http://dx.doi.org/10.1002/(SICI)1097-0312(199811/12)51:11/12¡1247::AID-CPA1¿3.0.CO;2-M

http://onlinelibrary.wiley.com/journal/10.1002/%28ISSN%291097-0312

The articles of this volume will be reviewed individually. For Part 1 see the following entry (Zbl 0928.00070).

Keywords : Memorial; Special issue; Dedication

Classification :

∗00B30 Festschriften
35-06 Proceedings of conferences (partial differential equations)
76-06 Proceedings of conferences (fluid mechanics)

Zbl 0928.00067

Nirenberg, Louis (ed.)

Special issue dedicated to the memory of Fritz John. Part 1. (English)

Commun. Pure Appl. Math. 51, No.9-10, 967-1246 (1998). ISSN 0010-3640
http://dx.doi.org/10.1002/(SICI)1097-0312(199809/10)51:9/10lt;969::AID-CPA1gt;3.0.CO;2-W
http://onlinelibrary.wiley.com/journal/10.1002/%28ISSN%291097-0312

The articles of this volume will be reviewed individually. For Part 2 see the preceding entry (Zbl 0928.00069).

Keywords : Memorial; Special issue; Dedication
Classification :
∗00B30 Festschriften
35-06 Proceedings of conferences (partial differential equations)
76-06 Proceedings of conferences (fluid mechanics)

Zbl 1079.35513

Berestycki, Henri; Caffarelli, Luis; Nirenberg, Louis

Further qualitative properties for elliptic equations in unbounded domains. (English)

Ann. Sc. Norm. Super. Pisa, Cl. Sci., IV. Ser. 25, No. 1-2, 69-94 (1997).
numdam:ASNSP_1997_4_25_1-2_69_0
http://www.sns.it/html/ClasseScienze/pubsci/

Summary: This article is one in a series by the authors [Commun. Pure Appl. Math. 50, 1089–1112 (1997; Zbl 0906.35035), Duke Math. J. 81, 467–494 (1996; Zbl 0860.35004)] to study some qualitative properties of positive solutions of elliptic second order boundary value problems of the type

$$\Delta u + f(u) = 0 \text{ in } \Omega, \quad u > 0 \text{ in } Q, \qquad u = 0 \text{ on } \partial\Omega \tag{1}$$

in various kinds of unbounded domains Ω of $\mathbb{R}^n$. Typically, we are interested in features like monotonicity in some directions and symmetry. In some cases, the positive solutions we consider are supposed to be bounded while in other cases boundedness is not assumed. The function f appearing in (1.1) will always be assumed to be (globally) Lipschitz continuous: $\mathbb{R}^+ \to \mathbb{R}$.
The present paper is devoted to the investigation of three main configurations. We consider a half space $\Omega = \{x = (x_1, \dots, x_n),\ x_n > 0\}$, infinite cylindrical or slab-like domains $\Omega = \mathbb{R}^{n-1} \times (0, h)$ and also the case when Ω is the whole plane. In the case of the half space, we derive some monotonicity and symmetry results establishing that a bounded solution of (1) actually only depends on one variable. This is related to a conjecture of De Giorgi on the classification of solutions to some problems of the type (1) in the whole space.

Classification :
∗35J65 (Nonlinear) BVP for (non)linear elliptic equations

35B05 General behavior of solutions of PDE

Zbl 0943.47049

Brézis, Haïm; Nirenberg, Louis

A Lyapunov-Schmidt procedure involving a nonlinear projection. (English)

Cordaro, Paulo D. (ed.) et al., Multidimensional complex analysis and partial differential equations. A collection of papers in honor of François Treves. Proceedings of the Brazil-USA conference, June 12-16, 1995, São Carlos, Brazil. Providence, RI: American Mathematical Society. Contemp. Math. 205, 25-32 (1997). ISBN 0-8218-0509-6/pbk

Let F be a smooth map from a neighbourhood U of the origin in a Banach space X into another Y, and consider the equation (1) $F(x) = 0$. The authors are concerned with finding a local family of solutions of (1). They assume that $X_2 = \ker F''(0)$ and $Y_1 = \text{Range } F''(0)$ (which is supposed to be closed) have closed complementing spaces X_1 and Y_2 in X and Y, respectively. Assuming moreover that the codimension of Y_1 is finite and $\text{dist}(F(x), Y_1) \leq \theta\|F(x)\|$ for some $\theta < 1$ and all $x \in U$ then there exists a unique smooth map u from a ball B in X_2 into X_1 such that $u(0) = 0$ and $F(x_2 + u(x_2)) = 0$ for $x_2 \in B$. The main tool in the proof is a convenient form of the implicit function theorem.
With the aid of this result the authors prove a generalization of a *V. I. Yudovich* theorem from [Mat. Zametki 49, No. 5, 142-148 (1991; Zbl 0747.47010); Engl. translation in Math. Notes 49, No. 5, 540-545 (1991)], relaxing the original assumptions on the codimension of Y_1 and properties of a cosymmetry map ensuring the existence of a family of solutions of (1).

M.Sablik (Katowice)

Keywords : bifurcation theory; nonlinear equation; Banach spaces; cosymmetry map; implicit function theorem; Yudovich theorem

Classification :

∗47J25 Methods for solving nonlinear operator equations (general)
47J15 Abstract bifurcation theory
58C15 Implicit function theorems etc. on manifolds

Zbl 0906.35035

Berestycki, H.; Caffarelli, L.A.; Nirenberg, L.

Monotonicity for elliptic equations in unbounded Lipschitz domains. (English)

Commun. Pure Appl. Math. 50, No.11, 1089-1111 (1997). ISSN 0010-3640
http://dx.doi.org/10.1002/(SICI)1097-0312(199711)50:11¡1089::AID-CPA2¿3.0.CO;2-6
http://onlinelibrary.wiley.com/journal/10.1002/%28ISSN%291097-0312

The authors investigate monotonicity properties for positive classical solutions u of the boundary value problem

$$(1) \qquad \Delta u + f(u) = 0 \quad \text{in } \Omega, \quad u = 0 \quad \text{on } \Gamma := \partial\Omega$$

where Ω is an unbounded set defined as $\Omega := \{x \in \mathbb{R}^n |\ x_n > \varphi(x_1, \dots, x_{n-1})\}$, with a globally Lipschitz continuous function $\varphi : \mathbb{R}^{n-1} \to \mathbb{R}$. Moreover u is assumed to satisfy the condition

$$(2) \qquad 0 < u < \sup u = M < \infty \quad \text{in } \Omega.$$

The principal results of the paper are as follows: Theorem 1.1. Under the following conditions:
(a) f is Lipschitz continuous on $\mathbb{R}^+$ and satisfies $f(s) > 0$ on $(0, \mu)$ and $f(s) \le 0$ for $s \ge \mu$ for some $\mu > 0$; (b) for some $0 < s_0 < s_1 < \mu$, $f(s) > \delta_0 s$ on $[0, s_0]$ for some $\delta_0 > 0$; (c) $f(s)$ is nonincreasing on (s_1, μ),
u is monotonic with respect to x_n, i.e. $\partial u/\partial x_n > 0$ in Ω.
Theorem 1.2. Under the assumptions of Theorem 1.1, the solution u of (1) has in addition the following properties:
(a) $u < \mu$ in Ω; (b) as $\text{dist}(x, \Gamma) \to \infty$, $u(x) \to \mu$ uniformly in Ω; (c) $u(x) \ge C[\text{dist}(x, \Gamma)]^\rho$ if $x_n - \varphi(x_1, \dots, x_{n-1}) < h_1$ for some positive constants C, ρ_1 and h_1; (d) u is the unique solution satisfying (1) and (2); (e) $\partial u/\partial x_n + \sum_{\alpha=1}^{n-1} a_\alpha \partial u/\partial x_\alpha > 0$ in Ω if $\sum a_\alpha^2 < \kappa^{-2}$, where κ is the Lipschitz constant of f.
The proofs of these results are established by use of the sliding method.

G.Philippin (Quebec)
Keywords : uniqueness; positive classical solutions; sliding method
Classification :
∗35J65 (Nonlinear) BVP for (non)linear elliptic equations
35B40 Asymptotic behavior of solutions of PDE
35B05 General behavior of solutions of PDE

Zbl 0905.35027

Brézis, Haïm; Nirenberg, Louis

Removable singularities for nonlinear elliptic equations. (English)

Topol. Methods Nonlinear Anal. 9, No.2, 201-219 (1997). ISSN 1230-3429
http://www-users.mat.uni.torun.pl/ tmna/

The authors study very general types of nonlinear elliptic equations containing gradient terms. They consider solutions which are defined in $D \setminus K$, where D is a bounded domain in $\mathbb{R}^N$ and K is a set of zero capacity. They show that if the solution is smooth it can be continued as a smooth solution in the whole domain D. The proof is based on the appropriate choice of test functions and very clever use of classical Sobolev inequalities and maximum principle. They also show by means of counter examples that the assumptions are in a certain sense optimal.

Catherine Bandle (Basel)
Keywords : Sobolev inequalities; maximum principle
Classification :
∗35J60 Nonlinear elliptic equations
35B60 Continuation of solutions of PDE

Zbl 0896.35001

Nirenberg, Louis

Lipman Bers and partial differential equations. (English)

Dodziuk, Józef (ed.) et al., Lipa's legacy. Proceedings of the 1st Bers colloquium, New York, NY, October 19–20, 1995. Providence, RI: American Mathematical Society. Contemp. Math. 211, 455-461 (1997). ISBN 0-8218-0671-8/pbk

This article offers a valuable overview of Lipman Bers' contributions in partial differential equations. These contributions arose initially in connection with problems of fluid dynamics, and received impetus from the exigencies of the military inspired curriculum at the Brown University Program in Applied Mathematics, where Bers enjoyed his first US position during the 1940's. Bers recognized the deep mathematical substance in the fluid dynamical problems and during the period 1942-1957 developed, originally in collaboration with A. Gelbart, the theory of pseudoanalytic functions. Essentially, the same theory was initiated and developed independently during those years by I. Vekua in the Soviet Union.

The present description provides an elaboration of that portion of the earlier article by *W. Abikoff* [Notices Am. Math. Soc. 1995, p. 8] which pertains to the indicated work. It is written by a world specialist who was active in similar directions and who himself worked jointly with Bers; it extracts nicely the essence of some of the major contributions, and it also indicates directions in which the theory has been further developed more recently by others.

The reviewer has had personal contact with some of the material covered in this article and in the earlier one of Abikoff and he was struck by some inaccurate descriptions and by the omission (in both articles) of some references. The circumstances of the omissions may reflect a more multifaceted personality of Bers than would be discerned from the descriptions in the present article and (to a larger extent) in those by Abikoff and by others that appeared in adjoining articles in the notices.

Bers was indeed the first to prove Theorem 1.3 on removability of isolated singularities for the minimal surface equation, however a very much stronger result for a broad class of equations and with a much simpler proof was obtained independently by the reviewer. That result became the reviewer's doctoral dissertation in 1951; it was published in Trans. Am. Math. Soc. 75, 385-404 (1953; Zbl 0053.39205) following unfortunate delays and changes that led to a misleading view of some of the history. The reviewer discovered the result early in 1950 while a student at Syracuse University. His advisor Abe Gelbart was then out of town, so he showed his result to Bers. Bers informed him of his own theorem for minimal surfaces, praised him for his new achievement, and told him he would see to it that the new result received recognition in the mathematical community. It was many years later when the reviewer learned that what Bers actually did was to tell his colleagues that things weren't working out with the advisor, that the student had come to him and that he (Bers) had helped him to obtain the new theorem.

The reviewer was not present when Bers delivered his invited lecture "Singularities of minimal surfaces" over half a year later to the International Congress in Cambridge, MA. He learned however from Gelbart who was there that the new theorem was not mentioned in that lecture; when Gelbert subsequently asked Bers why he ignored the

result, Bers responded that he hadn't had time to verify the proof. The theorem is also not mentioned in Bers' paper on the topic [Ann. Math., II. Ser. 53, 364-386 (1951; Zbl 0043.15901)] although Bers knew of the result before he submitted the paper. Perhaps that omission is in part responsible for the reference being overlooked in the present article.
In his article, the author comments that the removability theorem was extended to higher dimensions by *E. de Giorgi* and *G. Stampacchia* [Rend. Acc. Naz. Lincei, VIII.Ser. 38, 352-357 (1965; Zbl 0135.40003)]. That extension (in fact, a much more general one) appeared earlier in the reviewer's paper [Scripta Math. 26, 107-115 (1961; Zbl 0114.30401)], of which the present author seems not to have been aware. The extension of removability in the Lincei paper to sets of points of $(n-1)$-dimensional Hausdorff measure zero is obtained by formal application of the reviewer's method, although the relevant paper is not cited there.
The present article closes with a discussion of *Bers'* paper [Commun. Pure Appl. Math. 7, 441-504 (1954; Zbl 0058.40601)] on existence of two-dimensional subsonic flows past an obstacle. The reviewer is surprised that the earlier paper of *M. Shiffman* [J. Rational Mech. Anal. 1, 605-652 (1952; Zbl 0048.19301)] is not cited in this context. Although Bers produced a significant improvement in terms of determining the circulation from the Kutta-Joukowski condition, Shiffman's contribution was clearly the pathbreaking one; additionally the paper of Bers is tied to function-theoretic methods that do not extend as do those of Shiffman to the more physical three-dimensional case.
All the above remarks notwithstanding, the reviewer would not want to dispute the closing comments of the article, that Bers was a wonderful teacher, and that through his work and his warm personality he had a great influence on many people. The reviewer himself profited as a student from Bers' infectious enthusiasm for mathematics, disarming informality and accessibility, and lively stimulating lectures, all of which had their impact toward developing his own scientific perspectives. Bers' contributions through his students and his writings will leave a permanent mark in the mathematical world, and it is appropriate that his memory be honored with this volume of papers recalling those contributions.

