
Zbl 1184.28007

Beliaev, D.; Järvenpää, E.; Järvenpää, M.; Käenmäki, A.; Rajala, T.; Smirnov, S.; Suomala, V.

Packing dimension of mean porous measures. (English)

J. Lond. Math. Soc., II. Ser. 80, No. 2, 514-530 (2009). ISSN 0024-6107; ISSN 1469-7750
http://dx.doi.org/10.1112/jlms/jdp040
http://jlms.oxfordjournals.org/

The corrected proof of the packing dimension [for earlier work, cf. *D. B. Beliaev, S. K. Smirnov*, Math. Ann. 323, No. 1, 123–141 (2002; Zbl 1037.28003); *E. Järvenpää, M. Järvenpää*, Proc. Am. Math. Soc. 130, No. 2, 419–426 (2002; Zbl 0994.28002)] estimate for mean porous measures is presented: Let $0 \leq \alpha \leq 1/2$ and $0 < p \leq 1$. Then there exists a constant C depending only on dimension d such that for all mean (α, p)-porous random measures on $\mathbb{R}^d$ the following inequality holds:

$$\dim_p \mu \leq d - p - \frac{C}{\log(1-2\alpha)}.$$

A construction showing that the upper bound ia asymptotically the best possible one as α tends to $1/2$ is also given. Moreover, it is clarified that mean porous measures cannot be estimated by mean porous sets.
Reviewer's remark: The notion of porosity comes from investigations of convergence sets of series of analytic functions and is obviously much older than the paper and its references seems to suggest.

Piero Villaggio (Pisa)

Keywords : mean porous Radon measure; mean porosity; mean porous set; packing dimension
Classification :
∗28A75 Geometric measure theory
28A80 Fractals

Zbl 1179.60054

Beliaev, D.; Smirnov, S.

Harmonic measure and SLE. (English)

Commun. Math. Phys. 290, No. 2, 577-595 (2009). ISSN 0010-3616; ISSN 1432-0916
http://dx.doi.org/10.1007/s00220-009-0864-7
http://link.springer.de/link/service/journals/00220/
http://projecteuclid.org/DPubS?service=UIversion=1.0verb=Displaypage=pasthandle=euclid.cm

Summary: We study the multifractal structure of Schramm's SLE curves. We derive the values of the (average) spectrum of harmonic measure and prove Duplantier's prediction

for the multifractal spectrum of SLE curves. The spectrum can also be used to derive estimates of the dimension, Hölder exponent and other geometrical quantities. The SLE curves provide perhaps the only example of sets where the spectrum is non-trivial yet exactly computable.

Classification :
∗60J67
82B31 Stochastic methods in equilibrium statistical mechanics
28A80 Fractals

Zbl 1163.37008

Graczyk, Jacek; **Smirnov, Stanislav**

Non-uniform hyperbolicity in complex dynamics. (English)

Invent. Math. 175, No. 2, 335-415 (2009). ISSN 0020-9910; ISSN 1432-1297
http://dx.doi.org/10.1007/s00222-008-0152-8
http://link.springer.de/link/service/journals/00222/

The Poincaré series is a basic tool of Kleinian groups, which is used to construct and study conformal densities and dimensions of the limit set. The basic attention of this article are estimates of the Poincaré series in rational dynamics, i.e. the problem of regularity of conformal measures or to study rational maps satisfying the summability condition, which requires only a polynomial growth of the derivative along critical orbits. Rational maps with parabolic periodic points are non-generic and therefore are excluded from considerations.
In the class of rational maps satisfying the summability condition it is proved the counterpart of Sullivan's result [*D. Sullivan*, Conformal dynamical systems. Geometric dynamics, Proc. int. Symp., Rio de Janeiro/Brasil 1981, Lect. Notes Math. 1007, 725–752 (1983; Zbl 0524.58024)], that conformal measures with minimal exponent are ergodic (hence unique) and non-atomic. For study properties of the Poincaré series for rational maps the notion of a restricted Poincaré series is introduced which is also well-defined for points in the Julia set and leads to new estimates, particularly implying that the convergence property of the Poincaré series is "self-improving". This turns out to be an underlying reasons for regularity properties of conformal measures on Julia sets, together with the divergence of the Poincaré series with Poincaré exponent, infimum of exponents with converging Poincaré series, as the consequence.
One of the central problems in the theory of iteration of rational functions is to estimate the Hausdorff dimension of Julia sets which are not the whole sphere and investigate their fractal properties. It is proved here that the Poincaré exponent coincides with the Hausdorff dimension of Julia sets J and $H\,\mathrm{Dim}(J) > 2$ unless $J = \widehat{\mathbb{C}}$ for rational functions satisfying the summability condition with an exponent $\alpha < \frac{2}{\mu_{\max}+2}$. To study the continuity of Hausdorff dimension of Julia sets the concept of the uniform summability is introduced.
Also a conformal analogue of Jakobson's (Benedicks-Carleson's) theorem is derived and the external continuity of the Hausdorff dimension of Julia sets for almost all points c from the Mandelbrot set with respect to the harmonic measure is proved.