R.Finn (Stanford)

Keywords : pseudoanalytic functions; removability of isolated singularities; minimal surface equation; two-dimensional subsonic flows past an obstacle
Classification :
∗35-01 Textbooks (partial differential equations)
01A70 Biographies, obituaries, personalia, bibliographies
30G20 Generalizations of analytic functions of Bers or Vekua type
35A20 Analytic methods (PDE)

Zbl 0873.58014

Nirenberg, Louis

Degree theory beyond continuous maps. (English)

CWI Q. 9, No.1-2, 113-120 (1996). ISSN 0922-5366
http://www.cwi.nl/cwi/publications_bibl/QUARTERLY/in_quart.html

The author gives an introductory overview (without proofs) on recent extensions of finite-dimensional degree theory to the non-continuous case. This is of particular interest in dealing with the Ginzburg-Landau equations. Let X, Y be compact connected Riemannian manifolds of the same dimension with Y smoothly embedded in some $\mathbb{R}^N$. Let $u \in VMO(X,Y)$ ("vanishing mean oscillation"), i.e., $u : X \to \mathbb{R}^N$ is an integrable function (defined a.e.) with $u(X) \subset Y$ such that

$$|u|_{BMO} := \sup_{B \subset X} \frac{1}{(\operatorname{vol} B)^2} \int_B \int_B |u(y) - u(x)| dy dx < \infty$$

and $\lim_{\operatorname{vol} B \to 0} \frac{1}{\operatorname{vol} B} \int_B |u - \frac{1}{\operatorname{vol} B} \int_B u(y) dy| dx = 0$ where B ranges over all geodesic balls in X with radius smaller than the injectivity radius r_0 of X. If $0 < \epsilon < r_0$ then the function $u_\epsilon : X \to \mathbb{R}^N$ defined by $u_\epsilon := \frac{1}{\operatorname{vol} B_\epsilon(x)} \int_{B_\epsilon(x)} u(y) d(y)$ (where $B_\epsilon(x)$ is the geodesic ball of radius ϵ around x) is continuous and $|u - u_\epsilon|_{BMO} \to 0$ as $\epsilon \to 0$. Now $u_\epsilon(x)$ need not be in Y, but a theorem of Saranson implies that $d(u_\epsilon(x), Y) \to 0$ as $\epsilon \to 0$, so one defines $\deg(u, X, Y) := \deg(Pu_\epsilon, X, Y)$ (where P is the projection onto the nearest point in Y) for $\epsilon > 0$ small enough. Details can be found in two articles by *H. Brézis* and the author [Sel. Math., New Ser. 1, No. 2, 197-263 (1995; Zbl 0852.58010); ibid. 2, No. 2, 309-368 (1996; Zbl 0868.58017)].

Ch.Fenske (Gießen)

Keywords : VMO; BMO; degree theory
Classification :
∗58C30 Fixed point theorems on manifolds
46E35 Sobolev spaces and generalizations
47H11 Degree theory

Zbl 0868.58017

Brézis, Haïm; Nirenberg, Louis (Mironescu, Petru)

Degree theory and BMO. II: Compact manifolds with boundaries. (Appendix with Petru Mironescu). (English)

Sel. Math., New Ser. 2, No.3, 309-368 (1996). ISSN 1022-1824; ISSN 1420-9020
http://dx.doi.org/10.1007/BF01587948
http://link.springer.de/link/service/journals/00029/

In an earlier paper *H. Brezis* and *L. Nirenberg* [Sel. Math., New Ser. 1, No. 2, 197-263 (1995; Zbl 0852.58010)] studied the degree theory for VMO (vanishing mean oscillation) maps between compact n-dimensional oriented manifolds without boundaries.
In this paper they study a class of maps u from a bounded domain $\Omega \subset \mathbb{R}^n$ into $\mathbb{R}^n$. A real function $f \in L^1_{\text{loc}}(\Omega)$ is said to be in BMO(Ω) (bounded mean oscillation) if

$$(*) \qquad |f|_{\text{BMO}(\Omega)} := \sup_B ⨍_B |f - ⨍_B f| < \infty,$$

where sup is taken over all balls with closure in Ω. Now VMO is the closure of $C^0(\overline{\Omega})$ in the BMO norm. In addition to the bounded domains they also consider domains Ω in a smooth open n-dimensional Riemannian manifold X_0. BMO(Ω) is defined as in

$(*)$; the sup is now taken over geodesic balls $B_\varepsilon(x)$ with $x < r_0$, the injectivity radius of $\overline{\Omega}$. Furthermore, the space BMO(Ω) is independent of the Riemannian metric on X_0. VMO is defined as above. They then consider VMO maps of Ω into an n-dimensional smooth open manifold Y (which is smoothly embedded in some $\mathbb{R}^N$). If X_0 and Y are oriented, and $p \in Y$ is such that, in a suitable sense, $p \notin u(\partial\Omega)$ then they define by approximation $\deg(u, \Omega, p)$.
The content of the paper is as follows: In §II.1 BMO and VMO are introduced together with associated properties. §II.2 takes up the definition of degree, various properties of degree are established, the invariance of degree under continuous deformations in the BMO topology provided some additional assumptions is proved. In §II.3 the behaviour of functions in VMO(Ω) on $\partial\Omega$ is considered. Section §II.4 extends to a certain class of maps a standard result for continuous maps $u : \overline{\Omega} \to \mathbb{R}^n$ with $u\,|_{\partial\Omega} = \varphi$, and with $\varphi \neq p$ on $\partial\Omega$ for some point $p \in \mathbb{R}^n$ that

$$\deg(u, \Omega, p) = \deg\left(\frac{\varphi - p}{|\varphi - p|},\ \partial\Omega, S^{n-1}\right).$$

Appendix 1 contains the proofs of results of §II.1. In Appendix 2 written with P. Mironescu the authors consider Toeplitz operators on S^1. Appendix 3 deals with properties of the harmonic extension of BMO and VMO maps.

W.Mozgawa (Lublin)

Keywords : vanishing mean oscillation; bounded mean oscillation; degree theory; VMO; BMO
Classification :
∗58C35 Integration on manifolds
57N65 Algebraic topology of manifolds

Zbl 0860.35004

Berestycki, H.; Caffarelli, L.A.; Nirenberg, L.

Inequalities for second-order elliptic equations with applications to unbounded domains. I. (English)

Duke Math. J. 81, No.2, 467-494 (1996). ISSN 0012-7094
http://dx.doi.org/10.1215/S0012-7094-96-08117-X
http://www.dukemathjournal.org
http://projecteuclid.org/handle/euclid.dmj

In recent papers, the authors have studied symmetry and monotonicity properties for positive solutions u of elliptic equations of the form

(1) $$u > 0,\ \Delta u + f(u) = 0 \text{ in } \Omega, \quad u = 0 \text{ on } \partial\Omega$$

in several classes of unbounded domains Ω in $\mathbb{R}^n$. Here they continue this program by considering another type of domain, $\Omega = \mathbb{R}^{n-j} \times \omega$, where ω is a smooth bounded domain in $\mathbb{R}^j$.
Denote by $x = (x_1, \ldots, x_{n-j})$ the coordinates in $\mathbb{R}^{n-j}$, and by $y = (y_1, \ldots, y_j)$ the coordinates in ω. The goal is to establish symmetry of solutions of (1) corresponding to

symmetries of ω. For example, if ω is a ball $\{|y| < R\}$, they prove that any solution of (1) depends only on $|y|$ and x, and is decreasing in $|y|$. Note that u is not assumed to be bounded. Throughout the paper it is assumed that f is Lipschitz continuous, with Lipschitz constant k, on $\mathbb{R}^+$ (or on $[0, \sup u]$ in the case where u is bounded).

V.Mustonen (Oulu)

Keywords : semilinear elliptic equation; symmetry of solutions

Classification :

∗35B05 General behavior of solutions of PDE
35J65 (Nonlinear) BVP for (non)linear elliptic equations
35B40 Asymptotic behavior of solutions of PDE

Zbl 0851.00010

Kuhn, Harold W. (ed.); Nirenberg, Louis (ed.); Sarnak, Peter (ed.); Weisfeld, Morris (ed.)

Issue 2 of a special volume: A celebration of John F. Nash jun. (English)

Duke Math. J. 81, No.2, 251-494 (1996).
http://www.dukemathjournal.org
http://projecteuclid.org/handle/euclid.dmj

The articles of this volume will be reviewed individually.

Keywords : Dedication

Classification :

∗00B15 Collections of articles of miscellaneous specific interest

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Zbl 0851.55004

Nirenberg, Louis

Degree theory beyond continuous maps. (English)

Hörmander, Lars (ed.) et al., Partial differential equations and mathematical physics. The Danish-Swedish analysis seminar, Copenhagen, Denmark, Lund, Sweden, March 17-19, May 19-21, 1995. Proceedings. Boston, MA: Birkhäuser. Prog. Nonlinear Differ. Equ. Appl. 21, 262-263 (1996). ISBN 0-8176-3906-3/hbk

Summary: This is a report of joint work with H. Brezis to appear in Selecta Mathematica.

Classification :

∗55M25 Degree, etc.

Zbl 0882.35019

Nirenberg, Louis

The maximum principle and related topics. (English)

Bloom, Thomas (ed.) et al., Modern methods in complex analysis. The Princeton conference in honor of Robert C. Gunning and Joseph J. Kohn, Princeton University, Princeton, NJ, USA, Mar. 16-20, 1992. Princeton, NJ: Princeton University Press. Ann. Math. Stud. 137, 283-289 (1995). ISBN 0-691-04428-7/pbk

This is a short report on the main results in the paper of *H. Berestycki*, *L. Nirenberg*, and *S. R. S. Varadhan* [Comm. Pure Appl. Math. 47, No. 1, 47-92 (1994; Zbl 0806.35129)], where full proofs and many more interesting results can be found. One of the main theorems concerns the existence of a first eigenvalue $\lambda_1 > 0$ with a positive eigenfunction for the linear eigenvalue problem

$$Lu = \sum_{i,j} a_{ij}(x) u_{x_i x_j} + \sum_i b_i(x) u_{x_i} + c(x)u = \lambda u \quad \text{in } D, \qquad u = 0 \quad \text{on } \partial D,$$

where D is a bounded domain in $\mathbb{R}^N$, $a_{ij} \in C(D)$, b_i, $c \in L^\infty(D)$, and L is uniformly elliptic. We emphasize that no smoothness condition is satisfied by D. Moreover, λ_1 is simple. There are some delicate points concerning these fine results. One is that some care is necessary in order to deal with the boundary conditions if D is not regular. A refined maximum principle is proved to hold if and only if $\lambda_1(-L) > 0$. A refined estimate of Alexandrov-Bakelman-Pucci type plays an important role in the proofs, and the same happens with a variational characterization of λ_1. The Krylov-Safonov version of Harnack's inequality is also used. An interesting improvement of the Alexandrov-Bakelman-Pucci estimate due to X. Cabré is announced here, it was published later in *X. Cabré* [Commun. Pure Appl. Math. 48, No. 5, 539-570 (1995; Zbl 0828.35017)].

J.Hernandez (Madrid)

Keywords : non-smooth domain; existence of a first eigenvalue; positive eigenfunction; linear eigenvalue problem; Alexandrov-Bakelman-Pucci estimate
Classification :
∗35B50 Maximum principles (PDE)
35J25 Second order elliptic equations, boundary value problems
47F05 Partial differential operators
35P05 General spectral theory of PDE
35B65 Smoothness of solutions of PDE

Zbl 0852.58010

Brézis, Haïm; Nirenberg, Louis

Degree theory of BMO. I: Compact manifolds without boundaries. (English)

Sel. Math., New Ser. 1, No.2, 197-263 (1995). ISSN 1022-1824; ISSN 1420-9020
http://dx.doi.org/10.1007/BF01671566
http://link.springer.de/link/service/journals/00029/

The authors consider the degree theory for mappings u from a compact smooth manifold X to a connected compact smooth manifold Y of the same dimension. The notion of degree can be extended to continuous maps from X to Y because if $u, v \in C^1(X, Y)$ are close in the C^0 topology then they have the same degree. For a C^1-map there is an integral formula for the degree. The integral formulas suggest the possibility of extending degree theory to another class of maps which need not be continuous namely maps in appropriate Sobolev spaces. This was done by several authors and the list of references is given in the paper. Among them, L. Boutet de Monvel and O. Gabber introduced a degree for maps $u \in H^{1/2}(S^1, S^1)$ and made an observation that this notion makes sense for maps in the class VMO (vanishing mean oscillation): the closure of the set of smooth maps in the BMO (bounded mean oscillation) topology. Namely, if $u \in \mathrm{VMO}(S^1, S^1)$ and $\overline{u}_\varepsilon(\theta) = \frac{1}{2\varepsilon}\int_{\theta-\varepsilon}^{\theta+\varepsilon} u(s)ds$ then $|\overline{u}_\varepsilon(\theta)| \to 1$ uniformly in θ, in spite of the fact that u need not be continuous. Then, for ε small,

$$u_\varepsilon(\theta) = \frac{\overline{u}_\varepsilon(\theta)}{|\overline{u}_\varepsilon(\theta)|}$$

has a well defined degree which is independent of ε. In the paper under review, the authors develop this concept for maps between n-dimensional manifolds X, Y and establish its basic properties. The degree is defined via approximation, in the BMO topology. The content of the paper is as follows:
In Section I.1 they recall the notion of BMO and VMO maps on Euclidean spaces and describe its extension to maps between manifolds. The next section takes up various examples of BMO and VMO maps. The degree for VMO maps is defined in Section I.3 and its standard properties are described in the next section. In Section I.5 the authors consider a natural question concerning maps from X to Y not necessarily of the same dimension. The last section deals with the question of the possibility of lifting a map $u \in \mathrm{BMO}(X, S^1)$ to $\mathrm{BMO}(X, \mathbb{R})$. The proofs of many technical statements are given in Appendix A. The proofs of results of Section I.6 are technical and use the John-Nirenberg inequality, various forms of which are presented in Appendix B. The authors announce that Part II of this paper will consider the degree theory for VMO maps on manifolds with boundary.

W.Mozgawa (Lublin)

Keywords : Sobolev space; VMO vanishing mean oscillation; BMO bounded mean oscillation; degree theory; BMO topology; John-Nirenberg inequality
Classification :
∗58C35 Integration on manifolds
58C25 Differentiable maps on manifolds (global analysis)
46E35 Sobolev spaces and generalizations
58D15 Manifolds of mappings

Zbl 0843.00014

Kuhn, Harold W. (ed.); Nirenberg, Louis (ed.); Sarnak, Peter (ed.); Weisfeld, Morris (ed.)

Special issue: a celebration of John F. Nash Jr. (English)

Duke Math. J. 81, No.1, 250 p. (1995).
http://www.dukemathjournal.org
http://projecteuclid.org/handle/euclid.dmj

The articles of this volume will be reviewed individually.