Irina V. Konopleva (Ul'yanovsk)

Keywords : Poincaré series for rational maps; critical points; Julia set; Hausdorff dimension

Classification :

∗37C45 Dimension theory of dynamical systems

37F50 Small divisors, rotation domains and linearization

37F45 Holomorphic families of dynamical systems, etc.

37F35 Conformal densities and Hausdorff dimension

Zbl 1112.82014

Smirnov, Stanislav

Towards conformal invariance of 2D lattice models. (English)

Sanz-Solé, Marta (ed.) et al., Proceedings of the international congress of mathematicians (ICM), Madrid, Spain, August 22–30, 2006. Volume II: Invited lectures. Zürich: European Mathematical Society (EMS). 1421-1451 (2006). ISBN 978-3-03719-022-7/hbk

This paper deals with the mathematical treatment of an important conjecture in conformal field theory and other areas of theoretical physics, the conformal invariance conjecture, saying that 2D statistical mechanics lattice models at criticality have a continuum scaling limit that is conformally invariant and universal, in the sense that this limit is the same for the same model on different lattices. The author reviews known results and conjectures, describes briefly the characterization of the possible scaling limits in terms of the Schramm Lowener's evolution (SLE) of a single parameter κ, and thereafter focuses on the sketch of a new proof he gave in another paper for the existence and the conformal invariance of the scaling limit for a single interface in Ising and Ising random cluster models on the square lattice at critical temperature. It identifies the corresponding scaling limits as SLE(3) and SLE(16/3) respectively, and also uses the same method to get SLE(6) as the scaling limit of percolation. Other conjectures and new directions are also developped for two families of lattice models which have nice loop representations, so-called loop models on hexagonal lattices and Fortuyn-Kasteleyn random cluster models, for which martingale principles and conformal martingales can be used to identify the parameter κ of the SLE in the limit.

Arnaud Le Ny (Orsay)

Keywords : statistical physics; conformal invariance; universality; Ising model; percolation

Classification :

∗82B20 Lattice systems

60K35 Interacting random processes

82B43 Percolation

30C35 General theory of conformal mappings

81T40 Two-dimensional field theories, etc.

Zbl 1089.00008

Benedicks, Michael (ed.); Jones, Peter W. (ed.); Smirnov, Stanislav (ed.)
Perspectives in analysis. Essays in honor of Lennart Carleson's 75th birthday. Proceedings of the conference, Stockholm, Sweden, May 26–28, 2003. (English)
Mathematical Physics Studies 27. Berlin: Springer. xiv, 376 p. with DVD-ROM. EUR 69.95 (2005). ISBN 3-540-30432-0/hbk

The articles of this volume will be reviewed individually.

Classification :
 ∗00B25 Proceedings of conferences of miscellaneous specific interest
 00B30 Festschriften

Zbl 1084.30017

Beliaev, D.; Smirnov, S.
On Littlewood's constants. (English)
Bull. Lond. Math. Soc. 37, No. 5, 719-726 (2005). ISSN 0024-6093; ISSN 1469-2120
http://dx.doi.org/10.1112/S0024609305004522
http://blms.oxfordjournals.org/
http://www.journals.cambridge.org/journal$_B$*ulletinoftheLondonMathematicalSociety*

Let Ω be a simply connected domain in the extended complex plane. Suppose that $\infty \in \Omega$ and denote by G the Green function of Ω with pole at ∞. Littlewood introduced the constant
$$\beta = \sup_{\Omega} \limsup_{r\to 0} \frac{\log length\{z : G(z) = r\}}{\log(1/r)}.$$
It was proved by *L. Carleson* and *P. W. Jones* [Duke Math. J. 66, 169-206 (1992; Zbl 0765.30005)] that β is equal to the constant
$$\gamma = \sup_{\phi} \limsup_{n\to\infty} \frac{\log|b_n|}{\log n} + 1,$$
where the supremum is taken over all functions
$$\phi(z) = z + \sum_{n=1}^{\infty} b_n z^{-n}$$
that are univalent outside the unit disk. Another result of *N. G. Makarov* [St. Petersburg Math. J. 10, 217–268 (1999; Zbl 0909.30016)] relates the constant β with the fine properties of harmonic measure and in fact shows that $\beta = \pi(1)$, where $\pi(t)$ is the so called universal packing spectrum of planar domains. Littlewood studied another constant in a different context. Motivated by problems in the value distribution theory of entire functions, he set
$$A_n = \sup \int_{\{|z|<1\}} \frac{|p'|}{1+|p|^2}\, dxdy,$$

where the supremum is taken over all polynomials p of degree n. Let α be the best possible constant in the inequality $A_n \leq Cn^\alpha$. In the present paper the authors prove that $\alpha \leq \pi(1)$. This result, combined with recent deep work of various authors (including Beliaev, Binder, Eremenko, Jones, Makarov, Smirnov), implies that amazingly

$$\alpha = \beta = \gamma.$$

It has been conjectured by Carleson and Jones that the value of this constant is 1/4. The best known bounds are

$$0.23 < \beta < 0.46.$$

The upper bound is due to *H. Hedenmalm* and *S. Shimorin* [Duke Math. J. 127, 341–393 (2005; Zbl 1075.30005)] and the lower bound is due to *D. Beliaev* and *S. Smirnov* [Proceedings of the 4th European congress of mathematics (ECM), Stockholm, Sweden, June 27-July 2, 2004. Zurich: European Mathematical Society (EMS). 41-59 (2005; Zbl 1079.30026)].

Dimitrios Betsakos (Thessaloniki)

Keywords : univalent function; harmonic measure; Green function; packing spectrum; universal spectrum; polynomial

Classification :

∗30C55 General theory of univalent and multivalent functions

30C10 Polynomials (one complex variable)

30C85 Capacity and harmonic measure in the complex plane

30D35 Distribution of values (one complex variable)

Zbl 1079.30026

Beliaev, D.; Smirnov, S.

Harmonic measure on fractal sets. (English)

Laptev, Ari (ed.), Proceedings of the 4th European congress of mathematics (ECM), Stockholm, Sweden, June 27–July 2, 2004. Zürich: European Mathematical Society (EMS). 41-59 (2005). ISBN 3-03719-009-4/hbk

The authors investigated some estimations for the supremum of multifractal spectra of harmonic measure for all planar domains. They started off with a brief introduction for classes of univalent functions and describe briefly the coefficient problems involved. Later, the authors discuss in more detail the estimates of the supremum of multifractal spectra.

Maslina Darus (Selangor)

Keywords : harmonic measure; fractal sets; univalent functions; planar domains

Classification :

∗30C85 Capacity and harmonic measure in the complex plane

28A80 Fractals

31A15 Potentials, etc. (two-dimensional)

Zbl pre05130611

Smirnov, Stanislav

Critical percolation and conformal invariance. (English)

Zambrini, Jean-Claude (ed.), XIVth international congress on mathematical physics (ICMP 2003), Lisbon, Portugal, 28 July – 2 August 2003. Selected papers based on the presentation at the conference. Hackensack, NJ: World Scientific. 99-112 (2005). ISBN 981-256-201-X/hbk

Classification :

∗82B43 Percolation
60K35 Interacting random processes
82-02 Research monographs (statistical mechanics)

Zbl 1058.37032

Przytycki, Feliks; Rivera-Letelier, Juan; Smirnov, Stanislav

Equality of pressures for rational functions. (English)

Ergodic Theory Dyn. Syst. 24, No. 3, 891-914 (2004). ISSN 0143-3857; ISSN 1469-4417
http://dx.doi.org/10.1017/S0143385703000385
http://journals.cambridge.org/action/displayJournal?jid=ETSbVolume=y

From authors' abstract: We prove that for all rational functions f on the Riemann sphere and the potential $-t\ln|f'|$, $t \geq 0$, all notions of pressure introduced in [*F. Przytycki*, Trans. Am. Math. Soc. 351, 2081–2099 (1999; Zbl 0920.58037)] coincide. In particular, we get a new simple proof of the equality between the hyperbolic Hausdorff dimension and the minimal exponent of conformal measure on a Julia set. We prove that these pressures are equal to the pressure defined with the use of periodic orbits under an assumption that there are not many periodic orbits with Lyapunov exponent close to 1 moving close together, in particular under the topological Collet-Eckmann condition. In Appendix A, we discuss the case $t < 0$.