Keywords : Dedication

Classification :

∗00B15 Collections of articles of miscellaneous specific interest

00B30 Festschriften

Zbl 0840.35035

Berestycki, Henri; Capuzzo-Dolcetta, Italo; Nirenberg, Louis

Variational methods for indefinite superlinear homogeneous elliptic problems. (English)

NoDEA, Nonlinear Differ. Equ. Appl. 2, No.4, 553-572 (1995). ISSN 1021-9722; ISSN 1420-9004

http://dx.doi.org/10.1007/BF01210623

http://link.springer.de/link/service/journals/00030/index.htm

The authors study the existence of positive solutions to the semilinear elliptic problem

$$-\Delta u + (q(x) - \tau)u = a(x)u^p \quad \text{in } \Omega, \quad Bu = 0 \quad \text{on } \partial\Omega, \tag{1}$$

where $\Omega \subset \mathbb{R}^N$, is a bounded domain, $1 < p < (N+2)/(N-2)$ (if $N \geq 3$), $1 < p$ (if $N = 1, 2$), q and a are continuous functions, $\tau \in \mathbb{R}$, and B is either the Dirichlet, Neumann, or Robin boundary operator. The function a is not assumed to be positive, so that classical methods cannot be applied directly to prove existence of a nontrivial solution of (1). The authors show that there exists a positive solution in the following cases:

(i) if $(q(x) - \tau) = 0$, then it is necessary and sufficient that a changes sign and $\int_\Omega a(x)dx < 0$.

(ii) if both sets $\{x \mid a(x) > 0\}$ and $\{x \mid a(x) < 0\}$ are not empty and $\int_\Omega a(x)\phi^{p+1}dx < 0$ (where $\phi > 0$ is a solution of $-\Delta\phi + q(x)\phi = 0$, $B\phi = 0$), then there exists $\tau^* > 0$, such that for $0 \leq \tau < \tau^*$ there is a solution of (1), while for $\tau > \tau^*$ no solution exists.

The also prove other necessary conditions which are based on a generalized Picone identity. The existence proofs rely on a constrained maximization procedure, but (i) can also be obtained by an application of the mountain pass theorem.

K.Pflüger (Berlin)

Keywords : indefinite nonlinearity; existence of positive solutions; nonexistence; principle eigenvalue; generalized Picone identity

Classification :

∗35J65 (Nonlinear) BVP for (non)linear elliptic equations

35J20 Second order elliptic equations, variational methods

Zbl 0816.35030

Berestycki, H.; Capuzzo Dolcetta, I.; Nirenberg, L.

Superlinear indefinite elliptic problems and nonlinear Liouville theorems. (English)

Topol. Methods Nonlinear Anal. 4, No.1, 59-78 (1994). ISSN 1230-3429

http://www-users.mat.uni.torun.pl/ tmna/

The authors study the boundary value problem

$$\sum_{i,j=1}^{N} a_{ij}(x)\frac{\partial^2 u}{\partial x_i \partial x_j} + \sum_{i=1}^{N} b_i(x)\frac{\partial u}{\partial x_i} + a(x)g(u) = 0 \quad \text{in } \Omega,$$

$$\sum_{j,k=1}^{N} \nu_j a_{jk}\frac{\partial u}{\partial x_k} + \alpha(x)u = 0 \quad \text{on } \partial\Omega.$$

Here the above differential operator is uniformly elliptic, $\alpha(x) \geq 0$ on $\partial\Omega$, but the coefficient $a(x)$ may change sign. The nonlinearity g is assumed to be C^1 with $g(0) = g'(0) = 0$, $g(s) > 0$ for large $s > 0$, and such that the limit $s^{-p}g(s)$ exists, as $s \to \infty$, for some $p > 1$. The main existence result then states that the above boundary value problem has a solution if $1 < p < (N+2)/(N-1)$. Many interesting additional statements are given, mainly for the model equation $\Delta u - m(x)u + a(x)g(u) = 0$.

J.Appell (Würzburg)
Keywords : nonlinear Liouville theorems
Classification :
∗35J65 (Nonlinear) BVP for (non)linear elliptic equations

Zbl 0807.01017

Nirenberg, Louis
Partial differential equations in the first half of the century. (English)
Pier, Jean-Paul (ed.), Development of mathematics 1900-1950. Based on a symposium organized by the Luxembourg Mathematical Society in June 1992, at Château Bourglinster, Luxembourg. Basel: Birkhäuser. 479-515 (1994). ISBN 3-7643-2821-5/hbk

If considering – according to Prof. Gelfand's opinion – that mathematics may be viewed as having two faces, the field of Partial Differential Equations (PDE) belongs to the former one, the one related to physics and other sciences (as compared with the strictly mathematical ones); actually, several scientific and engineering problems may get mathematical expression by means of differential equations. The main topics discussed in the study involve the existence of solutions under various boundary conditions (BC), or initial conditions (IC), i.e. conditions at some initial time t_0; uniqueness of solutions; estimates, and regularity of solutions. The essential problem in treating PDE is represented by inequalities – estimates of all types. In the 19th century, a significant part of the PDE study was connected with particular problems from both physics and mathematics, and along with the general theorem of Cauchy-Kowalewsky. The study also analyzes the so- called well-posed problem (1. if a solution exists, 2. if it is unique, 3. the solution (if unique) depends continuously on the data) elliptic equations (linear equations with regular coefficients and a priori estimates); hyperbolic equations, fluid dynamics, singular integral operators and Fourier transform, geometry, etc.

C.Cusmir (Iaşi)
Keywords : inequality; Gelfand; Cauchy-Kowalewsky theorem; elliptic equation
Classification :
∗01A60 Mathematics in the 20th century

35-03 Historical (partial differential equations)

Zbl 0806.35129

Berestycki, H.; Nirenberg, L.; Varadhan, S.R.S.

The principal eigenvalue and maximum principle for second-order elliptic operators in general domains. (English)

Commun. Pure Appl. Math. 47, No.1, 47-92 (1994). ISSN 0010-3640

http://dx.doi.org/10.1002/cpa.3160470105

http://onlinelibrary.wiley.com/journal/10.1002/%28ISSN%291097-0312

Let L be a uniformly elliptic operator in a general bounded domain (i.e., open connected set) $\Omega \subset \mathbb{R}^n$, of the form $L = M + c(x) = a_{ij}(x)\partial_{ij} + b_i(x)\partial_i + c(x)$, where for some positive constants c_0, C_0, $c_0|\xi|^2 \leq a_{ij}(x)\xi_i\xi_j \leq C_0|\xi|^2$ for all $\xi \in \mathbb{R}^n$. It is assumed that $a_{ij} \in C(\Omega)$, b_i, $c \in L^\infty$, $(\sum b_i^2)^{1/2}$, $|c| \leq b$ for some constant $b \geq 0$. The authors find a principal eigenvalue λ_1 and eigenfunction φ_1 for the Dirichlet problem for $-L$ and study their relationship with a refined maximum principle.
A brief outline of their work is the following: The principal eigenvalue is defined by $\lambda_1 = \sup\{\lambda \mid \exists\varphi > 0$ in Ω satisfying $(L + \lambda)\varphi \leq 0\}$. Various bounds on λ_1 are established, the dependence of λ_1 on Ω and on the coefficients b_i and c is studied and a principal eigenfunction φ_1 is constructed. L is said to satisfy the refined maximum principle in Ω if for any function $w(x)$ on Ω, $w \leq 0$ in Ω is implied by the conditions $Lw \geq 0$ in Ω, w bounded above, and $\limsup w(x_j) \leq 0$ for every sequence $x_j \to \partial\Omega$ for which $u_0(x_j) \to 0$. Here, u_0 is a special function which is constructed in the paper and is a positive function in Ω for which $Mu_0 = -1$ and u_0 vanishes, in a suitable sense, on $\partial\Omega$. It is proved that the refined maximum principle holds for L if and only if $\lambda_1 > 0$.

R.C.Gilbert (Placentia)

Keywords : bounded domain; Dirichlet problem; principal eigenvalue; principal eigenfunction; refined maximum principle

Classification :

∗35P15 Estimation of eigenvalues for PD operators
35J25 Second order elliptic equations, boundary value problems
35B50 Maximum principles (PDE)

Zbl 0820.35056

Berestycki, Henri; Capuzzo-Dolcetta, Italo; Nirenberg, Louis

Indefinite elliptic equations and nonlinear Liouville theorems. (Problèmes elliptiques indéfinis et théorèmes de Liouville non linéaires.) (French. Abridged English version)

C. R. Acad. Sci., Paris, Sér. I 317, No.10, 945-950 (1993). ISSN 0764-4442

Summary: We consider the semilinear equation $-\Delta u + m(x)u = a(x)g(u)$, where a may change sign in Ω, an open bounded set in $\mathbb{R}^N$, and g has superlinear growth. We present several results about the existence of positive solutions satisfying Neumann or

Dirichlet-type boundary conditions. In the homogeneous case $g(u) = u^p$ these solutions are obtained by a variational approach and we derive some necessary and sufficient conditions. In the general case, we obtain existence results under certain conditions on the term g. These are proved with the aid of a priori estimates. To carry this method through, we prove some new theorems of Liouville type for equations of the form $\Delta u + h(x)u^p = 0$.

Keywords : Dirichlet problem; Neumann problem; semilinear elliptic equation; existence of positive solutions
Classification :
∗35J65 (Nonlinear) BVP for (non)linear elliptic equations
35B45 A priori estimates

Zbl 0803.35029

Brézis, Haïm; Nirenberg, Louis

H^1 versus C^1 local minimizers. (English. Abridged French version)

C. R. Acad. Sci., Paris, Sér. I 317, No.5, 465-472 (1993). ISSN 0764-4442

Summary: We consider functionals of the form $\Phi(u) = (1/2)\int_\Omega |\nabla u|^2 - \int_\Omega F(x,u)$. Under suitable assumptions we prove that a local minimizer of Φ in the C^1 topology must be a local minimizer in the H^1 topology. This result is especially useful when the corresponding equation admits a sub and super solution.

Keywords : local minimizers of nonlinear functionals
Classification :
∗35J20 Second order elliptic equations, variational methods
35J65 (Nonlinear) BVP for (non)linear elliptic equations
35D10 Regularity of generalized solutions of PDE

Zbl 0798.35024

Nirenberg, Louis

The maximum principle and principal eigenvalue for second order elliptic equations in general bounded domains. (English)

Ricci, Paolo Emilio (ed.), Actual problems in analysis and mathematical physics. Proceedings of the international symposium dedicated to Gaetano Fichera on the occasion of his 70th birthday, Taormina, Italy, 15-17 October, 1992. Roma: Dipartimento di Matematica, Università di Roma "La Sapienza", 189-194 (1993).

This paper presents a selection of results on eigenvalue problems for second order elliptic equations published by *H. Berestycki*, the author and *S. R. S. Varadhan* [Commun. Pure Appl. Math. 47, No. 1, 47-92 (1994)].

G.Philippin (Quebec)

Keywords : maximum principle; principal eigenvalue
Classification :
∗35B50 Maximum principles (PDE)

35P99 Spectral theory and eigenvalue problems for PD operators

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Zbl 0798.35038

Berestycki, Henri; Nirenberg, Louis; Varadhan, Srinivasa

The ground state and maximum principle for second order elliptic operators in general domains. (État fondamental et principe du maximum pour les opérateurs elliptiques du second ordre dans des domaines généraux.) (French. Abridged English version)

C. R. Acad. Sci., Paris, Sér. I 317, No.1, 51-56 (1993). ISSN 0764-4442

Summary: For an elliptic operator L in a general bounded domain $\Omega\subset\mathbb{R}^N$ (no assumption of smoothness is made here), we define the principal eigenvalue by

$$(1)\quad \lambda_1=-\inf_{\{\phi>0\}}\sup_{x\in\Omega}\big\{L\phi(x)/\phi(x)\big\}=\sup\big\{\lambda;\ \exists\varphi>0 \text{ such that } L\varphi+\lambda\varphi\le 0 \text{ in } \Omega\big\}.$$

We show that the Krein-Rutman theory extends to this general setting. Indeed, we show that there exists a function $\phi_1\in L^\infty(\Omega)$ such that $(L+\lambda_1)\phi_1=0$ in Ω, ϕ_1 vanishes on $\partial\Omega$ in a sense which is made precise. This function is unique up to a multiplicative constant. Furthermore, the Maximum Principle (in a conveniently refined formulation) holds for L in Ω if and only if $\lambda_1>0$. We establish several properties of λ_1 about the dependence on the coefficients, the domain, etc. and several estimates which are new – even in the case of a regular domain Ω. In deriving these estimates we emphasize the structural aspect of the various constants – independently of the particular operator under consideration. In particular we show that the maximum principle holds for domains which are sufficiently "narrow" or have small measure.

Keywords : principal eigenvalue; Krein-Rutman theory

Classification :

- ∗35J15 Second order elliptic equations, general
- 35R05 PDE with discontinuous coefficients or data
- 35B50 Maximum principles (PDE)
- 35B30 Dependence of solutions of PDE on initial and boundary data
- 35B45 A priori estimates

Zbl 0793.35034

Berestycki, H.; Caffarelli, L.A.; Nirenberg, L.

Symmetry for elliptic equations in a half space. (English)

Lions, Jacques-Louis (ed.) et al., Boundary value problems for partial differential equations and applications. Dedicated to Enrico Magenes on the occasion of his 70th birthday. Paris: Masson. Res. Notes Appl. Math. 29, 27-42 (1993). ISBN 2-225-84334-1/pbk

In the course of studying regularity in some free boundary problems extending their work [Analysis and partial differential equations, Lect. Notes Pure Appl. Math. 122, 567-619 (1990; Zbl 0702.35252)] the authors are led to consider positive bounded solutions u in a half space H in $\mathbb{R}^n$, satisfying

$$\Delta u + \beta(x_n, u) = 0 \text{ in } H = \{x_n > 0\}, \quad u = 0 \text{ on } x_n = 0.$$

Here β is continuous on $[0,\infty] \times [0, M]$, $M = \sup u$, and for any finite t-interval, β is Lipschitz in u on $[0, M]$. (In the free boundary problem $\beta = \beta(u)$). The main result is the following Theorem. Assume $\beta(t, u)$ is nondecreasing in t, and $\beta(t, M) \le 0$ $\forall t \ge 0$. Then u is a function of x_n alone, and $u_{x_n} > 0$ if $x_n > 0$. Furthermore $\beta(\infty, M) = 0$.
If $M = \infty$, the conclusion of the theorem does not hold: in $\mathbb{R}^2$, $u = x_2 e^{x_1}$ satisfies $\Delta u - u = 0$.