Alois Klíč (Praha)

Keywords : tree pressure; hyperbolic pressure; hyperbolic variational pressure; variational pressure; conformal pressure

Classification :

∗37F10 Polynomials; rational maps; entire and meromorphic functions
37D35 Thermodynamic formalism, variational principles, equilibrium states

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Zbl 1050.37014

Makarov, N.; Smirnov, S.

On thermodynamics of rational maps. II: Non-recurrent maps. (English)

J. Lond. Math. Soc., II. Ser. 67, No. 2, 417-432 (2003). ISSN 0024-6107; ISSN 1469-7750
http://dx.doi.org/10.1112/S0024610702003964
http://jlms.oxfordjournals.org/

This paper studies analyticity properties of the pressure function of nonrecurrent maps. The approach is based on well-known techniques adapted to the complex dynamics situation. The authors show that it is not always possible to get analyticity on the whole positive axis.
For Part I, see the authors [Commun. Math. Phys. 211, 705–743 (2000; Zbl 0983.37033)].

Messoud A. Efendiev (Berlin)
Keywords : analyticity; complex dynamics; tower technique; pressure function
Classification :
∗37D35 Thermodynamic formalism, variational principles, equilibrium states
37F10 Polynomials; rational maps; entire and meromorphic functions
30C85 Capacity and harmonic measure in the complex plane
37E05 Maps of the interval

Zbl 1038.37035

Przytycki, Feliks; **Rivera-Letelier, Juan**; **Smirnov, Stanislav**
Equivalence and topological invariance of conditions for non-uniform hyperbolicity in the iteration of rational maps. (English)
Invent. Math. 151, No. 1, 29-63 (2003). ISSN 0020-9910; ISSN 1432-1297
http://dx.doi.org/10.1007/s00222-002-0243-x
http://link.springer.de/link/service/journals/00222/

Summary: We show the equivalence of several standard conditions for nonuniform hyperbolicity of complex rational functions, including the topological Collet-Eckmann condition (TCE), uniform hyperbolicity on periodic orbits, exponential shrinking of components of pre-images of small discs, backward Collet-Eckmann condition at one point, positivity of the infimum of Lyapunov exponents of finite invariant measures on the Julia set. The condition TCE is stated in purely topological terms, so we conclude that all these conditions are invariant under topological conjugacy.
For rational maps with one critical point in the Julia set, all the conditions above are equivalent to the usual Collet-Eckmann and backward Collet-Eckmann conditions. Thus, the latter ones are invariant by topological conjugacy in the unicritical setting. We also prove that neither part of this stronger statement is valid in the multicritical case.

Classification :
∗37F10 Polynomials; rational maps; entire and meromorphic functions
37F15 Expanding maps; hyperbolicity; structural stability

Zbl 1036.30017

Binder, I.; **Makarov, N.**; **Smirnov, S.**
Harmonic measure and polynomial Julia sets. (English)
Duke Math. J. 117, No. 2, 343-365 (2003). ISSN 0012-7094

http://dx.doi.org/10.1215/S0012-7094-03-11725-1
http://www.dukemathjournal.org
http://projecteuclid.org/handle/euclid.dmj

Let Ω be a domain in the Riemann sphere $\widehat{\mathbb{C}}$, let $E \subset \widehat{\mathbb{C}}$, and let $a \in \Omega$. Let $\omega(a, E, \Omega)$ denote the harmonic measure in Ω of the set E evaluated at a. An important theorem of *P. W. Jones* and *T. H. Wolff* [Acta Math. 161, 131–144 (1988; Zbl 0667.30020)] asserts that the Hausdorff dimension of the support of ω is at most 1. In the paper under review the authors study stronger estimates involving the dimension of harmonic measure. Let

$$\alpha_{a,\Omega}(z) = \liminf_{\delta \to 0} \frac{\log \omega(a, \{w : |w - z| \leq \delta\}, \Omega)}{\log \delta}.$$

The universal dimension spectrum of harmonic measure is the function

$$\Phi(\alpha) = \sup \dim\{z : \alpha_{a,\Omega}(z) \leq \alpha\},$$

where dim denotes Hausdorff dimension and the supremum is taken over all planar domains Ω and all evaluation points $a \in \Omega$. Let $\Phi_{\text{sc}}(\alpha)$ be the corresponding quantity with the supremum taken over all simply connected domains. The authors conjecture that for $\alpha \geq 1$, $\Phi(\alpha) = \Phi_{\text{sc}}(\alpha)$. They propose the application of methods of Complex Dynamics for the study of the conjecture. The main result of the paper is that the conjecture is true when $\partial\Omega$ is the Julia set of a polynomial such that the orbits of all critical points escape to ∞. The proof of this deep theorem uses earlier results of *N. G. Makarov* [Algebra Anal. 10, 1–62 (1998); translation in St. Petersbg. Math. J. 10, 217–268 (1999; Zbl 0909.30016)] (involving the pressure function of a polynomial and the universal integral means spectrum), and a construction of *B. Branner* and *J. H. Hubbard* [Acta Math. 160, 143–206 (1988; Zbl 0668.30008)].
Recently I. Binder and P. Jones proved the conjecture mentioned above.

Dimitrios Betsakos (Thessaloniki)

Keywords : harmonic measure; Hausdorff dimension; Julia set; conformal Cantor set
Classification :
∗30C85 Capacity and harmonic measure in the complex plane
37F10 Polynomials; rational maps; entire and meromorphic functions
37F35 Conformal densities and Hausdorff dimension

Zbl 1037.28003

Beliaev, D. B.; Smirnov, S. K.

On dimension of porous measures. (English)

Math. Ann. 323, No. 1, 123–141 (2002). ISSN 0025-5831; ISSN 1432-1807
http://dx.doi.org/10.1007/s002080100299
http://link.springer.de/link/service/journals/00208/

Porosity of a set is a quantity that describes the relative size of holes in the set at all small scales. It is well known that if the porosity of a set $A \subset \mathbb{R}^n$ is close to its

maximum value, then the dimension of A cannot be much bigger that $n-1$. This was first proved for Hausdorff dimension by *P. Mattila* [J. Lond. Math. Soc., II. Ser. 38, No. 1, 125–132 (1988; Zbl 0618.28005)] and later generalized for packing dimension with the correct asymptotics in terms of large porosity by *A. Salli* [Proc. Lond. Math. Soc., III. Ser. 62, No. 2, 353–372 (1991; Zbl 0716.28006)]. These results were extended to mean porous sets having holes of certain size at a certain percentage of small scales by *P. Koskela* and *S. Rohde* [Math. Ann. 309, 593–609 (1997; Zbl 0890.30013)]. They found the correct asymptotics for the dimension estimate of such sets when the holes are small.
In the paper under review the authors study corresponding questions for measures. They extend the earlier results by *J.-P. Eckmann*, *E. Järvenpää* and *M. Järvenpää* [Nonlinearity 13, 1–18 (2000; Zbl 1022.28002)] concerning dimensional behaviour of porous measures satisfying the doubling condition by considering general measures and mean porosity. The results of the paper may be regarded as analogues of the results of Salli and Koskela-Rohde for measures. The proofs combine new ideas to the methods of Salli, Koskela-Rohde and Eckmann-Järvenpää-Järvenpää in an interesting way. An important tool of independent interest is to verify that for a mean porous measure there exists a mean porous set of positive measure with asymptotically the same percentage of porous scales as the measure itself.

Maarit Järvenpää (Jyväskylä)

Keywords : mean porous measure; porous measure; dimension
Classification :
∗28A75 Geometric measure theory
28A78 Hausdorff measures

Zbl 1116.37309

Smirnov, Stanislav K.

On supports of dynamical laminations and biaccessible points in polynomial Julia sets. (English)

Colloq. Math. 87, No. 2, 287-295 (2001). ISSN 0010-1354; ISSN 1730-6302
http://dx.doi.org/10.4064/cm87-2-11
http://journals.impan.gov.pl/cm/
http://matwbn.icm.edu.pl/spis.php?wyd=8jez=pl

Summary: We use Beurling estimates and Zdunik's theorem to prove that the support of a lamination of the circle corresponding to a connected polynomial Julia set has zero length, unless f is conjugate to a Chebyshev polynomial. Equivalently, except for the Chebyshev case, the biaccessible points in the connected polynomial Julia set have zero harmonic measure.

Keywords : lamination; external ray; Julia set; harmonic measure
Classification :
∗37F20 Combinatorics and topology
30C85 Capacity and harmonic measure in the complex plane
30D05 Functional equations in the complex domain

Zbl 1009.60087

Smirnov, Stanislav; Werner, Wendelin

Critical exponents for two-dimensional percolation. (English)

Math. Res. Lett. 8, No.5-6, 729-744 (2001). ISSN 1073-2780
http://www.mrlonline.org/mrl/2001-008-005/index.html
http://www.mathjournals.org/mrl/