Keywords : semilinear elliptic equation in a half space; symmetry; positive bounded solutions
Classification :
∗35J65 (Nonlinear) BVP for (non)linear elliptic equations
35A25 Other special methods (PDE)

Zbl 0799.35073

Berestycki, Henri; Nirenberg, Louis
Travelling fronts in cylinders. (English)
Ann. Inst. Henri Poincaré, Anal. Non Linéaire 9, No.5, 497-572 (1992). ISSN 0294-1449
numdam:AIHPC_1992__9_5_497_0
http://www.sciencedirect.com/science/journal/02941449

The authors are concerned with travelling wave solutions in an infinite cylinder $\Sigma := \mathbb{R} \times \omega$ with $\omega \subseteq \mathbb{R}^{n-1}$ a bounded domain. They consider equations of the form $\Delta u - \beta(y, c)\partial_{x_1} u + f(u) = 0$ in Σ $(x = (x_1, y))$ under homogeneous Neumann boundary conditions on $\partial\Sigma$ and asymptotic conditions $u(-\infty, \cdot) = 0$ and $u(+\infty, \cdot) = 1$. As far as β is concerned, they assume: β continuous on $\omega \times \mathbb{R}$ and strictly increasing in its second argument, $\beta(y, c) \to \pm\infty$ as $c \to \pm\infty$ uniformly for $y \in \omega$. One may think of f as being in $C^2([0, 1])$ with $f(0) = 0 = f(1)$ and $f'(1) < 0$.
Three cases are considered: (A) $f > 0$ on $(0, 1)$; (B) $\exists\theta > 0 : f|_{[0,\theta]} \equiv 0$ and $f|_{(\theta,1)} > 0$; (C) $\exists\theta > 0 : f|_{(0,\theta)} < 0$ and $f|_{(\theta,1)} > 0$. In case (A) they show that there exists a $c^* \in \mathbb{R}$ such that the above problem is solvable, iff $c \ge c^*$. If $f'(0) > 0$, then the solution is unique modulo translations. For case (B) they obtain a solution (c, u), whereas ω convex has to be additionally required in case (C) for that purpose.
There are many more significant results in this comprehensive investigation, which extends various classical results from combustion theory as well as the celebrated paper of Kolmogorov, Petrovsky and Piskounov to higher dimensions.

G.Hetzer (Auburn)
Keywords : travelling wave solutions; infinite cylinder
Classification :
∗35J65 (Nonlinear) BVP for (non)linear elliptic equations
35K99 Parabolic equations and systems
80A25 Combustion, interior ballistics

Zbl 0790.35001

Nirenberg, Louis

On the maximum principle. Videotape. (English)

AMS-MAA Joint Lecture Series. Providence, RI: American Mathematical Society. $ 49.95 (1992).

This videotape captures a lecture on the maximum principle. The method of moving planes is explained. It is shown how the maximum principle provides a simplified approach to this method. The symmetry and monotonicity of solutions of certain boundary value problems are discussed in this context.

Bernd Wegner (Berlin)

Keywords : maximum principle; method of moving planes; symmetry; monotonicity

Classification :

∗35-01 Textbooks (partial differential equations)

35J25 Second order elliptic equations, boundary value problems

35B50 Maximum principles (PDE)

Zbl 0840.35011

Berestycki, H.; Nirenberg, L.

Asymptotic behaviour via the Harnack inequality. (English)

Ambrosetti, A. (ed.) et al., Nonlinear analysis. A tribute in honour of Giovanni Prodi. Pisa: Scuola Normale Superiore, Quaderni. Universitá di Pisa. 135-144 (1991).

Let $Lu = a_{ij}u_{ij} + b_iu_i + cu$ be uniformly elliptic with L^∞ coefficients. The authors investigate solutions of $Lu = 0$ on the semi-infinite cylinder $[0,\infty) \times \omega$, $\omega \subset \mathbb{R}^{n-1}$, with $\partial u/\partial\nu = 0$ on $[0,\infty) \times \partial\omega$. They show that if u, v are positive solutions with $u, v \to 0$ as $x_1 \to \infty$, and if $c(x) \le 0$ then, for some constant $A > 0$, $v(x_1, y)/u(x_1, y) \to A$ as $x_1 \to \infty$, uniformly in ω. The same estimate is proved when v is as before and u satisfies the semilinear equation $Lu = f(x, u)$, provided $|f(x, u)| \le Cu^{1+\delta}$ for some $\delta > 0$, $0 < u$ small, and $c(x) \le -m < 0$. As a corollary, a similar asymptotic estimate is proved for solutions in $\mathbb{R}^n$ when $|x|b_i(x)$ and $|x|^2c(x)$ are bounded for $|x| \ge 1$.

G.Porru (Cagliari)

Keywords : Harnack inequality; semi-infinite cylinder; positive solutions; semilinear equation

Classification :

∗35B40 Asymptotic behavior of solutions of PDE

35J25 Second order elliptic equations, boundary value problems

35J65 (Nonlinear) BVP for (non)linear elliptic equations

Zbl 0784.35025

Berestycki, H.; Nirenberg, L.

On the method of moving planes and the sliding method. (English)

Bol. Soc. Bras. Mat., Nova Sér. 22, No.1, 1-37 (1991). ISSN 0100-3569

http://dx.doi.org/10.1007/BF01244896
http://www.springerlink.com/content/110365/

Summary: The method of moving planes and the sliding method are used in proving monotonicity or symmetry in, say, the x_1 direction for solutions of nonlinear elliptic equations $F(x,u,Du,D^2u)=0$ in a bounded domain Ω in $\mathbb{R}^n$ which is convex in the x_1 direction. Here we present a much simplified approach to these methods; at the same time it yields improved results. For example, for the Dirichlet problem, no regularity of the boundary is assumed. The new approach relies on improved forms of the Maximum Principle in "narrow domains". Several results are also presented in cylindrical domains – under more general boundary conditions.

Keywords : monotonicity and symmetry in one direction; maximum principle in "narrow domains"; method of moving planes; sliding method; nonlinear elliptic equations
Classification :
∗35J60 Nonlinear elliptic equations
35B50 Maximum principles (PDE)
35B05 General behavior of solutions of PDE

Zbl 0780.35054

Berestycki, H.; Nirenberg, L.

Travelling front solutions of semilinear equations in n dimensions. (English)
Frontiers in pure and applied mathematics, Coll. Pap. Ded. J.-L. Lions Occas. 60th Birthday, 31-41 (1991).

[For the entire collection see Zbl 0722.00015.]
An infinite cylindrical domain $\Sigma=\mathbb{R}\times\omega\subset\mathbb{R}^N$, where ω is a bounded domain in $\mathbb{R}^{N-1}$ with smooth boundary, is considered. An element $x\in\Sigma$ is written in the form $x=(x_1,y)$, $x_1\in\mathbb{R}$, $y=(x_2,\dots,x_n)\in\omega$ and by ν is denoted the outward unit normal vector on $\partial\omega$ as well as the outward unit normal to $\partial\Sigma$.
Travelling front solutions in Σ are solutions of problems of the following type:

$$-\Delta u+(c+\alpha(y))u_{x_1}=f(u)\qquad(\text{or }-\Delta u+c\alpha(y)u_{x_1}=f(u))\text{ in }\Sigma,$$

with $\partial u/\partial\nu=0$ on $\partial\Sigma$, $u(-\infty,y)=0$, $u(+\infty,y)=1$, uniformly in $y\in\overline{\omega}$. Here $\alpha:\overline{\omega}\to\mathbb{R}$ is a given continuous function assumed to be positive and c is a real parameter, the velocity, usually an unknown in the problem. The function f will be assumed to be Lipschitz, and to vanish outside the interval [0,1]; on the interval [0,1] it is assumed that $f\in C^{1,\delta}$ for some $0<\delta<1$ on some neighbourhood of 0 and 1, respectively, and $f'(1)<0$. The existence and uniqueness theorems of (c,u) and the exponential behaviour of u as $x\to-\infty$ are presented.

I.Onciulescu (Iaşi)
Keywords : reaction-diffusion; travelling front solutions; existence; uniqueness; exponential behaviour
Classification :
∗35K60 (Nonlinear) BVP for (non)linear parabolic equations

35K57 Reaction-diffusion equations

Zbl 0751.58006

Brézis, Haïm; Nirenberg, Louis

Remarks on finding critical points. (English)

Commun. Pure Appl. Math. 44, No.8-9, 939-963 (1991). ISSN 0010-3640

http://dx.doi.org/10.1002/cpa.3160440808

http://onlinelibrary.wiley.com/journal/10.1002/%28ISSN%291097-0312

Let F be a real C^1 function defined on a Banach space X. In the first part of the paper there are presented some applications of Ekeland's Principle in obtaining critical points of functions F which satisfy the Palais-Smale property. The main result is given by Theorem 1, proved in this part by using Ekeland's Principle. In the second part is presented a general deformation theorem (Theorem 3). Next a new proof of a recent theorem of Ghoussoub (Theorem 2) is given by using deformation Theorem 3. In the third part, the authors apply Theorem 2 to functions F which are bounded below and satisfy the Palais-Smale property. Finally, in the Appendix, the authors give a new proof of Theorem 1 based on deformation Theorem 3.

N.Papaghiuc (Iaşi)

Keywords : critical points; Palais-Smale property; Ekeland's Principle; deformation

Classification :

∗58E05 Abstract critical point theory

58E15 Appl. of variational methods to extremal problems in sev.variables

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Zbl 0705.35004

Berestycki, H.; Nirenberg, L.

Some qualitative properties of solutions of semilinear elliptic equations in cylindrical domains. (English)

Analysis, et cetera, Res. Pap. in Honor of J. Moser's 60th Birthd., 115-164 (1990).

[For the entire collection see Zbl 0688.00009.]

Let $S=\{x=(x_1,...,x_n);\ y=(x_2,...,x_n)\in\omega\}$ be a cylindrical domain in $\mathbb{R}^n$, where ω is a bounded domain in $\mathbb{R}^{n-1}=\{y=(x_2,...,x_n)\}$ with C^2- boundary and let ν denote the exterior unit normal to S at any boundary point. The authors consider equations of the form (here $u_1=u_{x_1}$)

$$(1)\quad \Delta u-\beta(u)u_1+f(y,u)=0 \text{ in } S$$

(and some more general ones) under Neumann condition (2) $u_\nu=0$ on ∂S or Dirichlet (3) $u=0$ on ∂S. The solutions are supposed to belong to $C^2(\bar{S})$ and, to satisfy for some constant k, (4) $u>k$ in S, (5) $\lim_{x_1\to-\infty}u(x_1,y)=k$ uniformly for $y\in\bar{\omega}$. In many of the results, conditions on u as $x_1\to+\infty$ are also imposed. The function f is supposed to be continuous where defined, and in many cases, to be differentiable in u; β (y) is assumed to be continuous.

In the present paper the authors take up questions of the following type: Is u monotonous in x_1? Is it symmetric in x_1 about some value? In case $\beta = 0$, and f is odd in u, is u antisymmetric in x_1, about some value? If the condition $u(x_1, y) \to K > k$ as $x_1 \to +\infty$ is required, is the solution unique - up to x_1- translation?
In the present paper the authors use: the method of moving planes of *B. Gidas*, *W. M. Ni* and *L. Nirenberg* [Commun. Math. Phys. 68, 209-243 (1979; Zbl 0425.35020)], and "the method of sliding domains": shifting a solution u along the x_1 axis and then comparing the shifted u with another solution, or with the original u. Both methods were used in their previous paper [J. Geom. Phys. 5, No.2, 237-275 (1988; Zbl 0698.35031)]. However a new ingredient is needed to carry out these procedures: some fairly precise knowledge of the asymptotic behaviour of the solution near $x_1 = \pm\infty$. The authors rely on some results of Agmon, Nirenberg and of Pasy, which are described in Section 2. These results involve "exponential solutions" of the form $v = e^{\lambda x_1}\phi(y)$ of linearized equations

$$(6) \quad (\Delta - \beta(y)\partial_1 - a(y))v = 0$$

under boundary condition (2) or (3), where $a(y) = -f_u(y, k)$. This means that ϕ (y)$\not\equiv 0$ satisfies

$$(7) \quad (-\Delta_y + a(y))\phi = (\lambda^2 - \lambda\beta(v))\phi$$

and ϕ satisfies $\phi_\nu = 0$ or $\phi = 0$ on $\partial\omega$.
Section 3 is devoted to the spectral analysis of equations (7). In Section 4 the results of Sections 2 and 3 are applied to obtain asymptotic behaviour near $(x_1 =) + \infty$ of solutions (1) under condition (2) or (3).
In Section 5 the authors study travelling front solutions in S satisfying (2) and (4), (5) with $k = 0$. These investigations are related to several models in biology, chemical kinetics and combustion (see *D. G. Aronson* and *H. F. Weinberger* [Lect. Notes Math. 446, 5-49 (1975; Zbl 0325.35050)] and *P. C. Fife* [Lect. Notes Biomath. 28 (1979; Zbl 0403.92004)]).
Section 6 is concerned with solitary wave solutions $u > 0$ in S, $u(x_1, y) \to 0$ as $|x_1| \to \infty$, of $\Delta u + f(y, u) = 0$ under condition (2) or (3). In Section 7 the authors study solutions of equations

$$(8) \quad u - c \cdot \alpha(y)u_1 + f(y, u) = 0 \text{ in } S$$

and

$$(9) \quad u - (c + \alpha(y))u_1 + f(y, u) = 0 \text{ in } S$$

under the condition

$$(10) \quad u_\nu = 0 \text{ on } \partial S.$$

In (8) α (y)≥ 0 in ω and in (9) α (y) is a given function and the constant c is to be determined. More precisely the authors study solutions of (8), (9) under (10) satisfying the assumptions: $k < u < K$; $u(x_1, y) \to K$ as $x_1 \to +\infty$. These investigations have connections with the work of the first author and *B. Larrouturou* [J. Reine Angew. Math. 396, 14-40 (1989; Zbl 0658.35036)].

I.J.Bakelman

Keywords : cylindrical domain; Neumann condition; Dirichlet; monotonous; symmetric; odd; antisymmetric; moving planes; sliding domains; asymptotic behaviour; travelling front solutions; solitary wave solutions
Classification :
∗35B05 General behavior of solutions of PDE

35J65 (Nonlinear) BVP for (non)linear elliptic equations

Zbl 0702.35252

Berestycki, H.; Caffarelli, L.A.; Nirenberg, L.

Uniform estimates for regularization of free boundary problems. (English)

Analysis and partial differential equations, Coll. Pap. dedic. Mischa Cotlar, Lect. Notes Pure Appl. Math. 122, 567-619 (1990).