The paper deals with problems of the existence and values of critical exponents for site percolation on the triangular lattice. Each vertex of a triangular lattice is open with probability p (and closed with probability $1-p$). Then $p=\frac{1}{2}$ is the critical value. The authors give, besides other results, asymptotic formulas (for $p\to\frac{1}{2}$) for the probability $\theta(p)$ that the origin belongs to an infinite cluster of open vertices, of the average cardinality $\chi(p)$ of finite clusters, and of the correlation length $\xi(p)$ corresponding to the typical radius of a finite cluster. In particular, it holds

$$\theta(p)=(p-1/2)^{5/36+o(1)} \quad \text{for } p\to 1/2,$$
$$\chi(p)=(p-1/2)^{-43/18+o(1)} \quad \text{for } p\to 1/2,$$
$$\xi(p)=(p-1/2)^{-4/3+o(1)} \quad \text{for } p\to 1/2.$$

The survey paper contains also many other results concerning the subject.

Martin Janžura (Praha)

Keywords : site percolation; triangular lattice; critical exponent

Classification :

∗60K35 Interacting random processes
82B26 Phase transitions (general)
82B43 Percolation

Zbl 0985.60090

Smirnov, Stanislav

Critical percolation in the plane: Conformal invariance, Cardy's formula, scaling limits. (English. Abridged French version)

C. R. Acad. Sci., Paris, Sér. I, Math. 333, No. 3, 239-244 (2001). ISSN 0764-4442
http://dx.doi.org/10.1016/S0764-4442(01)01991-7
http://www.sciencedirect.com/science/journal/07644442

Summary: We study critical site percolation on triangular lattice. We introduce harmonic conformal invariants as scaling limits of certain probabilities and calculate their values. As a corollary we obtain conformal invariance of the crossing probabilities [conjecture attributed to Aizenman by *R. Langlands*, *Ph. Pouliot* and *Y. Saint-Aubin*, Bull. Am. Math. Soc., New Ser. 30, No. 1, 1-61 (1994; Zbl 0794.60109)] and find their values [predicted by *J. Cardy*, J. Phys. A, Math. Gen. 25, L201–L206 (1992), we discuss simpler representation found by Carleson]. Then we discuss existence, uniqueness, and

conformal invariance of the continuum scaling limit. The detailed proofs appear in [the author, "Critical percolation in the plane" (http://www.math.kth.se/ stas/papers].

Keywords : critical site percolation; harmonic conformal invariants; crossing probabilities; continuum scaling limit

Classification :

∗60K35 Interacting random processes
82B27 Critical phenomena
82B43 Percolation

Zbl 1034.30014

Jones, Peter W.; Smirnov, Stanislav K.

Removability theorems for Sobolev functions and quasiconformal maps. (English)

Ark. Mat. 38, No. 2, 263-279 (2000). ISSN 0004-2080; ISSN 1871-2487
http://dx.doi.org/10.1007/BF02384320
http://www.arkivformatematik.org/
http://www.springerlink.com/content/0004-2080

Several sufficient conditions for a set to be (quasi)conformally removable and Sobolev removable are established. The (quasi)conformally removable and the Sobolev removable play very important roles in the holomorphic dynamics. Therefore the authors have given the applications of these two ideas in the holomorphic dynamics. We notice that the authors are mostly interested in the planar case, where all their theorems are considered.

Metin Bolcal (Istanbul)

Keywords : quasiconformal map; Sobolev space; removable set

Classification :

∗30C62 Quasiconformal mappings in the plane
30C35 General theory of conformal mappings
30C65 Quasiconformal mappings in R^n and other generalizations
37F15 Expanding maps; hyperbolicity; structural stability
46E35 Sobolev spaces and generalizations

Zbl 0983.37033

Makarov, N.; Smirnov, S.

On "thermodynamics" of rational maps. I: Negative spectrum. (English)

Commun. Math. Phys. 211, No.3, 705-743 (2000). ISSN 0010-3616; ISSN 1432-0916
http://dx.doi.org/10.1007/s002200050833
http://link.springer.de/link/service/journals/00220/
http://projecteuclid.org/DPubS?service=UIversion=1.0verb=Displaypage=pasthandle=euclid.cm

This paper is devoted to study the pressure spectrum $P(t)$ of the maximal measure for rational maps, and also some other related parametrics. The authors consider also a modified version $\widetilde{P}(t)$ which is defined by means of the variational principle with respect to non-atomic invariant measures and compute the spectrum of $P(t)$ in terms of $\widetilde{P}(t)$. The authors analyze important properties of $\widetilde{P}(t)$ for negative values of t.