The authors consider problems of the type

$$Lu = a_{ij}(x)u_{ij} + b_i(x)u_i + c(x)u = \beta_\epsilon(u) \text{ in } \Omega$$

where the nonlinearity β_ϵ has support in [0,ϵ] and $\beta_\epsilon \leq B/\epsilon$. An example of such a nonlinearity is $\beta_\epsilon(u) = (1/\epsilon)b(u/\epsilon)$, where β is continuous with support in [0,1], positive on (0,1) and $\int_0^1 \beta(s)ds = M > 0$. They obtain estimates up to the boundary. For the behavior near the boundary it is assumed that u satisfies $\mu(x) \cdot \nabla u = 0$ on $\partial\Omega$, μ pointing outside. The paper contains the following sections: a Harnack inequality up to the boundary; uniform Lipschitz continuity of u_ϵ on compact subsets of Ω, independent of ϵ ; non degeneracy of certain minimal solutions; study of $\lim_{\epsilon\to 0} u_\epsilon$; study of the regularity of the free boundary; application of the results to the free boundary in a flame propagation problem. Among many other interesting results they show that $v = \lim_{\epsilon\to 0} u_\epsilon$ satisfies $a_{ij}\nu_i\nu_j|\nabla v|^2 = 2M$ on σ. Here is a smooth portion of the free boundary and $v > 0$ on one side of σ and $v = 0$ on the other side.

R.Sperb

Keywords : free boundary problem; Harnack inequality; uniform Lipschitz continuity; non degeneracy

Classification :

∗35R35 Free boundary problems for PDE

35B45 A priori estimates

Zbl 0778.35035

Nirenberg, L.

On fully nonlinear elliptic equations of second order. (English)

Sémin. Équations Dériv. Partielles 1988-1989, Exp. No.16, 6 p. (1989).

numdam:SEDP_1988-1989____A17_0

Classification :

∗35J60 Nonlinear elliptic equations

35J15 Second order elliptic equations, general

Zbl 0763.46023

Brézis, Haïm; Nirenberg, Louis

A minimization problem with critical exponent and nonzero data. (English)

Symmetry in nature, Symp. in Honour of L. A. Radicati di Brozolo, Pisa/Italy 1989, 129-140 (1989).

Summary: [For the entire collection see Zbl 0718.00014.]
Let Ω be a smooth bounded domain in $\mathbb{R}^n$ with $n \geq 3$. Given $\varphi \in L^q(\Omega)$, consider the following minimization problem

$$J = \inf_{u \in H_0^1} \int_\Omega |\nabla u|^2, \qquad \|u + \varphi\|_q = \gamma, \tag{1}$$

where $\| \ \|_q$ denotes the norm in $L^q(\Omega)$, $\gamma > 0$ is a constant and $q = 2n/(n-2)$ is the limiting exponent for the Sobolev embedding. It is well-known that the infimum in (1) is not achieved if $\varphi = 0$. Our main result is: Theorem 1. Assume $\varphi \not\equiv 0$. Then the infimum in (1) is achieved.

Keywords : minimization problem; Sobolev embedding
Classification :
∗46E35 Sobolev spaces and generalizations
49J35 Minimax problems (existence)
47F05 Partial differential operators
46E30 Spaces of measurable functions

Zbl 0679.58021

Nirenberg, L.

Variational methods in nonlinear problems. (English)

Topics in calculus of variations, Lect. 2nd Sess., Montecatini/Italy 1987, Lect. Notes Math. 1365, 100-119 (1989).

[For the entire collection see Zbl 0668.00016.]
This is a popular lecture to serve old variational methods and to represent some new ones for the solution of nonlinear problems. Results for finding nontrivial stationary points of real C^1-functions defined in Banach spaces (e.g. mountain pass lemma) and applications to elliptic problems are discussed. Techniques for finding multiple stationary points of functionals with invariant properties are demonstrated on systems of ode's.

L.G. Vulkov

Keywords : popular lecture; variational methods; nonlinear problems; multiple stationary points
Classification :
∗58E30 Variational principles on infinite-dimensional spaces
58-01 Textbooks (global analysis)
01A99 Miscellaneous topics in history of mathematics

Zbl 0698.35054

Nirenberg, Louis

Fully nonlinear second order elliptic equations. (English)

Calculus of variations and partial differential equations, Proc. Conf., Trento/Italy 1986, Lect. Notes Math. 1340, 239-247 (1988).

[For the entire collection see Zbl 0641.00013.]
Exposé de synthèse des travaux de Caffarelli-Nirenberg-Spruck (et J. Kohn) sur les équations elliptiques de la forme $F(x, u, Du, D^2u) = 0$ [voir Commun. Pure Appl. Math. 37, 369-402 (1984; Zbl 0598.35047), 38, 209-252 (1985; Zbl 0598.35048) et Acta Math. 155, 261-301 (1985; Zbl 0654.35031)]. Ils couvrent en particulier des problèmes du type $f(\lambda) = \psi(x)$ sur Ω, $u = \phi$ sur $\partial\Omega$ où $\lambda = (\lambda_1, \lambda_2, ..., \lambda_n)$ sont les valeurs propres de la matrice (u_{ij}) et f est une fonction symétrique des λ_i.

H.Brezis

Keywords : fully nonlinear second order elliptic equations; eigenvalues of the Hessian
Classification :
∗35J60 Nonlinear elliptic equations

Zbl 0698.35031

Berestycki, H.; Nirenberg, L.

Monotonicity, symmetry and antisymmetry of solutions of semilinear elliptic equations. (English)

J. Geom. Phys. 5, No.2, 237-275 (1988). ISSN 0393-0440
http://dx.doi.org/10.1016/0393-0440(88)90006-X
http://www.sciencedirect.com/science/journal/03930440

This paper is concerned with various qualitative properties of solutions of semilinear or quasilinear second order elliptic equations. These properties include symmetry, antisymmetry and monotonicity properties. Proofs rely upon the so-called moving planes method and involve various extensions or variants of the work by B. Gidas, W. M. Ni and L. Nirenberg.

P.-L.Lions

Keywords : semilinear; quasilinear; symmetry; antisymmetry; monotonicity; moving planes method
Classification :
∗35B99 Qualitative properties of solutions of PDE
35J60 Nonlinear elliptic equations
35K55 Nonlinear parabolic equations
35B50 Maximum principles (PDE)

Zbl 0685.35045

Nirenberg, L.

Fully nonlinear elliptic equations. (English)

The mathematical heritage of Hermann Weyl, Proc. Symp., Durham/NC 1987, Proc. Symp. Pure Math. 48, 217-225 (1988).

[For the entire collection see Zbl 0644.00001.]
Ausgehend vom Weylschen Einbettungsproblem wird das Dirichlet-Problem für die allgemeine nichtlineare elliptische Differentialgleichung $F(x, u, Du, D^2u) = 0$ für Gebiete $\Omega \subset \mathbb{R}^n$ $(n > 2)$ betrachtet.
Wesentlich für die Anwendung der Kontinuitätsmethode ist die Gewinnung von a priori Schranken für die C^2- und die $C^{2,\mu}$-Normen der Lösung u in Ω bzw. $\bar{\Omega}$. Im allgemeinen Fall ist die Frage, ob eine C^2-Abschätzung immer eine $C^{2,\mu}$-Abschätzung impliziert, offen.
In dem folgenden Übersichtsartikel werden für spezielle Formen von F eine Reihe von Beiträgen zu diesem Thema diskutiert. Es handelt sich dabei vor allem um neuere Ergebnisse von Caffarelli, Nirenberg und Spruck. Diese beziehen sich auf den Fall, wo Funktionen der Eigenwerte der Hesseschen Matrix $\{u_{jk}\}$ bzw. der Hauptkrümmungen der Hyperfläche (x,u(x)) vorgegeben sind. Eine wesentliche Voraussetzung an F ist die Konkavität bezüglich $\{D^2u\}$.

E.Heinz

Keywords : Weyl embedding problem; continuity method; concave pde; principal; curvature
Classification :
∗35J65 (Nonlinear) BVP for (non)linear elliptic equations
35B45 A priori estimates
35J60 Nonlinear elliptic equations
35J25 Second order elliptic equations, boundary value problems
53A05 Surfaces in Euclidean space
35A07 Local existence and uniqueness theorems (PDE)
35B50 Maximum principles (PDE)
53C45 Global surface theory (a la A.D. Aleksandrov)
35J20 Second order elliptic equations, variational methods
35B60 Continuation of solutions of PDE
35M99 PDE of special type

Zbl 0672.35028

Caffarelli, Luis; Nirenberg, Louis; Spruck, Joel

Nonlinear second-order elliptic equations. V: The Dirichlet problem for Weingarten hypersurfaces. (English)

Commun. Pure Appl. Math. 41, No.1, 47-70 (1988). ISSN 0010-3640
http://dx.doi.org/10.1002/cpa.3160410105
http://onlinelibrary.wiley.com/journal/10.1002/%28ISSN%291097-0312

[For part IV see the preceding review.]
Here is studied the Dirichlet problem for a function u in a bounded domain Ω in $\mathbb{R}^n$ with smooth strictly convex boundary $\partial\Omega$. At any point x in Ω the principal curvatures $\kappa = (\kappa_1, ..., \kappa_n)$ of the graph (x,u(x)) satisfy a relation (1) $f(\kappa_1, ..., \kappa_n) = \psi(x) > 0$, where ψ is a given smooth positive function on $\bar{\Omega}$. The function u satisfies the Dirichlet boundary condition (2) $u = 0 \quad on \quad \partial\Omega$.

The existence and the uniqueness of the solution of (1), (2) with some special properties is proved under appropriate assumptions on f.

P.Drábek

Keywords : Weingarten hypersurfaces; Dirichlet problem; smooth strictly convex boundary; principal curvatures; Dirichlet boundary condition; existence; uniqueness

Classification :

∗35J65 (Nonlinear) BVP for (non)linear elliptic equations
35B65 Smoothness of solutions of PDE
35A05 General existence and uniqueness theorems (PDE)
53A05 Surfaces in Euclidean space

Zbl 0668.35028

Caffarelli, L.; Nirenberg, L.; Spruck, J.

On a form of Bernstein's theorem. (English)

Analyse mathématique et applications, Contrib. Honneur Jacques-Louis Lions, 55-66 (1988).

[For the entire collection see Zbl 0651.00008.]

Die Autoren zeigen, daß jede glatte Funktion in $\mathbb{R}^n$, die die Wachstumsbedingung $\nabla u(x) = o(|x|^{1/2})$ für $|x| \to \infty$ erfüllt, und deren Graph die mittlere Krümmung Null besitzt, eine affine Funktion ist.

W.Wendt

Keywords : smooth function; growth condition; mean curvature; affine function

Classification :

∗35J60 Nonlinear elliptic equations
35J15 Second order elliptic equations, general

Zbl 0641.35025

Caffarelli, L.; Nirenberg, L.; Spruck, J.

Correction to: The Dirichlet problem for nonlinear second-order elliptic equations. I. Monge-Ampère equation. (English)

Commun. Pure Appl. Math. 40, 659-662 (1987). ISSN 0010-3640

http://dx.doi.org/10.1002/cpa.3160400508

http://onlinelibrary.wiley.com/journal/10.1002/%28ISSN%291097-0312

Correction to the authors' paper [ibid. 37, 369-402 (1984; Zbl 0598.35047)].

Keywords : Dirichlet problem; second-order; Monge-Ampère equation

Classification :

∗35J65 (Nonlinear) BVP for (non)linear elliptic equations
35J25 Second order elliptic equations, boundary value problems

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Zbl 0611.35029

Caffarelli, L.; Nirenberg, L.; Spruck, J.

The Dirichlet problem for the degenerate Monge-Ampère equation. (English)

Rev. Mat. Iberoam. 2, No.1-2, 19-27 (1987). ISSN 0213-2230
http://projecteuclid.org/rmi
http://www.uam.es/departamentos/ciencias/matematicas/ibero/irevista.htm

The authors study the problem of finding a convex function u in Ω such that

$$(1) \quad \det(u_{ij}) = 0 \quad in \quad \Omega; \quad (2) \quad u = \phi \text{ given on } \partial\Omega,$$

where Ω is a bounded convex domain in R^n with smooth, strictly convex boundary $\partial\Omega$ and $u_i = \partial u/\partial x_i$, $u_{ij} = \partial^2 u/\partial x_i \partial x_j$ etc. The existence of a smooth solution in $\bar\Omega$ satisfying (2) of the corresponding elliptic problem

$$(1)' \quad \det(u_{ij}) = \psi > 0 \quad in \quad \Omega,$$

has been recently shown by *N. V. Krylov* [Math. USSR, Izv. 22, 67-97 (1984); translation from Izv. Akad. Nauk SSSR, Ser. Mat. 47, No.1, 75-108 (1983; Zbl 0578.35024)] and the authors [Commun. Pure Appl. Math. 37, 369-402 (1984; Zbl 0598.35047)] in case ψ and ϕ are sufficiently smooth. It is interesting to treat the degenerate problem (1), (2). The corresponding question for degenerate complex Monge-Ampère equation to find a plurisubharmonic function w in a bounded pseudoconvex domain Ω in $\mathbb{C}^n$ satisfying

$$(3) \quad \det(w_{z_j \bar z_k}) = 0 \quad in \quad \Omega,$$

and (2) is also interesting. The authors with *J. J. Kohn* [Commun. Pure Appl. Math. 38, 209-252 (1985; Zbl 0598.35048)] treated the equation

$$(3)' \quad \det(w_{z_j \bar z_k}) = \psi > 0 \quad in \quad \Omega,$$

and showed that there is a plurisubharmonic solution w belonging to $C^{1,1}(\Omega)$, provided $\psi \not\equiv 0$, ψ satisfies some other conditions, and ψ and ϕ are sufficiently smooth.
In the unique solution of (1), (2) is given by

$$(4) \quad u(x) = \max\{v(x) | \quad v \in C(\bar\Omega), \text{ v convex and } v \le \phi \text{ on } \partial\Omega\},$$

and several authors have studied the regularity of u. The authors prove an extension up to the boundary of the regularity in case ϕ is sufficiently smooth.

S.D.Bajpai

Keywords : existence; smooth solution; degenerate; complex Monge-Ampère equation; plurisubharmonic function; pseudoconvex; regularity

Classification :
 ∗35J65 (Nonlinear) BVP for (non)linear elliptic equations
 35J70 Elliptic equations of degenerate type
 35D05 Existence of generalized solutions of PDE
 35D10 Regularity of generalized solutions of PDE
 32U05 Plurisubharmonic functions and generalizations

Zbl 0672.35027

Caffarelli, L.; Nirenberg, L.; Spruck, J.

Nonlinear second order elliptic equations. IV. Starshaped compact Weingarten hypersurfaces. (English)

Current topics in partial differential equations, Pap. dedic. S. Mizohata Occas. 60th Birthday, 1-26 (1986).

[For the entire collection see Zbl 0604.00006.]
[For Part III see Acta Math. 155, 261-301 (1985; Zbl 0654.35031).]
The existence of the embedded Weingarten surface Y: $S^n \to R^{n+1}$ is studied, the principal curvatures $[k_1, ..., k_n]$ of which satisfy a relation (1) $f(-k_1, ..., -k_n) = \psi(Y)$. Under the suitable assumptions on f and ψ, the localization of Y as a graph of function v (i.e., $Y = [x, v(x)]$, $x = [x_1, ..., x_n]$, $x_{n+1} = v(x)$), transform (1) to the elliptic equation $G(Dv, D^2v) = \psi(x, v)$ (Section 1).
It is proved that there exists a C^∞-surface which solves (1), as well as the fact that any two solutions are endpoints of a one- parameter family of homothetic dilations, all of which are solutions (Theorem 1). The proof of this result is given by the continuity method (Section 2), which is based on a priori estimates, established in Sections 3,4.

O.John

Keywords : existence; embedded Weingarten surface; principal curvatures; homothetic dilations; continuity method; a priori estimates

Classification :

∗35J65 (Nonlinear) BVP for (non)linear elliptic equations
35B65 Smoothness of solutions of PDE
35A05 General existence and uniqueness theorems (PDE)
53A05 Surfaces in Euclidean space

Zbl 0654.35031

Caffarelli, L.; Nirenberg, L.; Spruck, J.

The Dirichlet problem for nonlinear second order elliptic equations. III: Functions of the eigenvalues of the Hessian. (English)

Acta Math. 155, 261-301 (1985). ISSN 0001-5962; ISSN 1871-2509
http://dx.doi.org/10.1007/BF02392544
http://www.springerlink.com/openurl.asp?genre=journalissn=0001-5962
http://www.actamathematica.org/

This paper is a continuation of parts I and II [Commun. Pure Appl. Math. 37, 369-402 (1984; Zbl 0598.35047) and 38, 209-252 (1985; Zbl 0598.35048)]. Here is studied the solvability of Dirichlet's problem in a bounded domain $\Omega \subset R^n$ with smooth boundary $\partial\Omega$:

$$F(D^2u) = \psi \quad in \quad \Omega; \quad u = \phi \quad on \quad \partial\Omega,$$

where the function F is defined by a smooth symmetric function $f(\lambda_1, ..., \lambda_n)$ of the eigenvalues $\lambda = (\lambda_1, ..., \lambda_n)$ of the Hessian matrix $D^2u = \{u_{ij}\}$. It is assumed that the equation is elliptic, i.e. $\partial f/\partial x_i > 0$, for all i, and that f is a concave function.

P.Drábek

Keywords : existence; multiplicity; Dirichlet's problem; bounded domain; smooth boundary; eigenvalues; Hessian matrix

Classification :

∗35J65 (Nonlinear) BVP for (non)linear elliptic equations
35A05 General existence and uniqueness theorems (PDE)
35J25 Second order elliptic equations, boundary value problems

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Zbl 0598.35048

Caffarelli, L.; Kohn, J.J.; Nirenberg, Louis; Spruck, J.

The Dirichlet problem for nonlinear second-order elliptic equations. II: Complex Monge-Ampère, and uniformly elliptic, equations. (English)

Commun. Pure Appl. Math. 38, 209-252 (1985). ISSN 0010-3640
http://dx.doi.org/10.1002/cpa.3160380206
http://onlinelibrary.wiley.com/journal/10.1002/%28ISSN%291097-0312

[For Part I, see ibid. 37, 369-402 (1984; Zbl 0598.35047).]
This is the second paper in a series of three papers devoted to the Dirichlet problem for second-order nonlinear elliptic equations. The third one will appear in Acta Mathematica. In this paper the authors treat the problem

$$F(x,u,Du,D^2u)=0 \quad in \quad \Omega, \quad u=\phi \quad on \quad \partial\Omega.$$

The function F is smooth for $x \in \bar{\Omega}$ in all the arguments,

$$\sum(\partial F/\partial u_{ij})\xi_i\xi_j > 0 \quad for \quad \xi=(\xi_1,..,\xi_n)\neq 0,$$

and F is a concave function of the second derivatives $\{u_{ij}\}$. (1) The paper contains three sections. In the first section the C^2 a priori estimates for elliptic complex Monge-Ampère equations are derived. The principal contribution of the second section is the derivation of a logarithmic modulus of continuity of u_{ij} near the boundary. The last section is a self-contained treatment of a rather general class of "uniformly elliptic" operators satisfying (1). It is worth mentioning that this paper is in close relation to works of N. V. Krylov and N. S. Trudinger which are stated in the references.

P.Drabek

Keywords : Dirichlet problem; uniformly elliptic equations; elliptic complex Monge-Ampère equations; logarithmic modulus of continuity

Classification :

∗35J65 (Nonlinear) BVP for (non)linear elliptic equations
35B50 Maximum principles (PDE)
35A05 General existence and uniqueness theorems (PDE)
35B45 A priori estimates

Zbl 0572.35043

Nirenberg, Louis

Uniqueness in the Cauchy problem for a degenerate elliptic second order equation. (English)

Differential geometry and complex analysis, Vol. dedic. H. E. Rauch, 213-218 (1985).

[For the entire collection see Zbl 0561.00010.]
Let $u \in C_2(\bar{\Omega})$ be a solution of a degenerate elliptic equation $Pu = -(a_{ij}u_{x_i})_{x_j} + a_i u_{x_i} + cu = 0$ (with a_{ij} differentiable and positive semidefinite) with zero Cauchy data on $\partial\Omega$ near some point $x_0 \in \partial\Omega$. Suppose there holds a Levi-condition $|\sum_i a_i\xi_i|^2 \leq C\sum_{i,j} a_{ij}\xi_i\xi_j$ and a kind of pseudo-convexity condition at x_0. Then it is shown that u vanishes near x_0.

M. Wiegner

Keywords : uniqueness; Cauchy data; Levi-condition; pseudo-convexity

Classification :
- ∗35J70 Elliptic equations of degenerate type
- 35A05 General existence and uniqueness theorems (PDE)
- 32T99 Pseudoconvex domains

Zbl 0598.35047

Caffarelli, L.; Nirenberg, Louis; Spruck, J.

The Dirichlet problem for nonlinear second-order elliptic equations. I: Monge-Ampère equation. (English)

Commun. Pure Appl. Math. 37, 369-402 (1984). ISSN 0010-3640
http://dx.doi.org/10.1002/cpa.3160370306
http://onlinelibrary.wiley.com/journal/10.1002/%28ISSN%291097-0312

In der vorliegenden Arbeit behandeln die Verff. in einem beschränkten, strikt konvexen Gebiet $\Omega \subset \mathbb{R}^n$ mit C^∞- Rand $\partial\Omega$ die Monge-Ampèresche Gleichung

$$\det(u_{ij}) = \psi \quad mit \quad u|\partial\Omega = \phi|\Omega.$$

Hierbei sind $u_{ij} := \partial_i\partial_j u$, $\psi \in C^\infty(\bar{\Omega})$, $\psi > 0$ und $\phi \in C^\infty(\bar{\Omega})$. Es wird die eindeutige Existenz der Lösung u des Dirichletproblems in der Klasse der strikt konvexen Funktionen und $u \in C^\infty(\bar{\Omega})$ gezeigt.
Zum Beweis wird die Kontinuitätsmethode benutzt. Dazu wird eine a priori-Abschätzung von der Form $|u|_{2+\alpha} \leq K(\Omega, \psi, \phi)$ benötigt. Der Nachweis dieser Abschätzung ist wesentlicher Inhalt dieser Arbeit. Dazu werden das Maximumsprinzip und einseitige Abschätzungen der dritten Ableitungen bis zum Rand verwandt. Abschließend werden auch allgemeinere Monge-Ampère-Gleichungen untersucht.

R. Leis

Keywords : Dirichlet problem; strictly convex domain; Monge-Ampère equations; continuation method

Classification :
∗35J65 (Nonlinear) BVP for (non)linear elliptic equations
35B50 Maximum principles (PDE)
35A05 General existence and uniqueness theorems (PDE)
35B45 A priori estimates

Zbl 0563.46024

Caffarelli, L.; Kohn, R.; Nirenberg, Louis
First order interpolation inequalities with weights. (English)
Compos. Math. 53, 259-275 (1984). ISSN 0010-437X; ISSN 1570-5846
numdam:CM_1984__53_3_259_0
http://www.journals.cambridge.org/journal_CompositioMathematica

The authors prove a necessary and sufficient condition for there to exist a constant C such that for each $u \in C_0^\infty\ (R^n)$,

$$\||x|^\gamma u\|_{L^r} \leq C\||x|^\alpha |Du|\|^a_{L^p}\||x|^\beta u\|^{1-a}_{L^q},$$

where α, β, γ, a, r, p, q, and n are fixed real numbers satisfying a number of specified relationships. Special cases of this inequality have appeared in a number of papers, including a previous paper of the authors [Comm. Pure Appl. Math. 35, 771-831 (1982; Zbl 0509.35067)] and a paper of *B. Muckenhoupt* and *R. Wheeden* [Trans. Am. Math. Soc. 192, 261-274 (1974; Zbl 0289.26010)]. The proof is lengthy but elementary, and consists of verifying a large number of cases.

P.Lappan
Classification :
∗46E35 Sobolev spaces and generalizations
26D10 Inequalities involving derivatives, diff. and integral operators
46M35 Abstract interpolation of topological linear spaces
26D20 Analytical inequalities involving real functions

Zbl 0561.53001

Nirenberg, Louis
The work of Yau, Shing-Tung. (English)
Proc. Int. Congr. Math., Warszawa 1983, Vol. 1, 15-19 (1984).

[For the entire collection see Zbl 0553.00001.]
Report on the work of Shing-Tung Yau including 16 references up to 1982.

Keywords : Calabi conjecture; positive mass conjecture; Monge-Ampere equation; elliptic equations
Classification :
∗53-02 Research monographs (differential geometry)
58-02 Research monographs (global analysis)
01A60 Mathematics in the 20th century

Zbl 0598.35046

Nirenberg, Louis

The Dirichlet problem for the Monge-Ampère equation. (English)

Methods of functional analysis and theory of elliptic equations, Proc. Int. Meet. dedic. Mem. C. Miranda, Naples/Italy 1982, 193-198 (1983).

[For the entire collection see Zbl 0583.00017.]

This talk is concerned with the Dirichlet problem for elliptic Monge- Ampère equations of the form

$$(1)\quad \det(u_{ij}) = \psi(x) > 0 \quad in \quad \Omega; \quad (2) \quad u = \phi \quad on \quad \partial\Omega.$$

Here Ω is a bounded convex domain in R^n, $n > 3$, with C^∞ strictly convex boundary. One seeks a strictly convex function u in Ω whose Hessian matrix $\{u_{ij}\} = \{u_{x_i x_j}\}$ satisfies (1), where ψ (x) is a given C^∞ positive function in $\bar{\Omega}$.

Keywords : Dirichlet problem; Monge-Ampère equations

Classification :

∗35J65 (Nonlinear) BVP for (non)linear elliptic equations

35A05 General existence and uniqueness theorems (PDE)

Zbl 0541.35029

Brézis, Haïm; Nirenberg, Louis

Positive solutions of nonlinear elliptic equations involving critical Sobolev exponents. (English)

Commun. Pure Appl. Math. 36, 437-477 (1983). ISSN 0010-3640

http://dx.doi.org/10.1002/cpa.3160360405

http://onlinelibrary.wiley.com/journal/10.1002/%28ISSN%291097-0312

Semilinear elliptic equations involving critical Sobolev exponents were considered being hard to attack because of the lack of compactness. Indeed the well known nonexistence results of Pokhozaev asserts that for a starshaped domain, there is no nontrivial solution for the BVP with critical Sobolev power function as nonlinear term. Surprisingly, it is proved in this paper that the lower term can reverse this situation.

The method used here is essentially close to that employed in Yamabe's problem by *Th. Aubin* [J. Math. Pures Appl., IX. Sér. 55, 269-296 (1976; Zbl 0336.53033)]. Namely, a versin of the mountain pass theorem without the Palais-Smale condition is applied. The decisive device in order to overcome this lack of compactness is to estimate the mountain pass value by a number associated with the best Sobolev constant. The following typical example is discussed in this paper: (∗) $\quad -\Delta u = u^p + \mu \quad u^q$ on Ω, $u > 0$ on Ω, $u = 0$ on $\partial\Omega$, $n = \dim\Omega$, where $p = (n+2)/(n-2)$, $1 < q < p$ and $\mu > 0$ is a constant. When $n \geq 4$, (*) has a solution for every $\mu > 0$. When $n = 3$, (a) if $3 < q < 5$ (*) has a solution for every $\mu > 0$; (b) if $1 < q \leq 3$ (*) possesses a solution only for $\mu \geq some\mu_0 > 0$. However, in case $1 < q \leq 3$, the problem is left open for $\mu < \mu_0$.

K.Chang
Keywords : positive solutions; best Sobolev constant; isoperimetric inequality; limiting Sobolev exponent; Semilinear elliptic equations; critical Sobolev exponents; mountain pass theorem
Classification :
∗35J60 Nonlinear elliptic equations
35J20 Second order elliptic equations, variational methods
35A05 General existence and uniqueness theorems (PDE)

Zbl 0528.49006

Nirenberg, Louis
On some variational methods. (English)
Bifurcation theory, mechanics and physics, Proc. Colloq., 169-176 (1983).

Keywords : minimax method; Palais-Smale condition; mountain pass theorem; nonlinear Dirichlet problem
Classification :
∗49J45 Optimal control problems inv. semicontinuity and convergence
58E30 Variational principles on infinite-dimensional spaces
58E05 Abstract critical point theory
49J10 Free problems in several independent variables (existence)
49J20 Optimal control problems with PDE (existence)
49J35 Minimax problems (existence)
49Q20 Variational problems in geometric measure-theoretic setting
35J60 Nonlinear elliptic equations
58J32 Boundary value problems on manifolds

Zbl 0524.47041

Nirenberg, Louis
Variational and topological methods in nonlinear problems. (English)
The mathematical heritage of Henri Poincare, Proc. Symp. Pure Math. 39, Part 2, Bloomington/Indiana 1980, 89-124 (1983).