Messoud Efendiev (Berlin)
Keywords : rational maps; maximal measure; invariant measure; spectrum
Classification :
∗37D35 Thermodynamic formalism, variational principles, equilibrium states
37C40 Smooth ergodic theory, invariant measures
37E05 Maps of the interval
37F10 Polynomials; rational maps; entire and meromorphic functions

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Zbl 0983.37052

Smirnov, Stanislav
Symbolic dynamics and Collet-Eckmann conditions. (English)
Int. Math. Res. Not. 2000, No.7, 333-351 (2000). ISSN 1073-7928; ISSN 1687-0247
http://dx.doi.org/10.1155/S1073792800000192
http://imrn.oxfordjournals.org/

The paper is devoted to the study of one dimensional complex dynamics generated by iterations of polynomial maps $P(z) := z^d + C$, $d \in \mathbb{N}$, $C \in \mathbb{C}$. It is proved that unicritical polynomials $P(z)$ with metrically generic combinatorics of the critical orbit satisfy Collet-Eckmann conditions. Here metrically generic means except of a set of Hausdorff dimension zero and combinatorics can be treated in the sense of Markov partition itineraries, kneading sequences, external angles or in the sense of a harmonic measure on the Mandelbrot set.
In particular, it is shown that except for a Hausdorff dimension zero set of angles all external rays for the degree d Mandelbrot set land at parameters C such that the corresponding polynomials are Collet-Eckmann.
Some generalizations of the obtained results to polynomials with many critical points and rational functions are also indicated.

Serguei Zelik (Moskva)
Keywords : one dimensional complex dynamics; Mandelbrot set; Collet-Eckmann conditions; kneading sequences; external angles
Classification :
∗37F10 Polynomials; rational maps; entire and meromorphic functions
37F45 Holomorphic families of dynamical systems, etc.
37B10 Symbolic dynamics

Zbl 0921.30004

Jones, Peter W.; Smirnov, Stanilav K.
On V. I Smirnov domains. (English)

Ann. Acad. Sci. Fenn., Math. 24, No.1, 105-108 (1999). ISSN 1239-629X
http://www.acadsci.fi/mathematica/

Let Γ be a closed rectifiable Jordan curve, Ω_+ the bounded component complementary to Γ, and F_+ any conformal map of $\{z : |z| < 1\}$ onto Ω_+; similarly, Ω_- denotes the unbounded component, F_- any conformal map of $\{z : |z| > 1\}$ onto Ω_-. Ω_+ (resp., Ω_-) is said to be a V. I. Smirnov domain if F'_+ (resp., F'_-) is an outer function. An old problem asked: If Ω_+ is a V. I Smirnov domain, must Ω_- also be a V. I. Smirnov domain? This note provides a negative answer to the question. Since the 1960's, examples have been known of Γ and Ω_+, with F_+ a singular function: (1) $F_+ = \exp\{-(\mu + i\widetilde{\mu})\}$, where μ is a positive measure on the unit circle, and μ is singular with respect to Lebesgue measure. Then Ω_+ is not a V. I. Smirnov domain. Theorem. If F_+ satisfies (1), then F_- satisfies (2) $|F'_-(z)| \geq c > 0$, $|z| > 1$. Corollary. Ω_- is a V. I. Smirnov domain. The proof of the theorem involves use of the harmonic measure ω_+ for Ω_+ (resp., ω_- for Ω_-), the evaluation of $d\omega_+$, and an inequality on the product (ω_+) (ω_-) obtained in *C. J. Bishop, L. Carleson, J. B. Garnett*, and *P. W. Jones* [Pac. J. Math. 138, 233-236 (1989; Zbl 0677.30017)].

Francis W.Carroll (Columbus / Ohio)

Keywords : Smirnov domain; harmonic measure
Classification :
∗30C20 Conformal mappings of special domains
30C45 Special classes of univalent and multivalent functions
31A15 Potentials, etc. (two-dimensional)
30C85 Capacity and harmonic measure in the complex plane

Zbl 0928.26010

Khavin, V.P.; Smirnov, S.K.

Approximation and extension problems for some classes of vector fields. (English. Russian original)

St. Petersbg. Math. J. 10, No.3, 507-528 (1999); translation from Algebra Anal. 10, No.3, 133-162 (1998). ISSN 1061-0022
http://www.ams.org/jourcgi/jrnl$_t$*oolbar*$_n$*av*/*spmj*$_a$*ll*

The authors establish interesting results concerning the approximation and the extension of continuous vector fields v on compact subsets $K \subset \mathbb{R}^N$ taking into account the geometric properties of v and K.