Keywords : existence; homotopy; topological degree; variational methods; stationary point; perturbation about a solution; Leray-Schauder degree theory; Fredholm maps; Palais-Smale condition; mountain pass lemma; Nash Moser implicit function technique
Classification :
∗47J05 Equations involving nonlinear operators (general)
47A53 (Semi-)Fredholm operators; index theories
49J40 Variational methods including variational inequalities
49J35 Minimax problems (existence)
35L20 Second order hyperbolic equations, boundary value problems
35J65 (Nonlinear) BVP for (non)linear elliptic equations
35B32 Bifurcation (PDE)
35B10 Periodic solutions of PDE

35A05 General existence and uniqueness theorems (PDE)
58E07 Abstract bifurcation theory
37J99 Finite-dimensional Hamiltonian etc. systems
58J45 Hyperbolic equations
58E15 Appl. of variational methods to extremal problems in sev.variables

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Zbl 0567.35033

Nirenberg, Louis

Elliptic equations with critical nonlinear exponent. (English)

Rend. Sem. Mat. Fis. Milano 52, 187-191 (1982). ISSN 0370-7377
http://dx.doi.org/10.1007/BF02925007
http://www.springerlink.com/content/109676/

From the author's summary: The positive solutions of a semilinear second order elliptic equation with critical nonlinear exponent are studied and existence and nonexistence theorems of a solution are given.

M.Biroli

Keywords : positive solutions; semilinear second order elliptic equation; critical nonlinear exponent; existence; nonexistence

Classification :

∗35J60 Nonlinear elliptic equations
35A15 Variational methods (PDE)
47J05 Equations involving nonlinear operators (general)

Zbl 0509.35067

Caffarelli, L.; Kohn, R.; Nirenberg, Louis

Partial regularity of suitable weak solutions of the Navier-Stokes equations. (English)

Commun. Pure Appl. Math. 35, 771-831 (1982). ISSN 0010-3640
http://dx.doi.org/10.1002/cpa.3160350604
http://onlinelibrary.wiley.com/journal/10.1002/%28ISSN%291097-0312

Keywords : partial regularity; weak solutions; singular points

Classification :

∗35Q30 Stokes and Navier-Stokes equations
35D10 Regularity of generalized solutions of PDE
35A20 Analytic methods (PDE)
76D05 Navier-Stokes equations (fluid dynamics)

Zbl 0544.58005

Nirenberg, Louis (Lin, Changshou)

Comments on nonlinear problems (with an appendix by Lin Changshou). (English)

Matematiche 36, 109-119 (1981). ISSN 0373-3505
http://www.dmi.unict.it/lematematiche/index.php?p=journal

The author gives an elegant proof of an S^1 version of the Borsuk- Ulam theorem. In the proof only the Brouwer degree and the transversality lemma were applied; but in previous proofs [*V. Benci*, Commun. Pure Appl. Math. 34, 393-432 (1981; Zbl 0447.34040); *E. Fadell, S. Husseini* and *P. Rabinowitz*, MRC Report (1981)], more complicate algebraic topology machinery was needed.
Let Ω be an open bounded neighbourhood of the origin in $R = C^a \times R^b$, $n = 2a + b$, with coordinates $z = (z', z'')$, $z' = (z_1, ..., z_a)$, $z'' = (z_{a+1}, ..., z_{a+b})$, z_α real for $\alpha < a$. For all real θ, consider the S^1-group action

$$z \mapsto T_\theta z = (e^{im_1\theta} z_1, ..., e^{im_a\theta} z_a, z_{a+1}, ..., z_{a+b}),$$

where the m_j are integers. The main theorem is the following: Let $f : \partial\Omega \to C^a \times R^b \backslash \{\theta\}$ be continuous. Assume that $f_j(T_\theta z) = e^{ik_j\theta} f_j(z)$, $k_j = integer \neq 0$, $j \leq a$; $f_\alpha(T_\theta z) = f_\alpha(z)$, real, $a < \alpha \leq a + b$; and that $z = (0, z'') \in \partial\Omega$, $f_\alpha(z) = z_\alpha$ for $a < \alpha$. Then $\deg(f, \Omega, \theta) = \prod_1^a (k_j/m_j)$. The proof depends on a beautiful application of the transversality lemma.
In the appendix, in combining this theorem with the Benci index, C. S. Lin gives a very simple proof of a basic property for maps which are equivariant under S^1-action, due to Fadell, Husseini and Rabinowitz.

K.Chang

Keywords : Borsuk-Ulam theorem; Brouwer degree; transversality
Classification :
∗58E05 Abstract critical point theory
55M25 Degree, etc.

Zbl 0492.35061

Nirenberg, Louis

Remarks on the Navier-Stokes equations. (English)

Journ. Équ. Dériv. Partielles, Saint-Jean-de-Monts 1981, Exp. No.13, 4 p. (1981).
numdam:JEDP_1981____A13_0

Keywords : three dimensions; incompressible Navier-Stokes equations
Classification :
∗35Q30 Stokes and Navier-Stokes equations
76D05 Navier-Stokes equations (fluid dynamics)
35D05 Existence of generalized solutions of PDE

Zbl 0477.35002

Nirenberg, Louis
Variational methods in nonlinear problems. (English)
Sémin. Goulaouic-Meyer-Schwartz 1980-1981, Équat. dériv. part., Exposé No.20, 5 p. (1981).
numdam:SEDP_1980-1981____A22_0

Keywords : survey; Mountain Pass Lemma; nonlinear string equation with a monotonic nonlinearity
Classification :
∗35A15 Variational methods (PDE)
35B10 Periodic solutions of PDE
58E05 Abstract critical point theory
47H05 Monotone operators (with respect to duality)
35L70 Second order nonlinear hyperbolic equations

Zbl 0469.35052

Gidas, B.; Ni, Wei-Ming; Nirenberg, Louis
Symmetry of positive solutions of nonlinear elliptic equations in $\mathbb{R}^n$. (English)
Adv. Math., Suppl. Stud. 7A, 369-402 (1981).

Keywords : symmetry of positive solutions; nonlinear elliptic equations; isolated singularities
Classification :
∗35J60 Nonlinear elliptic equations
35B40 Asymptotic behavior of solutions of PDE
35A20 Analytic methods (PDE)

Zbl 0468.47040

Nirenberg, Louis
Variational and topological methods in nonlinear problems. (English)
Bull. Am. Math. Soc., New Ser. 4, 267-302 (1981). ISSN 0273-0979; ISSN 1088-9485
http://dx.doi.org/10.1090/S0273-0979-1981-14888-6
http://www.ams.org/bull/
http://ProjectEuclid.org/bams

Keywords : nonlinear problems; degree of the mapping; minimax problems; bifurcation theory; implicit function theorem; Fredholm operators
Classification :
∗47J05 Equations involving nonlinear operators (general)
35J60 Nonlinear elliptic equations
58E07 Abstract bifurcation theory
49J35 Minimax problems (existence)

47A53 (Semi-)Fredholm operators; index theories

Zbl 0484.35057

Brézis, Haïm; Coron, Jean-Michel; Nirenberg, Louis
Free vibrations for a nonlinear wave equation and a theorem of P. Rabinowitz. (English)
Commun. Pure Appl. Math. 33, 667-684 (1980). ISSN 0010-3640
http://dx.doi.org/10.1002/cpa.3160330507
http://onlinelibrary.wiley.com/journal/10.1002/%28ISSN%291097-0312

Keywords : Palais-Smale condition; Dirichlet boundary conditions; mountain pass theorem
Classification :
∗35L70 Second order nonlinear hyperbolic equations
35B10 Periodic solutions of PDE

Zbl 0454.47051

Nirenberg, Louis
Remarks on nonlinear problems. (English)
Differential geometry, Proc. int. Chern Symp., Berkeley 1979, 189-197 (1980).

Keywords : degree theory; variational methods
Classification :
∗47J25 Methods for solving nonlinear operator equations (general)

Zbl 0436.32018

Nirenberg, Louis; Webster, S.; Yang, P.
Local boundary regularity of holomorphic mappings. (English)
Commun. Pure Appl. Math. 33, 305-338 (1980). ISSN 0010-3640
http://dx.doi.org/10.1002/cpa.3160330306
http://onlinelibrary.wiley.com/journal/10.1002/%28ISSN%291097-0312

Keywords : biholomorphic map; pseudoconvex boundary; reflection principle; Kobayashi metric; real hypersurfaces; smoothness of continuous extension
Classification :
∗32H99 Holomorphic mappings on analytic spaces
32T99 Pseudoconvex domains
32F45 Invariant metrics and pseudodistances
32D15 Continuation of analytic objects (several variables)
32V40 Real submanifolds in complex manifolds

Zbl 0438.35059

Nirenberg, Louis
The use of topological, functional analytic, and variational methods in non-linear problems. (English)
Conf. Semin. Mat. Univ. Bari 163-168, 391-398 (1979).

Keywords : topological methods; monotone operators; compact inverse; weak solvability; nonlinear vibrating string equation
Classification :
∗35R20 Partial operator-differential equations
35L70 Second order nonlinear hyperbolic equations
47H05 Monotone operators (with respect to duality)
47J05 Equations involving nonlinear operators (general)

Zbl 0433.53002

(Kotake, Takeshi; Ochiai, Takushiro; Bourguignon, Jean-Pierre; Cheng, S.Y.; Hitchin, Nigel J.; Inonue, A.; Kazdan, Jerry L.; Koiso, N.; Lemaire, L.R.; Nirenberg, L.; Nishikawa, S.; Omori, H.; Schoen, Richard; Sunada, T.; Tanno, S.; Yau, Shing-Tung; Ueno, K.)
Non-linear problems in geometry. Conference held at Katata, September 3-8, 1979. (English)
Proceedings of the sixth international Symposium, Division of Mathematics, The Taniguchi Foundation. Sendai/Japan: Tohoku University, Department of Mathematics. V, 23 p. (1979).

Keywords : Geometry; Conference; Proceedings; Symposium; Katata/Japan; collection of open problems; Einstein metrics; Young-Mills functional; hermitian metrics; conformally flat; eigenvalues of differential operators; Ricci curvature; energy of maps; harmonic maps; pseudo- differential operators; minimal surfaces; Laplacian
Classification :
∗53-02 Research monographs (differential geometry)

Zbl 0425.35097

Kinderlehrer, D.; Nirenberg, Louis; Spruck, J.
Regularity in elliptic free boundary problems. II: Equations of higher order. (English)
Ann. Sc. Norm. Super. Pisa, Cl. Sci., IV. Ser. 6, 637-683 (1979).
numdam:ASNSP_1979_4_6_4_637_0
http://www.sns.it/html/ClasseScienze/pubsci/

Keywords : free boundary; hodograph; Legendre transform; regularity; non-linear elliptic equation; overdetermined elliptic systems; analyticity of solutions
Classification :
∗35R35 Free boundary problems for PDE

35A22 Transform methods (PDE)
35D10 Regularity of generalized solutions of PDE
35J35 Higher order elliptic equations, variational problems
35N10 Overdetermined systems of PDE with variable coefficients, general

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Zbl 0425.35020

Gidas, B.; Ni, Wei-Ming; Nirenberg, Louis

Symmetry and related properties via the maximum principle. (English)

Commun. Math. Phys. 68, 209-243 (1979). ISSN 0010-3616; ISSN 1432-0916
http://dx.doi.org/10.1007/BF01221125
http://link.springer.de/link/service/journals/00220/
http://projecteuclid.org/DPubS?service=UIversion=1.0verb=Displaypage=pasthandle=euclid.cm

Keywords : symmetry; positive solutions of second order elliptic equations; maximum principle

Classification :

∗35B50 Maximum principles (PDE)
35J15 Second order elliptic equations, general

Zbl 0456.35090

Kinderlehrer, David; Nirenberg, Louis

Hodograph methods and the smoothness of the free boundary in the one phase Stefan problem. (English)

Moving boundary problems, Proc. Symp. Gatlinburg/Tenn. 1977, 57-69 (1978).

Keywords : free boundary problems; regularity properties; Stefan problem; parabolic variational inequalities; Gevrey class; analyticity of the boundary data; analyticity of the free surface; hodograph; Legendre transform

Classification :

∗35R35 Free boundary problems for PDE
35K99 Parabolic equations and systems
35B65 Smoothness of solutions of PDE
35A22 Transform methods (PDE)
35K05 Heat equation
49J40 Variational methods including variational inequalities

Zbl 0402.35045

Kinderlehrer, D.; Nirenberg, Louis; Spruck, J.

Regularity in elliptic free boundary problems. I. (English)

J. Anal. Math. 34, 86-119 (1978). ISSN 0021-7670; ISSN 1565-8538
http://dx.doi.org/10.1007/BF02790009
http://www.springerlink.com/content/120600/

Keywords : Regularity of Elliptic Systems; Free Hypersurface; Coerciveness; Elliptic Boundary Value Problems; Plasma Containment; Membranes; Liquid Edge; Minimal Surfaces

Classification :

∗35J55 Systems of elliptic equations, boundary value problems
35A22 Transform methods (PDE)
35D10 Regularity of generalized solutions of PDE
35J60 Nonlinear elliptic equations

Zbl 0391.35060

Kinderlehrer, David; Nirenberg, Louis

The smoothness of the free boundary in the one phase Stefan problem. (English)

Commun. Pure Appl. Math. 31, 257-282 (1978). ISSN 0010-3640
http://dx.doi.org/10.1002/cpa.3160310302
http://onlinelibrary.wiley.com/journal/10.1002/%28ISSN%291097-0312

Keywords : One Phase Stefan Problem; Free Boundary; Variational Inequalities; Inhomogeneous Heat Equation

Classification :

∗35R35 Free boundary problems for PDE
35K20 Second order parabolic equations, boundary value problems
35B30 Dependence of solutions of PDE on initial and boundary data
35K55 Nonlinear parabolic equations
49J40 Variational methods including variational inequalities

Zbl 0391.35045

Kinderlehrer, David; Nirenberg, Louis

Analyticity at the boundary of solutions of nonlinear second-order parabolic equations. (English)

Commun. Pure Appl. Math. 31, 283-338 (1978). ISSN 0010-3640
http://dx.doi.org/10.1002/cpa.3160310303
http://onlinelibrary.wiley.com/journal/10.1002/%28ISSN%291097-0312

Keywords : Local and Global Regularity; Solutions of Nonlinear Second-Order Parabolic Equations; Analyticity

Classification :

∗35K55 Nonlinear parabolic equations
35K20 Second order parabolic equations, boundary value problems
35G30 Boundary value problems for nonlinear higher-order PDE
35B30 Dependence of solutions of PDE on initial and boundary data
35B65 Smoothness of solutions of PDE

Zbl 0387.53023

Pogorelov, Aleksey Vasil'yevich (Nirenberg, L.)
The Minkowski multidimensional problem. Translated by Vladimir Oliker and introduced by Louis Nirenberg. (English)
Scripta Series in Mathematics. A Halstead Press Book. New York etc.: John Wiley & Sons. 106 p. £9.70; $ 17.50 (1978).