D.Motreanu (Iaşi)

Keywords : smooth maps; geometric measure theory; approximation; extension; continuous vector fields
Classification :
∗26B99 Functions of several real variables
46E35 Sobolev spaces and generalizations

49Q15 Geometric measure and integration theory, etc.
41A65 Abstract approximation theory

Zbl 0916.30023

Graczyk, Jacek; Smirnov, Stas

Collet, Eckmann and Hölder. (English)

Invent. Math. 133, No.1, 69-96 (1998). ISSN 0020-9910; ISSN 1432-1297
http://dx.doi.org/10.1007/s002220050239
http://link.springer.de/link/service/journals/00222/

[*L. Carleson*, *P. Jones* and *J.-C. Yoccoz*, Bol. Soc. Bras. Mat. Nova Sér. 25, 1-30 (1994; Zbl 0804.30023)] determined the dynamical property (semihyperbolicity) necessary and sufficient for the Fatou components of a polynomial to be John domains. The paper under review examines the more general question of the conditions for Fatou components of a rational map to be Hölder domains. A rational function F is said to be C-E (Collet-Eckmann) with constants $C_1 > 0$, $\lambda_1 > 1$ if for any critical point c whose forward orbit does not contain any other critical point and belongs to or accumulates on the Julia set we have $|(F^n)'(Fc)| > C_1\lambda_1^n$. One also says that a rational function F satisfies the second Collet-Eckmann condition C-E $2(z)$ at z with constants $C_2 > 0$, $\lambda_2 > 1$ if for any preimage $y \in F^{-n}z$ we have $|(F^n)'(y)| > C_2\lambda_2^n$. A periodic Fatou component G of F is called C-E if for any (equivalently some) point $z \in G$ away from the critical orbits we have $|(F^n)'(y)| > C\lambda^n$ for any preimage $y \in F^{-n}z \cap G$ with constants $C > 0$, $\lambda > 1$. The multiplicity of a critical point c is the order of c as a zero of $F(z) - F(c)$. For simplicity we assume that no critical point belongs to another critical orbit. The main theorem states that rational C-E maps have neither Siegel discs, Herman rings, nor parabolic or Cremer points. The Fatou components of such maps are Hölder domains. Further (i) C-E implies CE $2(c)$ for the critical points c of maximal multiplicity μ_m whose backward orbits do not contain any other critical points. (ii) C-E implies CE $2(z)$ for all z away from the critical orbits. (iii) An attracting or superattracting Fatou component is Hölder if and only if it is C-E. In particular polynomial C-E sets are locally connected if they are connected and their Hausdorff dimension is strictly less than two.

I.N.Baker (London)

Keywords : Fatou components

Classification :
∗30D05 Functional equations in the complex domain

Zbl 0852.58067

Makarov, N.; Smirnov, S.

Phase transition in subhyperbolic Julia sets. (English)

Ergodic Theory Dyn. Syst. 16, No.1, 125-157 (1996). ISSN 0143-3857; ISSN 1469-4417
http://dx.doi.org/10.1017/S0143385700008749
http://journals.cambridge.org/action/displayJournal?jid=ETSbVolume=y

The authors study the pressure function for critically finite polynomials and analyze the case when this function fails to be real analytic.

N.Papaghiuc (Iaşi)

Keywords : subhyperbolic Julia sets; phase transition; critically finite polynomials
Classification :
∗37D99 Dynamical systems with hyperbolic behavior
37D45 Strange attractors, chaotic dynamics
37B99 Topological dynamics

Zbl 0832.49024

Smirnov, S.K.

Decomposition of solenoidal vector charges into elementary solenoids and the structure of normal one-dimensional currents. (English. Russian original)

St. Petersbg. Math. J. 5, No.4, 841-867 (1994); translation from Algebra Anal. 5, No.4, 206-238 (1993). ISSN 0234-0852
http://www.mathnet.ru/php/journal.phtml?jrnid=aaoption$_lang = eng$

The article in devoted to the study of the structure of vector charges whose divergence is a measure (of normal one-dimensional currents). It is shown that an arbitrary vector charge with null divergence can be decomposed into elementary solenoids – the simplest charges of this type are representable as "averaging of a circulation" along a sufficiently good embedding of $\mathbb{R}$ into $\mathbb{R}^n$. In the paper the technic of geometric measure theory is used, but its knowledge is not necessary for the understanding of the results.

Keywords : decomposition; vector charge; divergence; solenoids
Classification :
∗49Q15 Geometric measure and integration theory, etc.
49Q20 Variational problems in geometric measure-theoretic setting
58A25 Currents (global analysis)
28A75 Geometric measure theory