Classification :
∗53C45 Global surface theory (a la A.D. Aleksandrov)
35Q99 PDE of mathematical physics and other areas
35A30 Geometric theory for PDE, transformations

Zbl 0386.47035

Brézis, Haïm; **Nirenberg, Louis**
Characterizations of the ranges of some nonlinear operators and applications to boundary value problems. (English)
Ann. Sc. Norm. Super. Pisa, Cl. Sci., IV. Ser. 5, 225-326 (1978).
numdam:ASNSP_1978_4_5_2_225_0
http://www.sns.it/html/ClasseScienze/pubsci/

Classification :
∗47J05 Equations involving nonlinear operators (general)
47H05 Monotone operators (with respect to duality)
35J60 Nonlinear elliptic equations
35D05 Existence of generalized solutions of PDE
35L60 First-order nonlinear hyperbolic equations
35K55 Nonlinear parabolic equations

Zbl 0386.35045

Kinderlehrer, David; **Nirenberg, Louis**; **Spruck, Joel**
Régularité dans les problèmes elliptiques à frontière libre. (French)
C. R. Acad. Sci., Paris, Sér. A 286, 1187-1190 (1978).

Classification :
∗35R35 Free boundary problems for PDE
35J25 Second order elliptic equations, boundary value problems
35N99 Overdetermined systems of PDE
35D10 Regularity of generalized solutions of PDE

Zbl 0378.35040

Brézis, Haïm; **Nirenberg, Louis**
Forced vibrations for a nonlinear wave equation. (English)
Commun. Pure Appl. Math. 31, 1-30 (1978). ISSN 0010-3640

http://onlinelibrary.wiley.com/journal/10.1002/%28ISSN%291097-0312

Classification :

∗35L05 Wave equation
35L60 First-order nonlinear hyperbolic equations
35B10 Periodic solutions of PDE
35B45 A priori estimates

Zbl 0426.47034

Nirenberg, Louis

Topics in nonlinear functional analysis. (Lektsii po nelinejnomu funktsional'nomu analizu). Transl. from the English by N. D. Vvedenskaya. (Russian)
Matematika. Novoe v Zarubezhnoj Nauke. 5. Moskva: Izdatel'stvo "Mir". 232 p. R. 0.96 (1977).

Keywords : nonlinear functional analysis

Classification :

∗47-02 Research monographs (operator theory)
47H05 Monotone operators (with respect to duality)
47H10 Fixed point theorems for nonlinear operators on topol.linear spaces
47J05 Equations involving nonlinear operators (general)
26B10 Implicit function theorems, etc. (several real variables)
26E15 Calculus of functions on infinite-dimensional spaces
35A10 Cauchy-Kowalewski theorems
35G20 General theory of nonlinear higher-order PDE
58C30 Fixed point theorems on manifolds
58E05 Abstract critical point theory
58E07 Abstract bifurcation theory
55M20 Fixed points and coincidences (algebraic topology)
54H25 Fixed-point theorems in topological spaces

Zbl 0361.35012

Nirenberg, Louis

Regularity of free boundaries. (English)
Conf. Semin. Mat. Univ. Bari 145(1976), 9 P. (1977).

Classification :

∗35D10 Regularity of generalized solutions of PDE
35J15 Second order elliptic equations, general
35K10 Second order parabolic equations, general
35B30 Dependence of solutions of PDE on initial and boundary data

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Zbl 0359.47035

Brézis, Haïm; Nirenberg, Louis
Image d'une somme d'opérateurs non linéaires et applications. (French)
C. R. Acad. Sci., Paris, Sér. A 284, 1365-1368 (1977).

Classification :
∗47J05 Equations involving nonlinear operators (general)
47H05 Monotone operators (with respect to duality)

Zbl 0352.35023

Kinderlehrer, D.; Nirenberg, Louis
Regularity in free boundary problems. (English)
Ann. Sc. Norm. Super. Pisa, Cl. Sci., IV. Ser. 4, 373-391 (1977).
numdam:ASNSP_1977_4_4_2_373_0
http://www.sns.it/html/ClasseScienze/pubsci/

Classification :
∗35D10 Regularity of generalized solutions of PDE
35J25 Second order elliptic equations, boundary value problems
35K20 Second order parabolic equations, boundary value problems
35J60 Nonlinear elliptic equations

Zbl 0335.35028

Brézis, Haïm; Nirenberg, Louis
Some first order nonlinear equations on torus. (English)
Commun. Pure Appl. Math. 30, 1-11 (1977). ISSN 0010-3640
http://dx.doi.org/10.1002/cpa.3160300102
http://onlinelibrary.wiley.com/journal/10.1002/%28ISSN%291097-0312

Classification :
∗35F20 General theory of first order nonlinear PDE
35A05 General existence and uniqueness theorems (PDE)

Zbl 0357.35034

Nirenberg, Louis
Nonlinear differential equations invariant under certain geometric transformations. (English)
Symp. Math. 18, Trasform. quasiconf. Quest. connesse, Convegno 1974, 399-405 (1976).

Classification :
∗35J60 Nonlinear elliptic equations
35A22 Transform methods (PDE)

35A05 General existence and uniqueness theorems (PDE)

Zbl 0335.35081

Nirenberg, Louis

Propagation of singularities for linear partial differential equtions and reflections at a boundary. (English)

Sémin. Goulaouic-Schwartz 1974-1975, Équat. dériv. part. Anal. fonct., Exposé NO. XXVI, 18 p. (1976).

numdam:SEDP_1975-1976____A1_0

Classification :

∗35S15 Boundary value problems for pseudodifferential operators
35L50 First order hyperbolic systems, boundary value problems
35D99 Generalized solutions of PDE

Zbl 0335.35045

Nirenberg, Louis

Monge-Ampère equations and some associated problems in geometry. (English)

Proc. int. Congr. Math., Vancouver 1974, Vol. 2, 275-279 (1975).

Classification :

∗35J60 Nonlinear elliptic equations
53B10 Projective connections
53C55 Complex differential geometry (global)

Zbl 0311.35001

Nirenberg, Louis

Vorlesungen über lineare partielle Differentialgleichungen. (Russian)

Usp. Mat. Nauk 30, No.4(184), 147-204 (1975). ISSN 0042-1316

http://www.mathnet.ru/php/archive.phtml?jrnid=rmwshow=contentsoption_lang=rus

Classification :

∗35-02 Research monographs (partial differential equations)
35F05 General theory of first order linear PDE
35G05 General theory of linear higher-order PDE

Zbl 0306.35019

Nirenberg, Louis

On a problem of Hans Lewy. (English)

Fourier Integr. Oper. part. differ. Equat., Colloq. int. Nice 1974, Lect. Notes Math. 459, 224-234 (1975).

Classification :

∗35F05 General theory of first order linear PDE
35A05 General existence and uniqueness theorems (PDE)

Zbl 0305.35017

Nirenberg, Louis
On a question of Hans Lewy. (English. Russian original)
Russ. Math. Surv. 29, No.2, 251-262 (1974); translation from Usp. Mat. Nauk 29, No.2(176), 241-251 (1974). ISSN 0036-0279
http://dx.doi.org/10.1070/RM1974v029n02ABEH003856
http://www.turpion.org/php/homes/pa.phtml?jrnid=rm
http://www.iop.org/EJ/journal/0036-0279

Classification :

∗35F05 General theory of first order linear PDE
35A05 General existence and uniqueness theorems (PDE)
35R20 Partial operator-differential equations
47F05 Partial differential operators

Zbl 0298.35018

Loewner, Charles; Nirenberg, Louis
Partial differential equations invariant under conformal or projective transformations. (English)
Contribut. to Analysis, Collect. of Papers dedicated to Lipman Bers, 245-272 (1974).

Classification :

∗35J25 Second order elliptic equations, boundary value problems
35G05 General theory of linear higher-order PDE
35B45 A priori estimates
35J15 Second order elliptic equations, general
53A55 Differential invariants (local theory), geometric objects
53B20 Local Riemannian geometry

Zbl 0286.47037

Nirenberg, Louis
Topics in nonlinear functional analysis. Notes by R. A. Artino. (English)
New York: Courant Institute of Mathematical Sciences, New York University. VIII, 259 p. $ 6.75 (1974).

Classification :

∗47J05 Equations involving nonlinear operators (general)
47-02 Research monographs (operator theory)
55M25 Degree, etc.
45G10 Nonsingular nonlinear integral equations

35J65 (Nonlinear) BVP for (non)linear elliptic equations
47H05 Monotone operators (with respect to duality)
58C15 Implicit function theorems etc. on manifolds
58A10 Differential forms

Zbl 0283.00007

Ahlfors, Lars V. (ed.); Kra, Irwin (ed.); Maskit, Bernard (ed.); Nirenberg, Louis (ed.)
Contributions to analysis. A collection of papers dedicated to Lipman Bers. (English)
New York - London: Academic Press, a subsidiary of Harcourt Brace Jovanovich, Publishers. XVII, 441 p. $ 36.50; £17.50 (1974).

Classification :
∗00Bxx Conference proceedings and collections of papers
30-06 Proceedings of conferences (functions of a complex variable)

Zbl 0272.35029

Nirenberg, Louis; Walker, Homer F.
The null spaces of elliptic partial differential operators in $\mathbf{R}^n$. (English)
J. Math. Anal. Appl. 42, 271-301 (1973). ISSN 0022-247X
http://dx.doi.org/10.1016/0022-247X(73)90138-8
http://www.sciencedirect.com/science/journal/0022247X
http://www.sciencedirect.com/science/journal/0022247X

Classification :
∗35J30 Higher order elliptic equations, general
47F05 Partial differential operators

Zbl 0267.35001

Nirenberg, Louis
Lectures on linear partial differential equations. (English)
Conference Board of the Mathematical Sciences. Regional Conference Series in Mathematics. No.17. Providence, R.I.: American Mathematical Society (AMS). V, 58 p. $ 4.00 (1973).

Classification :
∗35-02 Research monographs (partial differential equations)
35S05 General theory of pseudodifferential operators

Zbl 0248.32013

Kohn, J.J.; Nirenberg, Louis
A pseudo-convex domain not admitting a holomorphic support function. (English)

Math. Ann. 201, 265-268 (1973). ISSN 0025-5831; ISSN 1432-1807
http://dx.doi.org/10.1007/BF01428194
http://link.springer.de/link/service/journals/00208/

Classification :
 ∗32T99 Pseudoconvex domains
 32A10 Holomorphic functions (several variables)
 32B15 Analytic subsets of affine space

Zbl 0264.49013

Brézis, Haïm; Nirenberg, Louis; Stampacchia, Guido
A remark on Ky Fan's minimax principle. (English)
Boll. Unione Mat. Ital., IV. Ser. 6, 293-300 (1972).

Classification :
 ∗49K35 Minimax problems (necessity and sufficiency)
 49J35 Minimax problems (existence)
 49J45 Optimal control problems inv. semicontinuity and convergence

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Zbl 0257.35001

Nirenberg, Louis
An abstract form of the nonlinear Cauchy-Kowalewski theorem. (English)
J. Differ. Geom. 6, 561-576 (1972). ISSN 0022-040X
http://projecteuclid.org/jdg
http://www.intlpress.com/journals/JDG/

Classification :
 ∗35A10 Cauchy-Kowalewski theorems
 35G25 Initial value problems for nonlinear higher-order PDE

Zbl 0236.35020

Nirenberg, Louis; Trèves, François (Trev, F.)
A correction to: On local solvability of linear partial differential equations. II: Sufficient conditions. (Russian)
Matematika, Moskva 16, No.4, 149-152 (1972).

Classification :
 ∗35S05 General theory of pseudodifferential operators
 35G99 General higher order PDE

Zbl 0317.35036

Nirenberg, Louis
An application of generalized degree to a class of nonlinear problems. (English)
3ieme Coll. sur l'Analyse fonction., Liège 1970, 57-74 (1971).

Classification :
∗35J55 Systems of elliptic equations, boundary value problems
47J05 Equations involving nonlinear operators (general)

Zbl 0267.47034

Nirenberg, Louis
Generalized degree and nonlinear problems. (English)
Contrib. nonlin. functional Analysis, Proc. Sympos. Univ. Wisconsin, Madison 1971, 1-9 (1971).

Classification :
∗47J05 Equations involving nonlinear operators (general)
54H25 Fixed-point theorems in topological spaces
35A05 General existence and uniqueness theorems (PDE)

Zbl 0232.47019

Nirenberg, Louis; **Treves, J.F.**
Remarks on the solvability of linear equations of evolution. (English)
Sympos. math., Roma 7, Probl. Evoluz. 1970, Equ. ipoellitt. Spazi funzion. 1971, 325-338 (1971).

Classification :
∗47A50 Equations and inequalities involving linear operators
42A38 Fourier type transforms, one variable
47A05 General theory of linear operators
34A05 Methods of solution of ODE

Zbl 0221.35019

Nirenberg, Louis; **Treves, J.F.**
A correction to: On local solvability of linear partial differential equations. II: Sufficient conditions. (English)
Commun. Pure Appl. Math. 24, 279-288 (1971). ISSN 0010-3640
http://dx.doi.org/10.1002/cpa.3160240209
http://onlinelibrary.wiley.com/journal/10.1002/%28ISSN%291097-0312

Classification :
∗35A07 Local existence and uniqueness theorems (PDE)

Zbl 0221.35001

Nirenberg, Louis; Trèves, François (Nirenberg, Luis; Trev, Fransua)
On local solvability of linear partial differential equations. I: Necessary conditions. (Russian)
Matematika, Moskva, 15, No.3, 142-172 (1971).

Classification :
∗35A07 Local existence and uniqueness theorems (PDE)

Zbl 0213.11501

Nirenberg, Louis; Trèves, François
On local solvability of linear partial differential equations. Part II: Sufficient conditions. (Russian)
Matematika, Moskva 15, No.4, 68-110 (1971).

Classification :
∗35A07 Local existence and uniqueness theorems (PDE)

Zbl 0212.10702

Nirenberg, Louis
A proof of the Malgrange preparation theorem. (English)
Proc. Liverpool Singularities-Symp. I, Dept. Pure Math. Univ. Liverpool 1969-1970, 97-105 (1971).

Classification :
∗32B05 Analytic algebras and generalizations

Zbl 0218.35075

Nirenberg, Louis
Pseudo-differential operators. (English)
Global Analysis, Proc. Sympos. Pure Math. 16, 149-167 (1970).

Classification :
∗35S05 General theory of pseudodifferential operators

Zbl 0208.35902

Nirenberg, Louis; Trèves, François
On local solvability of linear partial differential equations. Part II: Sufficient conditions. (English)
Commun. Pure Appl. Math. 23, 459-509 (1970). ISSN 0010-3640
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